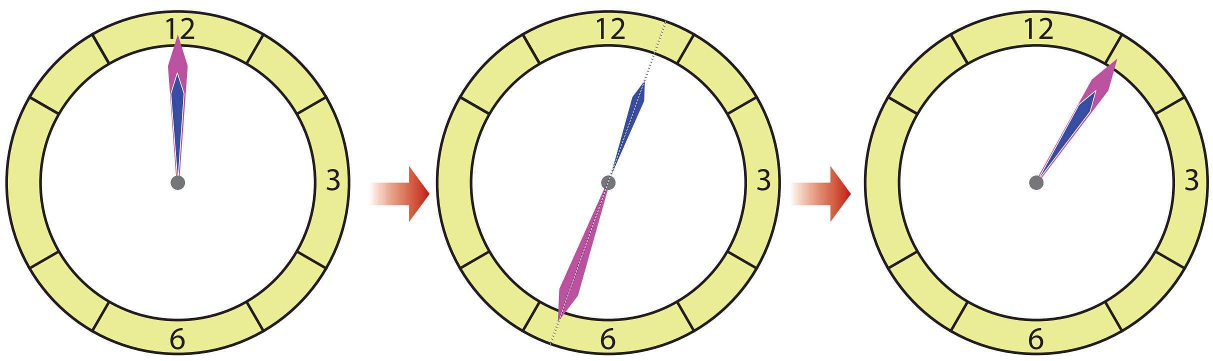
PUZZLE 1

At first 3:00 and 9:00 seem like the only times the clock hands form a right angle. But there are many more possibilities!

Let's try using the intermediate value theorem. The angle between the hands starts at noon at 0 degrees. Then they increase until the minute hand circles around and passes the hour hand again, a bit after 1:05 (refer image below).



If the angle went from 0 to 180, it must have hit 90 on the way. It then went back from 180 to 0 degrees the other way, so it hit 90 again. So each time the minute hand passes the hour hand accounts for two right angle moments.

So how many times does the minute hand pass the hour hand in a day?

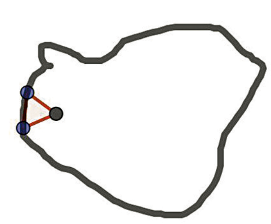
From midnight to noon, the minute hand makes 12 revolutions around the clock. Will it pass the hour hand 12 times? Not quite... the hour hand makes a revolution in those 12 hours as well, so that takes away one of the crossing.

That means we have 11 times when the minute hand and the hour hand are in the same place every 12 hours, which gives us 22 right angles. Double that to get 44 right angles in 24 hours. More than there seemed to be at first!

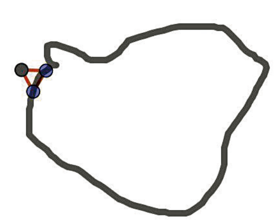
Interestingly, because the right angled clock hands come at regular intervals, we can say more: that every 24/44 hours, or about every 32.7 minutes, the hands of a clock will be at right angles.

PUZZLE 2

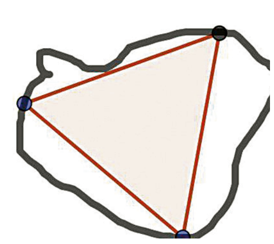
Imagine two points of an equilateral triangle drawn very close together on the boundary of the loop in question. The third point will be inside the loop.



If you move one of the boundary points around the loop, it will eventually be very close to the start point, but on the other side. And the third point will be outside the loop.



If the third point moves continuously from inside the loop to outside the loop, at some point it must have been on the loop.



There's your equilateral triangle with all three points on the loop! And the argument works for any loop!

PUZZLE 3

This one seems tricky, but the same trick will still work if we apply it right.

Let's imagine standing on the equator and we have a device which tells us how much warmer it is where we are than the spot on the opposite side of the globe.

If it reads "0," then there's no difference between where we are and the opposite point of the equator, and we're done!

But probably it won't be that easy. Probably it's some positive or negative number.

Let's say it reads a positive number. That means it's hotter where we are than on the opposite point on the equator.

Imagine walking around the equator, and when we arrive at the opposite point (we're imagining time is fixed in the moment), what does our device say? It must read a negative number.

So our device went from reading positive to negative. That means it must have read zero at some point, and that means the temperature must have been the same at the opposite ends of the equator at that point.

That's all for today. Hope you pay attention to the continuous movement happening all around you. Happy puzzling!