## 1-2 Nim

Topics: logic, patterns, addition, counting, subtraction, divisibility and remainders (optional) Materials: Counters (tiles, beans, pennies, etc.) or paper and pencil
Recommended Grades: K, 1, 2, 3, 4, 5
Common Core: K.CC.A.2, K.CC.B.4, K.CC.B.5, K.OA.A.1, K.OA.A.2, K.OA.A.5, 1.OA.B.4, 1.OA.C.5, 2.OA.B.2, 3.OA.D.9, 3.OA.A.2, 3.OA.A.3, 4.OA.C.5, MP1, MP2, MP3, MP5, MP7, MP8

You can take one or two counters from the pile. How do you get the last one?

## Why We Love 1-2 Nim

Nim is fun, challenging, and rewarding for a wide range of kids. Done right, it can engage everyone from Kindergarten to upper elementary kids, and connect to basic counting, and arithmetic up to division. Completely unlocking the game is an exciting and powerful achievement for a student.

## How to Play

Nim is a two-player game. You start with a pile of counters. On your turn, remove one or two counters from the pile. You must take at least one token on your turn, but you may not take more than two. Whoever takes the last token is the winner.

## Example Game

We start with 8 counters in the pile.
Player 1 takes one counter, leaving 7.
Player 2 takes two counters, leaving 5.
Player 1 takes one counter, leaving 4.
Player 2 takes one counter, leaving 3 .
Player 1 takes one counter, leaving 2.
Player 2 takes two counters, leaving $o$ and winning the game.

## Variations

1. Change the size of the pile.
2. 1-2-3 Nim: players may take one, two, or three counters per turn.
3. Poison: Whoever fills up the 10-Frame loses.

## Questions

The Central Question: how can you win 1-2 Nim? What would a perfect strategy look like?
Good questions for the teacher to ask students:

- What move should I (the teacher) make?
- How did you/they/I win that game?
- What do you think your/my opponent will do if you/I take two counters?
- Would you like to take back your move?
- What have you noticed about this game?

Possible student conjectures, true and false, that may arise:

- Whoever goes first wins.
- Whoever goes second wins.
- Odd vs. Even determines your strategy.
- It matters/doesn't matter what you do until there are less than six counters in the pile.
- Whoever can give their opponent four open spaces wins.

For students who have figured out the game, some challenge questions:

- How would you win 1-2 Nim if you start with a pile of 20? 30? 100 ? 217 ?
- These challenges can also apply to 1-2-3 Nim and Poison variations.


## Tips for the Classroom

1. Demonstrate the game with volunteers for at least three games (or many more!), until you are certain everyone understands it and is excited to play.
2.When demonstrating 1-2 Nim, narrate the game out loud, using mathematical language, and leaving empty space for students to chime in: "My opponent just took 2 leaving... [wait for students] 5 in the pile. Who has a advice for what I should do next?"
3.Remind students that they will lose many games as they play, and that every loss is an opportunity to learn. Can they steal the strategy of the person who just beat them? Point out how students are trying out new strategies as they play you in demonstration games.
4.As kids play each other, circulate to see what strategies they are developing. Challenge them to play you, and see if they can beat you.
5.Encourage student conjectures, but do not call them as true or false. Challenge students to break their own conjectures.
6.This game is great for station work once students already know how to play it.
2. We use the term "the 3 trap" to describe what happens when you give your opponent a pile of three counters. Understanding how to win boils down to understanding what pile sizes you want to leave your opponent with.
8.There are two incredibly powerful approaches to solving Nim. The first is to simplify. How could the game be easier? What if the pile had only one counter? From this place of almost absurd simplicity, we slowly raise the difficulty. What about two counters? Three counters?

The second approach is to organize the data in a coherent way. A table does this very nicely.
9. If kids want to play three-player, keep in mind that we discourage it. Normally trying out different numbers of players is a great impulse. In nim, it leads to spoilers, who can't win, but can choose who does win.
10. Homework: have kids teach 1-2 Nim to a friend or family member.

| Number of Counters | Winning Strategy |
| :---: | :---: |
| 1 | Go first. Take 1. |
| 2 | Go first. Take 2. |
| 3 | Go second. |
| 4 | $?$ |
| 5 | $?$ |

References: http://wordplay.blogs.nytimes.com/2011/o6/13/numberplay-1-2-nim/

