A MATHEMATICIAN AT PLAY

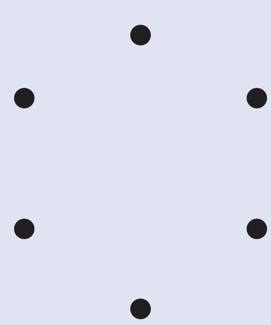
Time to play 'Don't make a triangle'

When everything seems to be going against you, can you still say for certain that something will surely happen? Daniel Finkel introduces you to Ramsey Theory, a study of maximum disorder, through a game called *Don't make a triangle*. So what are you waiting for? It's time to play and learn...

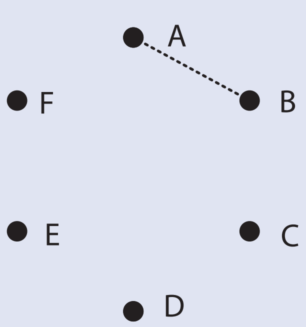
Today I want to share some classic puzzles about a truly fascinating field in mathematics known as Ramsey Theory. Ramsey Theory is a sort of study of maximum disorder: if everything goes against you, when can you still say for sure that something will happen? But we're going to approach this field from an entirely different angle: through a wonderful game called *Don't Make a Triangle*.

HOW TO PLAY DON'T MAKE A TRIANGLE

Don't Make a Triangle is a game for two players. Each takes a colour – let's say player 1 is red and player 2 is blue. Red and Blue take turns connecting the six points in the picture.



Each pair of points can be connected at most once, so if the edge drawn between them is red, it cannot be changed to blue, or vice versa.



If AB is red, then AB cannot be made blue. Similarly, if AB is blue, then AB cannot be made red.

Players take turns drawing in edges in their colour until one of them creates a triangle with corners on three of the original six points, and with its edges all red or all blue. Whoever creates that triangle first loses.

PUZZLE 1

Prove that there can never be a tie in Don't Make a Triangle.

In other words, prove that if you draw in all of the connecting edges between 6 points in blue and red, you'll either create a blue triangle or a red triangle.

(This is a tricky puzzle, so I'll put a hint at the very end of the article.)

PUZZLE 2

Can there be a tie if you were playing with 5 points instead of 6?

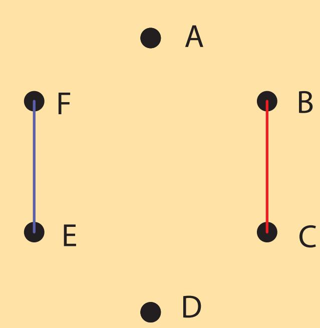
A version of this puzzle appeared in a math contest in the 1920s, and was written up in detail by the mathematician Frank Ramsey in a paper published in 1930.

The answer to puzzle 1 was so pretty, and the related problems so devious, that in effect, this single problem gave birth to an entire field known as Ramsey Theory. It's a fascinating and often bafflingly hard field, having to do with exactly this kind of problem: if we're avoiding shapes as we draw in colours between points, how many points does it take before we just can't avoid them anymore?

We can prove that with enough points, just about any shape or collection of shapes will have to emerge, but pinning down exactly how many points it will take is almost always elusive. Still, it's quite lovely: in a larger enough universe, order emerges from chaos.

This puzzle is sometimes known as the "friends and strangers" problem: if there are 6 people at a party, show that there is a group of 3 that are either all friends or all strangers.

There's a little translation required to see these two problems are one and the same: imagine drawing the 6 people at the party as dots, and connect a pair with red if they're friends, and with blue if they're strangers. Assuming every pair is either friends or strangers, this comes down to showing there are no ties in Don't Make a Triangle. (Of course, you have to assume people can only be friends or strangers. Maybe they're friends on Facebook or else not... that's more precise.)



In the above example, B and C are friends and hence we have used a red line to connect them. E and F are strangers and we have therefore employed a blue line to connect them.

Hint for Puzzle 1. Imagine you looked at just one dot, and considered the edges that came out of it: there are either 5 blue; 4 red and 1 blue; 3 red and 2 blue; and so on. Consider each of those cases, and what else needs to be true to avoid making a triangle. What do you notice?

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling *Prime Climb* and Tiny *Polka Dot*.