The Calendar Problem

Topics: Multiplication, addition, problem solving
Materials: Calendar, Hundreds chart, pencil and paper
Common Core: 4.OA.1, 4.OA.3, 4.OA.5, 4.NBT.5, MP1, MP2, MP3, MP6, MP7, MP8

How can you add up these numbers so quickly?

Why we love this lesson

Simple to launch, instantly engaging and accessible, this puzzle is a beautiful demonstration of the usefulness of multiplication. Not only that, figuring out why the trick works, even when you notice it does, requires exactly the kind of thinking we want our students to do.

The Launch

Draw a 3 by 3 square on a calendar (only getting days from a single month inside the square). Start by asking students what patterns they can find in that 3 b 3 square. You can spend 10 - 15 minutes on this or more, if appropriate.

6	7	8
13	14	15
20	21	22

Once students have had time to look for patterns, challenge your students to add up the numbers as quick as they can. No matter how fast they add, you can always beat them. What's the fast way to add these nine numbers together?

The Work

Once the students understand the question, let them work in pairs to try to figure it out. The teacher can circulate through the classroom, asking students to explain their ideas, and, as necessary, providing hints to students who are having trouble finding traction with the problem.

Prompts and Questions

Hint 1.

Just trying a bunch of examples and looking for patterns, either by listing or by coloring them in on a hundreds chart, can help a lot, especially if you organize the outcomes a bit. If we try different squares on the November page above, we'll get numbers like:

99

3	4	5
10	11	12
17	18	19

108

4	5	6
11	12	13
18	19	20

117

5	6	7
12	13	14
19	20	21

Shifting the square to the right seems to add 9. Will this always work? Why would it? What would happen for different months?

Hint 2

There may be a way to see something if you use a 2 by 2 square instead of a 3 by 3 square. Or you could try a 1 by 3 rectangle.

Hint 3

Maybe there's a way to break the addition problem down into manageable pieces. What if you added each row of the 3 by 3 square up, then added those sums? Or what if you added the columns first?

The Wrap

Bring students together to share their solutions. There are many ways to answer this question, and hopefully students will come up with several. One possible solution and argument is below.

The answer turns out always to be 9 times the number at the center of the square. This is because every opposite pair of numbers averages out to the number in the center. For example, instead of adding 10 and 12, we could just add two 11s. Instead of adding 3 and 19, we could just add two 11s. Adding all nine numbers comes out to the same as adding nine of the middle number, which is just a multiplication problem. That's how the teacher is able to do it so easily!

Extensions

- Is there a quick way to add up the numbers in a 4 by 4 square?
- What about rectangles?
- Calendars are a little small… if we try drawing squares on a 100s chart, is there still a quick way do it?
- What is the sum of <u>all</u> the numbers on a 100s chart?

Tips for the Classroom

1. For students, knowing that a quick way exists but that it's just out of reach is a big motivator. **Don't give away the trick!**
2. The extensions are so natural that it's definitely worth spending time on them. They may make up half or more of the time spent on this lesson. They're also a great opportunity for students to find their own versions of this argument.
3. For some squares, there's no clear "even" entry. Figuring out what to do in this situation will be an excellent challenge for kids who are ready for it.