Number Palindromes

Topics: Addition, Base 10, Logic
Materials: Paper and pencil, Number Palindrome sheet
Common Core: 3.NBT.A.2, 4.NBT.B.4, MP1, MP3, MP7, MP8

Palindromes are numbers that read the same forward as backward, like 10,501. When you add a number (24, say) to its reverse (42) you often get a palindrome (66). Does this always work? This lesson is an exploration of that question and its consequences.

Why We Love Number Palindromes

This extraordinary lesson hammers home how base ten and multi-digit addition work with an irresistible mystery. A pattern slowly reveals itself as the class compiles data, and calculations grow slowly more difficult as students make their way forward. The lesson ends with a serious challenge, and a connection to an unsolved problem in mathematics.

The Launch

Teacher: A palindrome is a word that is the same forward as backward. For example, mom, dad, and bob are all palindromes. Does anyone else know a palindrome? [Students contribute ideas and ask questions. Possible palindromes include words like *racecar*, or phrases like *taco cat*.]

Teacher: Numbers can be palindromes too, if they are the same forward and backward. For example, 313; 7997; 11; and 6 are palindromes. Can anyone else think of a number palindrome? [Students contribute number palindromes.]

Teacher: Now here is an amazing thing. Let's take a number that isn't a palindrome, say 24 [writes 24]. How do I know it is not a palindrome? Because if I turn it around, I get 42 [writes 42]. So I have these two numbers, 24 and 42… what happens if I add them together? [Adds 24 + 42] I get 66. And 66 is a palindrome! Will this always work? Let's try another, say 17. The reverse of 17 is 71, and if I add 17 + 71, I get 88, a palindrome! I'm going to go out on a limb and conjecture that whenever I add a number to its reverse I get a palindrome. Can anyone prove me wrong? Take 2 minutes and try to find a counterexample. [Students work independently for two minutes]

Teacher: Did anyone disprove my conjecture? Or does it hold up? [Take student comments and counterexamples.] So my conjecture isn't true. For example, 19 + 91 = 110, and 110 isn't a palindrome. But what if… what's the reverse of 110? I guess it is 011, or just 11. If we add 110 + 11, we get 121. And that's a palindrome! It took two steps to get there, but there it is! Let's call those first examples like 24 and 17 1-step palindromes, since they reached a palindrome in one step. And we can call 19 a 2-step palindrome.

Teacher: At this point, I want to change my conjecture. Does anyone else have any thoughts or conjectures before I do? [take student comments, writing down conjectures as appropriate.] Well, here is my conjecture:

Conjecture. Every number becomes a palindrome after some number of steps.

Teacher: We have palindromes, 1-step palindromes, and 2-step palindromes so far. There may be other kinds. What we are going to do now is try to collect some data. Everyone will get a 100s-chart to record their data. We can record by color. For example, let's say I want to check the number 18. I would add 18 to its reverse, 81, and get 99. That's a palindrome, and that took 1 step. So I can color 18 blue. Then I can try another number. I'm curious if everything will be a palindrome, and how many steps it will take to get to palindromes. Maybe there is some kind of pattern or structure. Let's find out.

The Work

Students work independently or in pairs to test for palindromes and record their answers. The teacher can be the custodian of a "master" copy, so that students can come up, show their work, and color in a spot on the master copy accordingly. If students need to use different colors than specified on the sheet, that's fine, as long as they are consistent in their own work.

The Wrap

The teacher gathers the students together to look at the master copy and discuss what students found. Some observations students may have:

1. The calculation to do a number and to do its reverse is the same. For example, to check 18, you calculate 18 + 81. To check 81, you calculate 81 + 18. But those will give you the same answer! So you only need to check half the numbers on the 100s chart, and the other half will be a mirror image.

2. Some students may have noticed that, in fact, every number on the diagonals are colored the same color. This is more subtle, but you can bring this question to the fore by asking how the check of the numbers 18, 27, 36, 45, 54, etc. will be similar?

3. The surprise at the end of this lesson is that 89 and 98 seem not to resolve into a palindrome. Or do they? In fact, you can reassure students that they will become palindromes eventually, but it takes a long time. This is a problem that motivated students can take home to try on their own.

4. Students may have experimented with three-digit numbers. In particular, it's true that if all the digits in a number are 4 or less, the number is a palindrome or a 1-step palindrome. Some students may have noticed this, and be able to articulate why it is true.

A final note: there is, in fact, a number that no one can determine whether it resolves into a palindrome or not. That number is 196. An interesting point of discussion: how could you ever *know* if 196 doesn't resolve into a palindrome? What if it took a million steps? How could you know such a thing?

Tips for the Classroom

1. Avoid certain trouble numbers during your opening demonstration. These are 95 and 59 (3-step palindromes), 96 and 69 (4-step palindromes) 97 and 79 (6-step palindromes), and 98 and 89 (24-step palindromes!!). Don't tell the kids how many steps it takes to do 98 until the end of class. You can also leave it vague (i.e., "98 takes somewhere between 20 and 30 steps to become a palindrome.") Steer students away from 98 and 89 if they try to do them too early in the class.

2. Students may notice that certain rows look like they are all 1-step, and not check certain numbers sufficiently. Make sure they are actually doing the arithmetic. The number 19 is a good check, since 12 through 18 are 1-step palindromes, and 19 is a 2-step palindrome.

3. Encourage students who need more practice to stick with low numbers first. Students who want a challenge can start with larger numbers (i.e. 60s, or even 90s).

4. An interesting extension for a future class: what if you took the differences between each number and its reverse instead of the sum? Would every number still eventually reach a palindrome?

5. You might want to have students note what palindrome each number ends at. Is there a pattern in these numbers as well?

Name______________________

Number Palindromes

Color Palindromes Red.
Color 1-step Palindromes Blue.
Color 2-step Palindromes Green.
Color 3-step Palindromes Yellow.
Color 4-step Palindromes Purple.
Color 5-step Palindromes Black.
Color 6-step Palindromes Orange.
Is that everything?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100