Subtracting Reverses

Topics: Subtraction, base 10, logic, patterns
Materials: Paper and pencil, Hundred Chart
Common Core: 3.NBT.A.2, 4.NBT.B.4, MP1, MP3, MP7, MP8

What happens when you take the difference of a number and its reverse?

Why We Love Subtracting Reverses

This extraordinary lesson combines base ten subtraction practice with an irresistible mystery. A pattern slowly reveals itself as the class compiles data.

Launch

Ask a student to give you a 2-digit number, i.e., 47. Take the reverse (74) and find the positive difference by subtracting the smaller from the larger (74 - 47 = 27). Then repeat:

The reversal of 27 is 72, so now we need to find the difference between those two numbers.
72 - 27 = 45

The reversal of 45 is 54, so we need to find the difference between those two numbers.
54 - 45 = 9

The reversal of 9 is 9, so we take the difference of 9 and itself.
9 - 9 = 0
And then we're done.

Conjecture. If you start with any 2-digit number and repeat this "subtracting reverses" process, you eventually end at 0.

Challenge the students to give you a counterexample to the conjecture, i.e., a 2-digit number that won't end at 0 if you continue this process. Suppose some give you 23:
32 - 23 = 9
9 - 9 = 0.

Do one or two more examples to make sure everyone understands the process. At this point, students may notice that the number 9 is occurring a lot. Go out on a limb and make another conjecture.

Conjecture. If you start with any 2-digit number and repeat this "subtracting reverses" process, you eventually end at 9. No other one-digit number from 1 - 8 ever occurs.

This is an aggressive conjecture, and students should feel motivated to disprove it. Give them each their own hundred chart (see below) to collect their work.

Prompts and Questions

- Which number do you think won't come to 9?
- What numbers do you know will go to 9 on their next step (i.e. 32 - 23 = 9). What if you color those in on your chart. What do you notice?
- What are other number aside from 9 that you arrive at on your first step (i.e., 27, since 27 = 74 - 47). Color those in in a different color. What do you notice about these numbers?
- You got a number that doesn't come to 9? That's a big deal! Double check it to make sure you got all the arithmetic right.

Wrap Up

Let the students share their findings. In this lesson, they are likely to have found a different surprise: every number that isn't the same as its reverse (like 66) to start will end at 9! Why?

You may not be able to arrive at a full solution with your students, but there is a good reason that this happens; you'll have to dig into the base 10 process and the nature of divisibility by 9 to find out. A direction that might be promising: students might have discovered that the numbers you arrive at after your first move subtracting a reverse are all multiples of 9 (0, 9, 18, 27, 36, 45, etc.). One way to think about why:

Consider a number like 74 = 7 tens + 4 ones.
It's reverse is 47 = 4 tens + 7 ones or equivalently 7 ones + 4 tens.

The difference 74 - 47 = (7 tens - 7 ones) - (4 tens - 4 ones) = 7 nines - 4 nines = 3 x 9. This argument may be too abstract for students; don't belabor it if so.

A great closing project is to try to do just enough experimenting to arrive at a conjecture for a question to send students home with: will three digit numbers end at 9 as well?

Example: 321 - 123 = 108. What next?
[It turns out that three digit numbers tend to end at 99.]

Tips for the Classroom

1. Make sure you open with a Number Talk or other exercise that let's you know that students are competent with two-digit subtraction.
2. Use base 10 blocks for students who have trouble with subtraction.
3. The hundred chart itself can also be a tool to help with subtraction. Take a number and its reverse (74 and 47) and find the difference by traveling between: 47, 57, 67, 77 (+30), 76, 75, 74 (-3)... total distance traveled: 30 - 3 = 27.
4. Try 3-digit numbers if students need additional challenges.

Name______________________

Subtracting Reverses

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100