*These meta-logic puzzles can be solved only by imagining what Abby and Bill actually know about their own numbers – even though you don't know what those numbers are.*

*The main thing to do to help students solve these is to try them out with actual examples!*

*If Abby asks Bill if his number is twice hers and he's thinking of the number 17, would he say "I don't know"? No. He would know that his number couldn't be twice hers, since he's got an odd number. The fact that he says "I don't know" means that he couldn't have an odd number, and now you (and Abby) know that fact. Meanwhile, Abby wouldn't have asked the question in the first place if her number was greater than half of the largest number in the range. So even asking the question gives a lot of information!*

*It would help to keep notes on what you learn with each question, and slowly the knowledge will reveal itself.*

## PUZZLE 1

What do we know about Abby's number? If she had chosen the number 9, then she would know that the sum of their numbers has two digits, and she would never have asked her first question. So Bill knows right away that Abby's number is not 9.

When Bill says he doesn't know if the sum is a two-digit number or not, he's communicating something too: that he doesn't have a 1. If he did, he would know that their numbers would add up to less than 10, since Abby has already communicated that she doesn't have a 9. So we could rephrase what they've told each other:

Abby and Bill each secretly choose a whole number between 1 and 9.

Abby: I don't know if the sum of our numbers is a 2-digit number.
Do you know? [I don't have a 9.]

Bill: I don't know. [I don't have a 1.]

Abby: I wasn't sure before, but now I know the sum is a 2-digit number. [I know because I picked 8, so whatever you picked, it sums to 10 or more when you add it to mine.]

So Abby has chosen 8. Get the hang of it? We can arrive at the solutions to the others in a similar manner.

## PUZZLE 2

Let's try translating what Abby and Bill actually communicate to each other into plain language. See if you agree. The key here is that asking if someone's number is twice yours means that your number must be at most half of the possible range of their number. Saying you don't know if your number is twice theirs means that you must have an even number (or twice an even number, as things progress).

Abby and Bill each secretly choose a whole number between 1 and 30.

Abby: Is your number twice my number?
[My number is at most 15.]

Bill: I don't know. [My number is even.]
Is your number twice my number?
[My number is at most 7.]

Abby: I don't know. [My number is divisible by 4 ] Did we pick the same number? [My number is in the same range as yours (1-7). In other words, my number is 4.]

Bill: We did. [My number is 4 too.]

So both Abby and Bill picked 4.

## PUZZLE 3

Asking whether the other's number is half yours means that you have an even number (else why would you ask?). If someone doesn't know, that means their number is in the bottom half of the possible range of numbers.
Let's translate the conversation into plain language once again.

Abby and Bill pick numbers from 1 to 40.

Abby: Is your number half mine? [My number is even.]

Bill: I don't know. [My number is at most 20] Is your number half mine? [My number could be twice yours. I know yours was even, so that means mine is a multiple of 4]

Abby: I don't know. [My number is at most 10] Is your number half mine? [My number is a multiple of 8. In other words, my number is 8.]

Bill: Yep. [My number is 4]

What numbers did Abby and Bill pick?

Solution: Abby picked 8, Bill picked 4.

## PUZZLE 4

We can make this problem easier by looking at the two numbers Abby doesn't choose rather than the 5 she does. Is there a pair that have the same produce, but different parity (evenness/oddness)?

Yes: {2,6) and {3,4} both have the product 12, but while the former sums to an even number, the latter sums to an odd number. Notice that {1,6} and {2,3} both sum to odd numbers, despite having the same product, so they can't be the missing pair.

We don't know what five numbers Abby picked, but we do know their product. It is 420, which equals both
1 x 3 x 4 x 5 x 7
and
1 x 2 x 5 x 6 x 7.

That's it for this week. Happy puzzling!