

# A Math Magic Trick

**Topics:** Arithmetic, algebra

**Material:** Pencil and Paper

**Grades:** 4, 5, 6, 7, 8, 9

**Common Core:** 4.OA.1, 4.OA.2, 4.OA.3, 4.OA.5, 5.OA.3, 6.EE.2, 6.EE.3, 6.EE.4, 7.EE.3, 8.NS.1, A-CED.1, A-CED.2, MP1, MP2, MP3, MP5, MP6, MP7, MP8

This trick is an extraordinary introduction to the power of algebraic thinking, and a reminder of what makes algebra so awe-inspiring.

## What we love about a Math Magic Trick

This deceptively simple trick can differentiate broadly. Some students can use it to practice arithmetic or get practice with the rules for, say subtracting negative numbers or dividing fractions, or even whole numbers. It can also be used to highlight the full power of the algebraic toolset. This one is a favorite.

## The Launch

Launch with the video at <https://www.youtube.com/watch?v=n2RTjqHUXGQ>.

Or follow the instructions below:

The teacher explains the trick to the class. Every student follows the instructions:

Pick a whole number between 1 and 10.

Add 2.

Multiply by 2.

Subtract 2.

Divide by 2.

Subtract your original number.

Everyone's final answer will be 1 (assuming they didn't make any arithmetic mistakes).

**Big Question 1.** Will this work for number that aren't whole numbers between 1 and 10? What numbers will it work for?

For many students, trying to break this pattern is a great challenge. They can try big numbers, negative numbers, decimals, fractions, on their own or with help from the teacher. Lo and behold, everything they try will work out.

After they'd done a multitude of examples, we have the next question:

**Big Question 2.** Why does this always work?

Taken at face value, this seems like an impossibly hard question. To show it always works, you need to test the trick for *every* number, and that means an infinite amount of work. How can you prove things like this are *always* true?

A common breakthrough is to try the trick with pi as the starting number. Since, as many students know, the digits of pi go on forever without repeating, we can't really write it out. But maybe we can just use the letter! This gives us:

$\pi$  (pick a number)  
 $\pi + 2$  (add two)  
 $2\pi + 4$  (multiply by 2... this step can be a little tricky to those who don't know algebra)  
 $2\pi + 2$  (subtract 2)  
 $\pi + 1$  (divide by 2)  
1 (subtract your original number)

Note: Video on this transition available at <https://youtu.be/HMSmt5yOl6E>

But here's the big idea of algebra: we didn't need to know anything about that  $\pi$  symbol to find out that the answer was 1. In fact,  $\pi$  could have stood for *any* number, and the answer would have been the same. Wait... if it could have stood for any number that the trick would still have given the answer 1, then the trick works for any number! This is actually an algebraic proof, and it gives an argument that the trick works. Always.

It may be nice to mention that folks will commonly use x rather than  $\pi$ .

x (pick a number)  
 $x + 2$  (add two)  
 $2x + 4$  (multiply by 2... this step can be a little tricky to those who don't know algebra)  
 $2x + 2$  (subtract 2)  
 $x + 1$  (divide by 2)  
1 (subtract your original number)

**Big Question 3.** What happens if you tinker with the trick?

A natural way to tinker is to double all the numbers.

#### Variation

Pick a number.

Add 4.

Multiply by 4.

Subtract 4.

Divide by 4.

Subtract your original number.

Do you always get the same answer?

Of course, students may have other ideas of how to tinker. Figuring out the relationship between what's changed in the trick and how the answer is affected is a great exploration, and students can go to town on it. We might even be able to introduce a second variable if they're ready, and have something like

### Generalized Variation

Pick a number.

Add  $m$ .

Multiply by  $m$ .

Subtract  $m$ .

Divide by  $m$ .

Subtract your original number.

Does it always give the same answer? What it is (in terms of  $m$ )?

## **Tips for the Classroom**

1. This is a fantastic lesson in terms of showing off the real power of algebra. The main trick to this one is not to get students out of trouble too early. As long as they're confounded by the fact that the answer is always the same, you can keep goading them into trying another example (will it work for  $-4.17$ ?) and underlining how difficult the task of testing every number is. Don't rush students who aren't ready for the conceptual leap, and keep priming them to be ready for it when it comes.
2. For younger grades, consider using visuals like tape diagrams, shapes, or other pictorial representations of algebra, rather than letters like  $x$  and  $N$ .