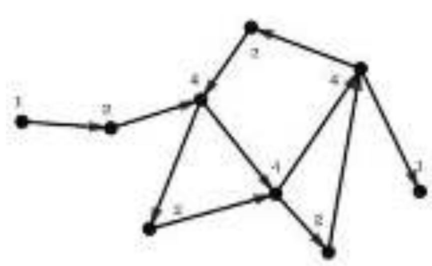
At the heart of all these puzzles is a remarkably simple observation, which comes down, once again, to parity, or the idea of evenness and oddness.

The fundamental observation that solves all these puzzles is that there are two special points in a path drawn with a single, unbroken, pencil line: the beginning and the end. Consider any other point along the path, and count the number of edges connected to that edge. While the beginning and ending points connect to a single edge, every other point connects to two edges, the one we drew into the point, and the one we drew going out of that point.

So far, so good, but what if our path loops back through these points?

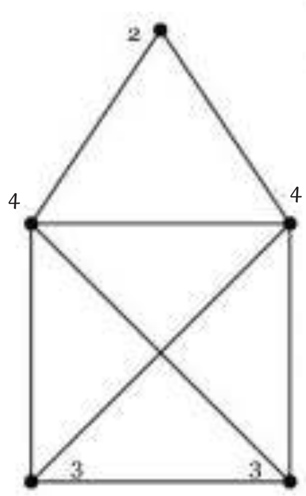


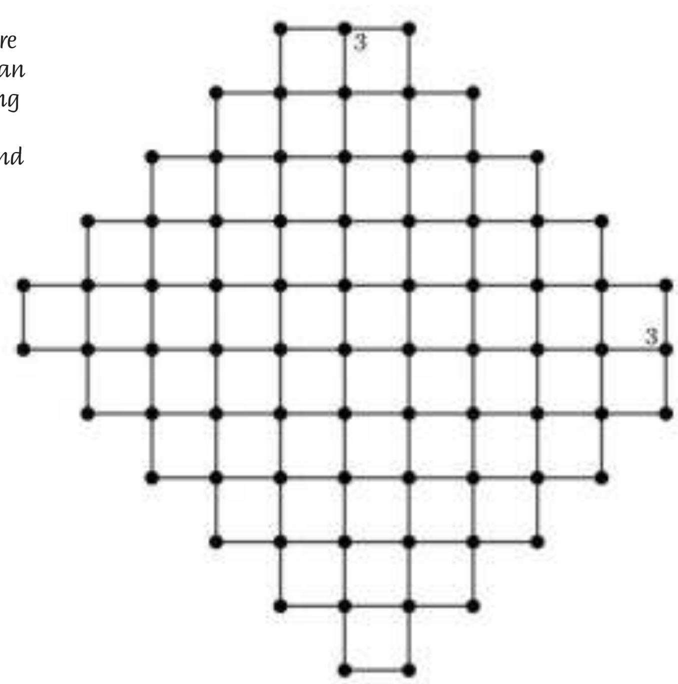
Again, every time the path winds back through a point it has already visited, it adds another two edges to that point. So if the point was even before, it stays even. If the point was odd before, it stays odd.

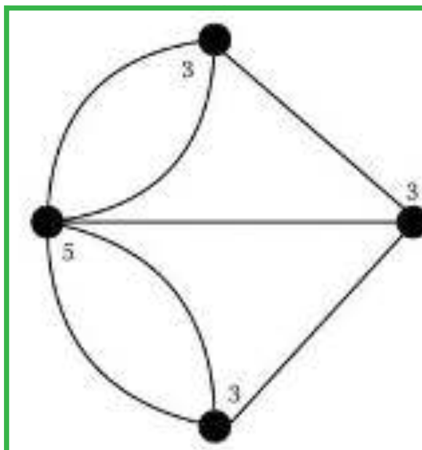
And that simple insight completely solves the problem! And also gives us the solution for the fourth puzzle. For the path to be drawable with a single pencil according to our rules, it must have no more than two points with an odd number of edges touching them, and those two points are the beginning and end (or end and beginning). This gives us a remarkably easy way to solve the first three problems as well.

Puzzles 1 and 2

For these two puzzles there are exactly two points with an odd number of edges coming from them. Those are the places to start drawing or end drawing.







Puzzle 3

For the bridges of Königsberg, there are four points with an odd number of edges coming from them. That means it can't be drawn! There would need to be two beginnings and two ends, so it can't be done.

This is one of my favourite examples of how deeper mathematics hides in unexpected places, and how taking a step back and looking at the structure of things can save us from tedious work.