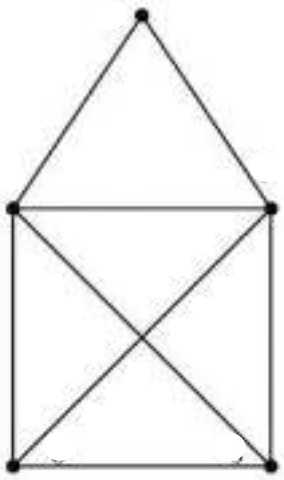
A MATHEMATICIAN AT PLAY

Do this without lifting your pencil

Have you ever tried your hand at those puzzles where you are asked to draw a figure without lifting your pencil or redrawing the same stretch of line? What if I were to tell you that all these problems have a simple observation that will hand you the answer? Or that the secret lies in viewing them as networks. Daniel Finkel points you in the right direction…

When I was five years old, a kid in my neighborhood challenged me with a puzzle that I've remembered ever since. As it turns out, there's much more to this puzzle than meets the eye.

***Puzzle 1**: Draw this shape without picking up your pencil or redrawing the same stretch of line more than once.*

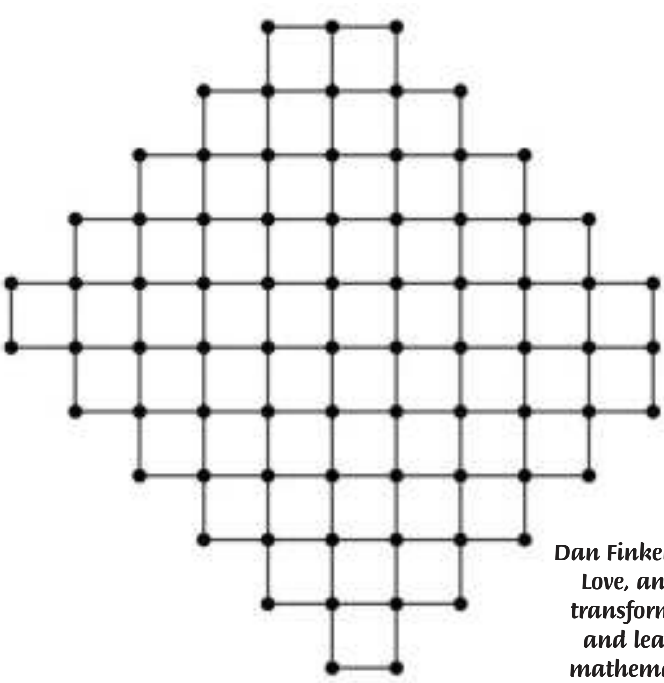


Puzzle 1 seems relatively benign. Mess around with it for a bit and you'll get it. A less obvious version comes from the Kuba children in the Congo, who took this puzzle much, much farther. The story, related by anthropologist Emil Torday, goes like this: the children were drawing complicated networks in the sand. When they saw Torday, they challenged him to draw the pictures in the sand without picking up his finger or drawing the same stretch more than once.

"The children were drawing," said Torday, "and I was at once asked to perform certain impossible tasks; great was their joy when the white man failed to accomplish them."

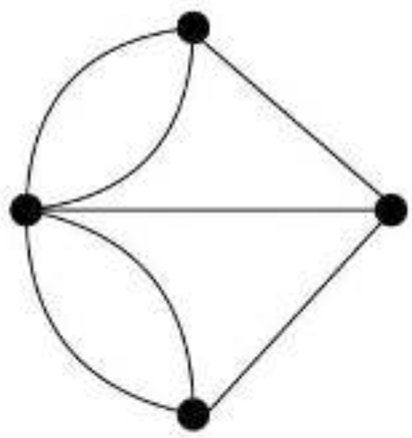
The picture the children challenged Torday with is the next puzzle. And it is, as those children knew, possible to draw.

***Puzzle 2**: Draw this shape without picking up your pencil or redrawing the same stretch of line more than once.*



What those children knew is the same thing that mathematician Leonard Euler figured out when he tackled the famous Bridges of Konigsberg puzzle.

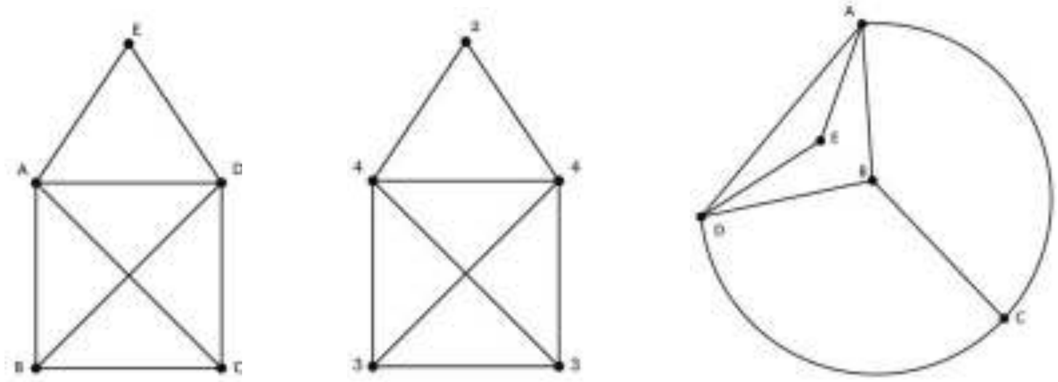
***Puzzle 3**: Why is it impossible to to take a walking tour of Konigsberg that crosses all seven bridges (marked in green) exactly once?*



What's extraordinary is that Euler and the Kuba children looked deeper into the structure of these puzzles and asked what about the fundamental structure would make it possible to know whether a network could be drawn without lifting your pencil or repeating lines, but without tedious trial and error. How did they do it?

These types of questions are part of an area of mathematics known as graph theory. It's a fascinating area of study. The objects of interest are precisely these kinds of networks. It's worth noticing that the walking tour of Konigsberg is quite different on the surface than the "house" image in puzzle 1. However, if we put dots on each side of the bridges, and replace the bridges with connecting lines, Konigsberg is just another graph, or network.

Does it matter if the connecting lines are curved or straight? Not for our purposes. In fact, we could abstract the information in a graph like the "house" in puzzle 1 as follows. Label the five points A, B, C, D, and E, and keep track of which pairs have an edge drawn between them: AB, AC, AD, AE, BC, BD, CD, DE. Then we could hand that information over to someone else who could redraw that image however they liked. They could move the points around, or draw curved connecting edges, or squiggles. As long as the points and connections encode the same way, it's essentially the same graph.



This flexibility of information makes graph theory surprisingly important in mathematics. It is possible to use graphs to solve deep problems in geometry and topology. Graphs have also been critical in understanding networks of people and organizations, and can be used to model the spread of disease and forest fires.

Here's a final, more open puzzle to conclude today's column.

***Puzzle 4**: Given any graph, find a simple way to check whether it is possible to draw the graph without picking up your pencil or redrawing an edge.*

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling Prime Climb and Tiny Polka Dot.