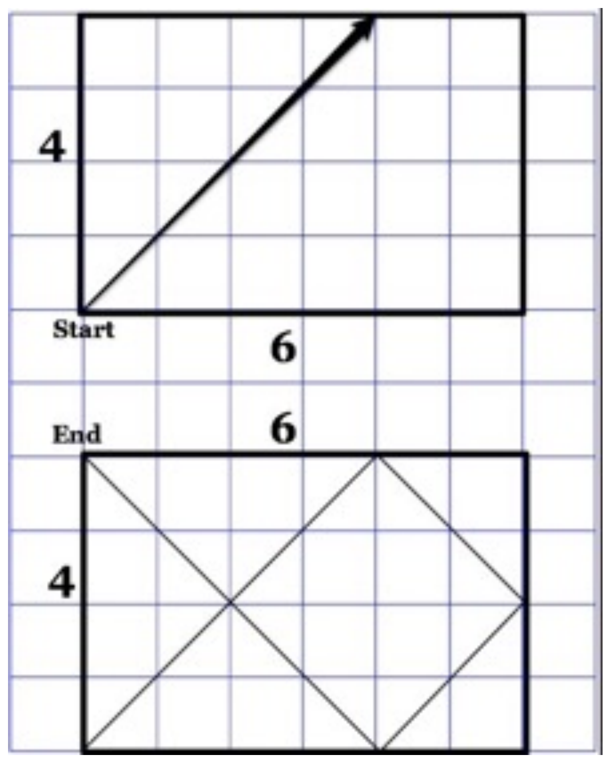
The Billiard Ball Problem

Math Concepts: Geometry, modeling, scaling, fractions, ratio and proportion
Materials: Graph paper, preferably with a larger sized grid, and writing utensils
Common Core: MP1, MP2, MP3, MP5, MP6, MP7

Which corner will the billiard ball end up in?

Why we love the Billiard Ball Problem

This is a beautiful puzzle, with a deceptively simple and satisfying answer. As soon as students can draw straight lines at 45 degrees, they have access to the puzzle, but cracking it is much trickier!



The Launch

Get two numbers between 3 and 8 from the class, and draw a table on graph paper (or a relatively accurate white board version of one); now you're ready to demonstrate how the billiard ball problem works.

There are four corners, one in each corner of the rectangle (none on the sides). A "ball," modeled by a straight line, is launched at 45 degrees from the bottom left corner, which means it cuts through every square of your graph paper exactly along the diagonal. You can ask the class: what happens when it hits the side of the table? The answer is that it bounces off at the same angle and continues along the diagonal of the square grids.

At this point, the question is almost immediate: will the ball end up in a "pocket," i.e., corner? If so, what corner will the billiard ball end up in? In the example on the right, it ends up in the top left corner. Do as many examples as you need to until the mechanism is clear to the class.

Big Questions

- How can you predict what corner the billiard ball will end up in?
- How many bounces does it take to get there?

The Work

Students catch on quick to how the ball bounces, and from there, they can work on their own. There are plenty of discoveries for them to make along the way.

First, the billiard tables scale. You can check that the path of the ball on a 4 by 6 table looks identical on the 2 by 3 table; both paths look the same as the 6 by 9 table. Only the proportion matters. Some kids will see this by looking at square tables.

Second, there's a great lesson about how to attack hard problems: if kids don't know where to start, you can steer them toward easy cases. And what are the easy cases? Make the vertical side 1, so you have a 1 by n table. We get some nice subquestions:

Can you predict the corner for a 1 by _____ table?

A 2 by _____ table?

A 3 by _____ table?

A 4 by _____ table?

Third, there's a question of how to collect the data so you can use it. I like to keep track using a table. A bigger one for students is included on the next page.

TL = top left corner; TR = top right; BR = bottom right; BL = bottom left

height/ length	1	2	3	4	5	6	7	8
1	TR	BR	TR	BR				
2	TL	TR						
3								
4						TL		
5								

What corner does the ball end in?

This allows the students to know what still needs to be done, have a way to look at all their results at once, and look for patterns.

You can use the handout at the end of this packet to let students keep track of their data.

The Wrap

There will be plenty of conjectures to discuss and disprove in the wrap up discussion of this lesson. Finding the full answer may take multiple days.

There are many ways to solve this problem—so many, in fact, that we won't try to summarize them. The main idea is that tables with the same ratios have the same paths, and beyond that, everything comes down to the evenness and oddness of the dimensions of the table. However, don't give this away to students! Let them convince themselves what's true, and keep finding counterexamples. In this case, articulating a conjecture that's actually true requires some care.

Tips for the Classroom

1. Some students have a surprising amount of trouble drawing the path of the billiard ball. Make sure all your students can actually, correctly derive the path. Graph paper helps!

2. Certain conjectures are easy to come by, but can also be instructive. The 1 by 1, the 2 by 2, the 3 by 3 table, etc., are all similar shapes, and clearly the billiard ball ends up in the top right corner. Use these observations from students to ask them what's like the 1 by 2, or the 2 by 3, etc. table.

3. The "4" row of the table provides lots of counterexamples. You can encourage students to try that one if they're overly convinced in their conjectures.

4. There's a deep idea, related to attacking this problem, of imagining the billiard ball of going into an identical table "through the looking glass" of the surface it reflects off of. This idea can provide a completely different, very powerful approach, if you unpack it.

Name________________________________

Billiard Ball Results

	1	2	3	4	5	6	7	8	9	10
1	TR	BR								
2										
3										
4										
5										
6										
7										
8										
9										

What Corner does the ball end up in?

TL = Top Left

TR = Top Right

BL = Bottom Left

BR = Bottom Right