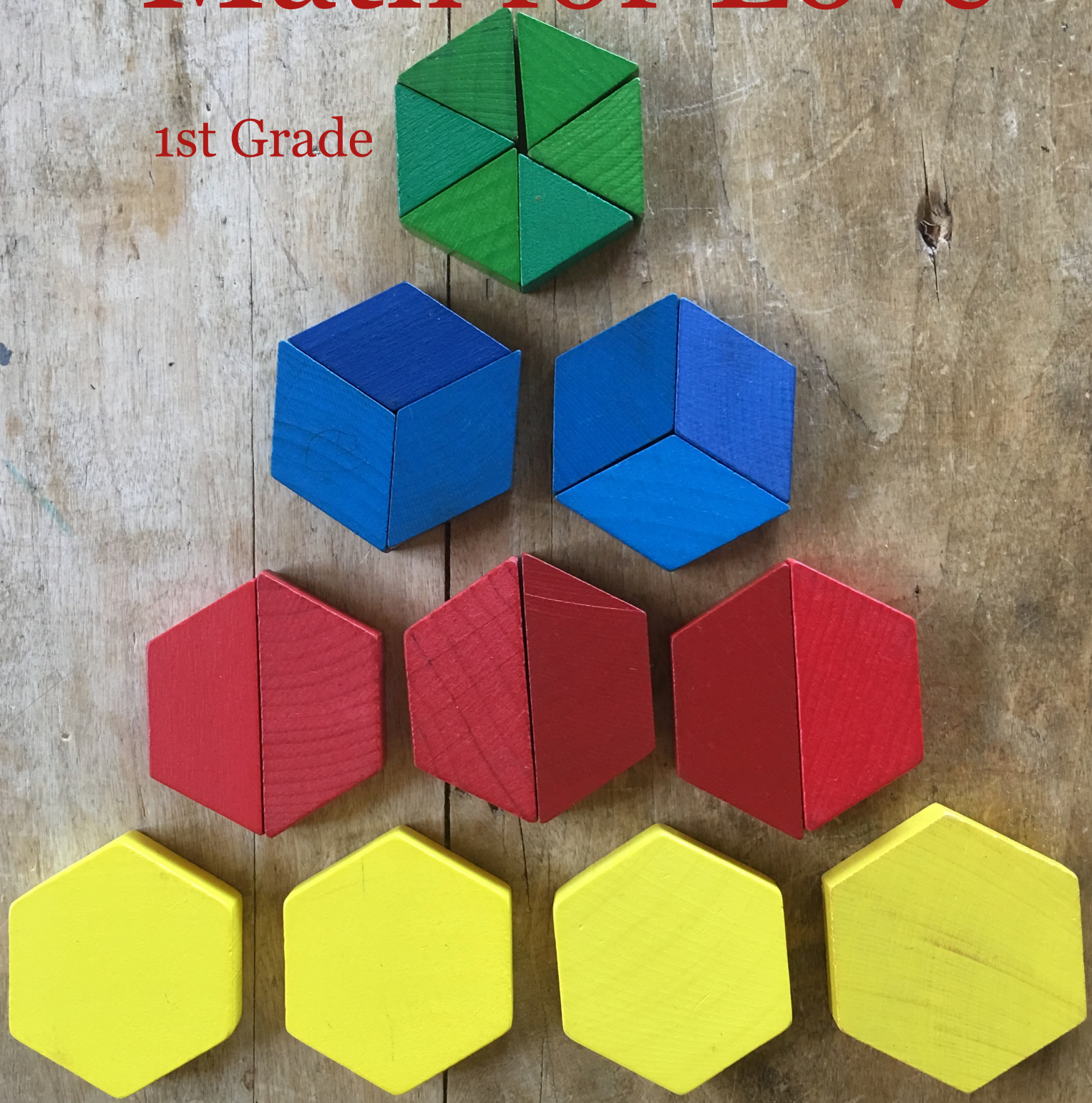
Math for Love

1st Grade



by Dan Finkel & Katherine Cook

A word about using this book

This book was designed to support a summer math program lasting sixteen 75 - 90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. Games can be played for 5 - 30 minutes. Deeper tasks might be good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this curriculum that should be helpful: mathforlove.com/video/math-for-love-video-pd.

Enjoy!

A word about the copyright

We want this book to be used by teachers to help students explore math in a positive way. Feel free to make photocopies, share ideas with parents and colleagues, and use this as a resource draw on. In general, we support this kind of fair use of our materials. Please don't post elements from this book online without citing the source, share large chunks of the book electronically, or sell parts of the book anyone.

Table of Activities

Where to find each activity the FIRST time it's used in this curriculum

Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

Goals of the Math For Love Curriculum

We wrote this program to be both *play-based* and *rigorous*. The goals of the program are two-fold:

- Improve conceptual understanding of and fluency in mathematics
- Give everyone an opportunity to have fun and enjoy math

Many students haven't had enough time working with conceptual models of mathematics before being pushed into abstraction. To remedy this, the curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorially while making connections to abstract reasoning.

Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas.
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement**. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation**. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.

3. **Thinking**. Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. **Positive Environment**. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Be ready with questions**
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value *their* thinking and contributions, not just the answer.

★ **Model how to play games, and teach how to win and lose**
Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Dot Talk or opening game (see the Opener in the daily plan). Use the Math Games and Station Breaks for transitions between Activities. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Have a *growth mindset* classroom**
Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you *learn* to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Encourage conjectures and counterexamples**
Establish a habit of supporting students' conjectures, hypotheses and predictions, and students will learn more and commit to the thinking process. Help them use

counterexamples as a tool to break and improve conjectures (see the **Counterexamples** lesson plan on Day 1), and they'll begin behaving like true mathematicians. Making conjectures and counterexamples normalizes mistakes as part of the learning process, and gives students a practical way to learn from them. It also makes doing and thinking mathematics the central activity of your class.

★ **Give your students *time* to think and explore**
Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**
We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

★ **Value play**
It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less, you can end sooner. If you need something longer, you should find many of the activities extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- *Goals*
- *Optional Warm Up*
- *Opener*
- *Activity*
- *Game*
- *Choice Time*
- *Closer*

Goals

These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students.

Optional Warm Up

Young students get so much out of free play that we recommend starting class with 5 - 10 minutes of free play with the blocks and math manipulatives. The discoveries they make (e.g., 6 pattern block triangles can make a yellow hexagon!) end up being fundamental to the mathematics we'll ask them to do later. If you have the time, it's time well spent. If not, block free play should be a standard option for Choice Time.

Opener

The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

In general, the Opener should last about 5 - 10 minutes.

Activity, Game, Choice Time

Following the opener, there is a suggestion for an activity, a game, and Choice Time. This is where the bulk of class time will be spent. There are two recommended ways to approach these three elements.

1. Have students rotate between three stations. This is especially recommended when you have additional adults (instructional aides, parent volunteers, tutors) in the room aside from the teacher.
2. Take the whole class through the activities one by one. This is recommended when the teacher is the only adult in the classroom.

Either way you run your classroom, the elements are designed to give students the maximum opportunity to think & engage, practice skills, explore questions, and have fun.

Choice Time includes a suggestion of a small group of past games and activities for the students to try. This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

Closer

The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer should take 5 minutes or less.

Other Notes and Best Practices

★ **Math Breaks and Physical Games**
Check out the math-based movement breaks in **Appendix 2**. These are excellent as transitions.

★ **Folder for Worksheets**
Give each student a folder where they can keep their worksheets. If they finish an activity early, they can turn back to their unfinished worksheets and finish them. They can also work on them during Choice Time.

★ **Choice Time**
Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should both choose the activity that is right for them, and then stick with it for at least half of the time.

★ **Challenge Problems**
Challenge problems (see **Appendix 3**) are great options for Choice Time any day. Offering “spicy" variations of worksheets or unfinished activities as Choice Time activities can be another nice option.

★ **Station Transitions**
If you use stations, provide 1-2 minute warnings before station transitions, to apply a gentle transition, cleanup, and—especially at Activity 1—a brief reflection or wrap-up. If you use an alarm, make it a gentle sound (i.e., a gong) rather than an abrasive one (i.e., a clock radio alarm).

★ **Games to send home**
See **Appendix 1** for games to send home. These will help parents/guardians and students play math games at home. You can always send other favorite games home, or encourage students to share games they've learned with people at home. Note that there is no homework for this program otherwise.

★ **Block Free Play**
Giving students 5 - 10 minutes to play freely with the blocks and manipulatives from the class is an excellent way to start every day, if you can spare the time. This will help behavior and focus, and help students develop intuition for the blocks. Block Free Play is always available as a Choice Time option, but consider it as an entry routine as well.

★ **Build Dot Talk images with magnetic ten frames**
For Dot Talks, we recommend you create physical versions of them with magnetic ten frames or other manipulatives. Physical versions are often preferable, since students can manipulate the blocks directly.

Day 1

Goals

1. Establish class norms and community.
2. Give Preassessment, and observe student comfort and ability with mathematics.
3. Play math games and explore situations involving counting, adding, and comparing.

Warm Up (Optional)

Block Free Play

If you have time as students are entering the classroom, it's highly recommended that you make blocks available at desks, or at stations, and let students explore the materials and chat with each other for the first 5 - 10 minutes of class. This immediately establishes the classroom as a place where students engage with materials and each other. Consider this as something to do every day.

Opener

Mingle and/or Guess My Number

Note: if you have a name game you like to use to get acquainted with your students, feel free to use it instead.

Preassessment

Make sure to let students know that this preassessment is not a "test," and not something they're expected to know any or all of the answers to. It's just a way for you, the teacher, to see what ideas they are familiar with, so you can make sure you keep them challenged and interested. They definitely shouldn't worry if they can't get all, or even most, of the answers. (We've included some very challenging questions!) So just tell them to do their best and not to sweat it.

Activity

Exploration of Materials

Closer

Today the class focused on playing games and exploring blocks. Ask students what they think those things have to do with learning math? Let students share and discuss.

Some points to emphasize, or bring up if no one else does:

- Math includes learning about a lot of things! Numbers, shapes, and patterns are all part of math.
- You can play to learn math! In fact, it's one of the best ways to do it.

Mingle

The teacher calls out a number (e.g., 3), and the students get themselves into groups of that size as quickly as they can. It might be impossible for everyone to get in a group every time, but each new number gives everyone another chance.

Once students get in groups, they can learn each other's names.

In the basic game, just call out single numbers. Once students get the gist, you can call out addition or subtraction problems (i.e., "get into groups of 7-4"). Don't forget to call out a group of 1 and a group of however many students are in the entire class at some point in the game.

Tips for the Classroom

1. The teacher can get into or out of the groups in order to make sure no student is by themselves.

Guess My Number

Topics: Greater than/less than, logic
Materials: Whiteboard or paper and pencil
Common Core: MP1, MP3, K.CC.C7, 1.NBT.B.3

Guess My Number is a quick, fun, and easy opening game to lead with a small group or the entire class. Students try to guess the number you're thinking of in the fewest number of guesses possible.

Why We Love Guess My Number

It's hard to think of a game more natural than Guess My Number. Kids can immediately start playing, and have a great time doing it. The game is also a perfect way to introduce the basics of logical thought and strategy, and let the game do the teaching with only minimal extra observations from the teacher. Surprisingly replayable and enjoyable, this is a great game to play early and often.

The Launch

As you write the numbers from 1 to 10 down on the board, tell your students that you are going to think of a number from 1 to 10, and they will try to guess it in the fewest number of guesses possible. After every guess, you will tell them whether your number is greater or less than their guess.

Example
Teacher: Who would like to make the first guess? [Students raise hands. The teacher calls on different students for each guess.]
Student: 3.
Teacher: My number is not 3. But my number is greater than 3. [Optional: write "My number >3".] Are there any numbers I can cross off my list?
Student: It's not 3. [Teacher crosses off the 3]
Student: It's not 2 or 1 either.
Teacher: Right. Because my number is greater than 3, but those numbers are less than 3 [Teacher crosses off 1 and 2.] Who has another guess?
Student: 9.
Teacher: My number is not 9, but my number is less than 9. [optional: write "My number <9".] Can I cross any more numbers off the list?
Student: The 9 and the 10.
Teacher: Because my number is less than 9, so it can't be 9 or 10. [Crosses them off. Looks at the board.] So the only options left are 4, 5, 6, 7, or 8. Take a minute to think about what would be a good next number to guess. Then tell someone sitting next to you what you would guess next, and why. [Students pair and share.] Who has another guess?
Student: Is it 7?
Teacher: My number is not 7, but my number is less than 7 [optional: write "My number

<7]. What numbers can I cross off?
Student: 7 and 8.
Teacher: Right. Because my number is less than 7, so it can't be 7 or 8. [Crosses them off.] So the only options are 4, 5, or 6. Who has another guess?
Student: Is it 5?
Teacher: My number is not 5, but it is… greater than 5. [Students' hands shoot up.] Whoever thinks they know my number, say it together.
Students: 6!
Teacher: You got it! Now that took you [counts] 1, 2, 3, 4, 5 guesses. Who thinks they could do it in fewer? [Optional: play again.]

Prompts and Questions

- How can you guess my number in the fastest, most efficient way possible?
- Talk to a neighbor about what you think the next guess should be, and why.
- How many numbers do you think that guess will cross out?

Tips for the Classroom

1. **Cheat**! By which I mean, don't actually choose your number ahead of time. If students guess 1, tell them your number is more than 1. Always make each guess give them the least amount of information possible, and deny them the lucky guess. Make them work for it, and they'll be more invested in working smarter.

   AVOID:
   Student: Is it 8?
   Teacher: It is! Lucky guess! You got it on the first try.

   BETTER:
   Student: Is it 8?
   Teacher: My number is not 8, but my number is less than 8.

2. You can slowly expand up to larger ranges of numbers as students are ready for them. I'll usually go up to 12 after a few games, and soon to 20. Kids love to see the game get harder, as long as it doesn't get too hard too fast. And because you write all the numbers up on the board at the start, they can always see what needs to happen.

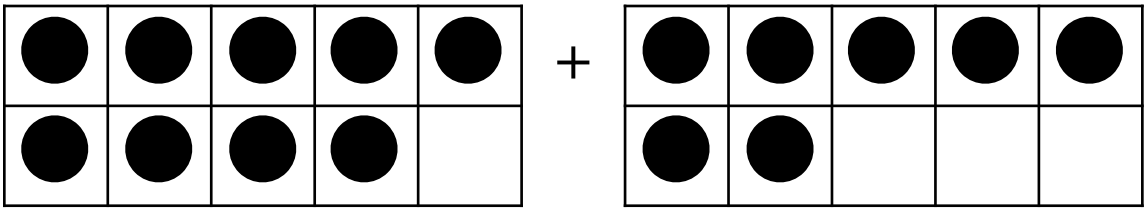
3. If kids make a bad guess, don't try to steer them toward a good guess right away. But you can ask the students after you write the guesses down which guesses were most helpful, or whether they would make a different guess if they could take it back.

4. Don't play for too long at one time. One or two games is usually enough to get the kids mentally alert and ready for whatever is coming next.

Name_____

Preassessment

1) How many dots?
Show how you figured it out.



2) There are three dots below.
How many *more* dots do you need to make 10?

Name_____

Fill in the blank.

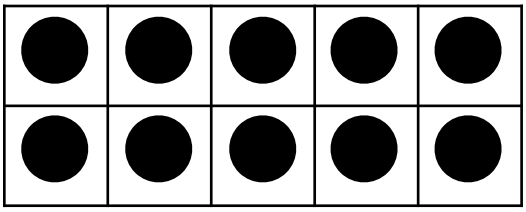
3) 6 + 13 = _____

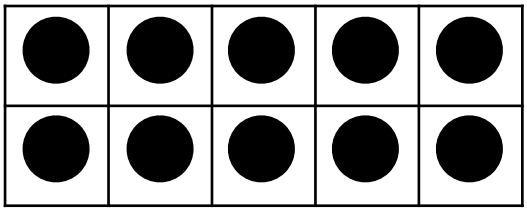
5) 19 - 11 = _____

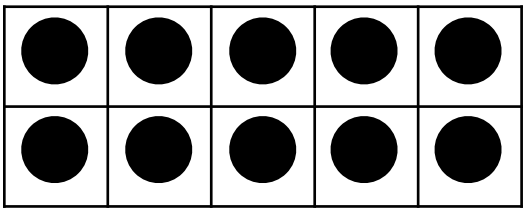
4) 17 + 8 = _____

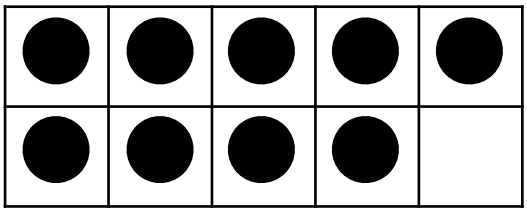
6) 5 + _____ = 23

7) How many <u>more</u> dots do you need to have **50 dots**?









8) I have 15 books. Then I got 9 more books. How many books do I have?

9) Julie started with 19 dollars.
She got 5 dollars from her mom.
Later, she spent 8 dollars buying a present.

How much money does Julie have now?

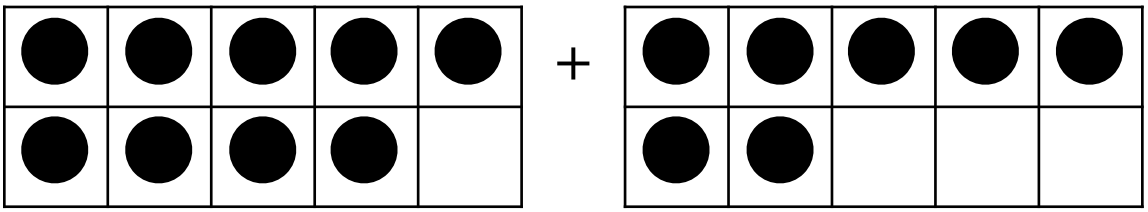
10) I had $60.
I bought a bike for $45.
Then I sold it for $50.
How much money do I have now?

Name_____

Preassessment
Solutions and Rubric

Each question worth up to 10 points.
Some partial credit available.

1) How many dots?
Show how you figured it out.



Answer: 9 + 7 = 16
5 points for correct equation/explanation,
5 points for correct answer

2) There are three dots below.
How many *more* dots do you need to make 10?

Answer: 7 dots
10 points for correct answer

Name_____

Fill in the blank.

3) 6 + 13 = ___**19**_____

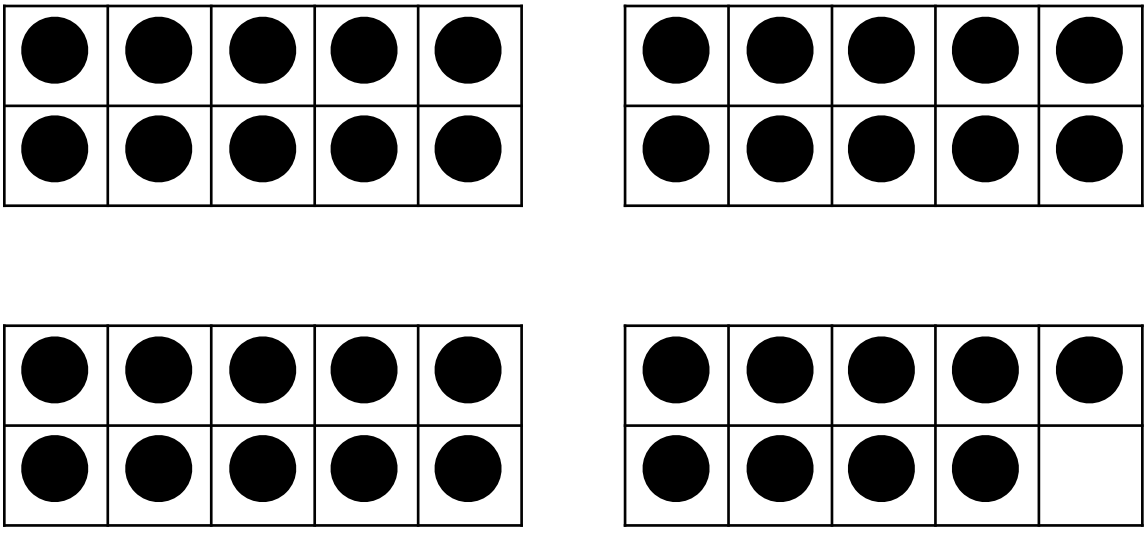
5) 19 - 11 = ___**8**_____

4) 17 + 8 = ___**25**_____

6) 5 + _**18**____ = 23

10 points for each correct answer

7) How many <u>more</u> dots to you need to have **50 dots**?



Answer: 11

Up to 5 points partial credit for a correct picture answer that students miscount (i.e., drawing in 1 ten frame and 1 extra dot)

8) I have 15 books. Then I got 9 more books. How many books do I have?

24 books
full 10 points for correct answer, including unit

-1 point for leaving off "books"
up to 5 points partial credit for setting up correct equation (15 + 9 = ____)

9) Julie started with 19 dollars.
She got 5 dollars from her mom.
Later, she spent 8 dollars buying a present.

How much money does Julie have now?

$16
Full 10 points for correct answer

-1 point for leaving off "$" or "dollars"
up to 5 points partial credit for setting up correct equation or picture model,
i.e., 19 + 5 - 8 = ____

10) I had $60.
I bought a bike for $45.
Then I sold it for $50.
How much money do I have now?

Answer: $65
10 points for correct answer

Up to 5 points partial credit for setting up an equation or representation correctly, i.e., 60 - 45 + 50 = _____

Exploration of Materials

Students are encourage to explore and play with the materials they'll be using in the class this summer. Free play is ideal, as long as the students stay relatively focused.

If you need to focus them further, consider using these challenges to motivate students who need extra guidance. Make sure ten frames are available for students who want to count tiles, snap cubes, or counters.

Pattern Block Building Challenges

Pattern Blocks Building Challenges: first, students take option 1, and free build. For students who need additional challenges, look to the next two options.

- **Option 1**: Free build with pattern blocks.
- **Option 2**: Cut out the 12 cards on the following page, and separate into two piles. For a basic building challenge, a student picks a card at random, and then builds according to what is on the card.

Example 1: Build a *Person*.
Example 2: Build something with *12 blocks*.

- **Option 3**: Advanced challenge: a student gets two cards, one which tells them what to build, and the other how many blocks to use.

Example: Build a *Triangle* using *15 blocks*.

Piles of Tiles

with the color tiles and/or the snap cubes

Let students free play and build with the tiles and snap cubes. For students who need more direction, give them a pile of tiles, and challenge them to figure out which color occurs most.

Challenge Prompt: In this pile of tiles, which color is there more of? Write down your guess, and then find out.

Tiny Polka Dot Games and Challenges

with the Tiny Polka Dot decks

Challenge 1: Pick a suit (i.e., a color). Arrange the Tiny Polka Dot cards in that suit from smallest to biggest. What will it look like if you organize an entire deck?

Challenge 2: Pick a suit. Count all the Tiny Polka Dot cards of that color (in one deck).

Challenge 3: Pick a suit. Count all the dots in that suit!

Two-Color Counters Challenge

Challenge Prompt: Shake a handful of counters and then spill them out. Without counting, are there more black counters or white counters?

Once you've guessed, count the counters of each color and see which has more.

Try again with more counters!

Dice

Challenge Prompt: pick up six dice. If you roll them all, which number will get rolled the most? Make your precision, then try and see.

Pattern Block Building Challenges

Flower	Bird	Person
Pattern	Triangle	House
5 blocks	10 blocks	15 blocks
20 blocks	23 blocks	25 blocks

Piles of Tiles

Which color tile occurs the most in this pile?

Write your name under the color you think has the most.

Red	Green	Blue	Yellow

When you have voted, count how many of each color there are.

Day 2

Goals

1. Practice counting with Counting Collections routine.
2. Match numbers with collections of dots.

Warm Up (optional)

Block Free Play

Opener

Dot Talk - see Appendix 4 for Dot Talk images

Activity

Counting Collections

Game

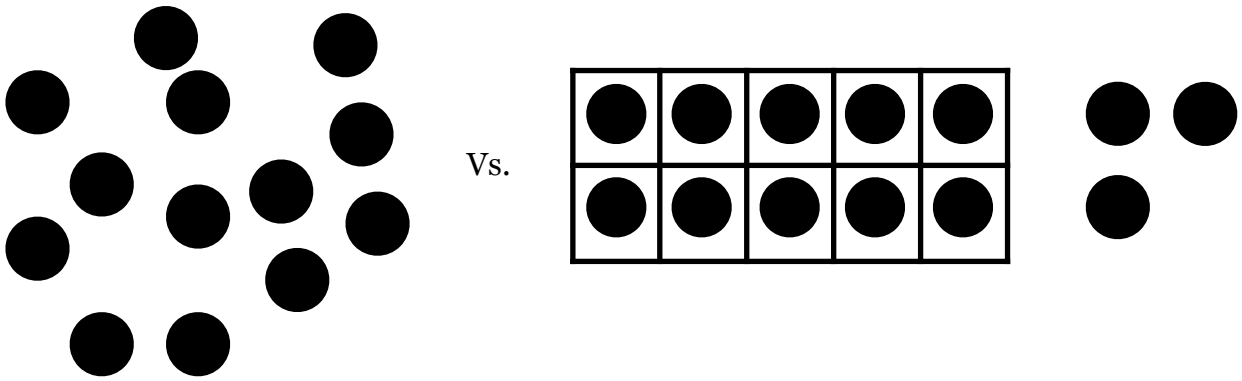
Match the Dots

Choice Time

Block Free Play
Match the Dots (or other Tiny Polka Dot games)
Counting Collections

Closer

Ask students what kind of arrangements were easiest to count in Counting Collections, and which were hardest. If you have magnetic dot ten frames, place a jumble of 13 dots on the board, and ask students to count them. Then arrange the dots in a ten frame with three outside it. Is that easier to count? What strategies might students use next time they do Counting Collections? (You can draw these otherwise. Alternatively, you can use Tiny Polka Dot cards from the green, unordered suit, vs. the blue, ten frame suit)



Dot Talks

Topics: Mental math, numerical fluency; argument & critique
Materials: White board & projector
Common Core: 1.OA5, 1.OA.6, 1.OA.7, 1.NBT.1, 1.NBT.2, 1.NBT.4, and especially MP3

This mental math routine creates powerful positive habits for students.

Why We Love Dot Talks

In just 5 - 10 minutes, these openers get all students involved, help strengthen fluency, intuition, and mental math strategies, improve students' ability to explain and critique solutions, and allow teachers a valuable window into their students' thinking.

The Launch

The talk starts very simply. The teacher projects the dot image on the board where all students can see it, and asks students to figure out how many dots there are.

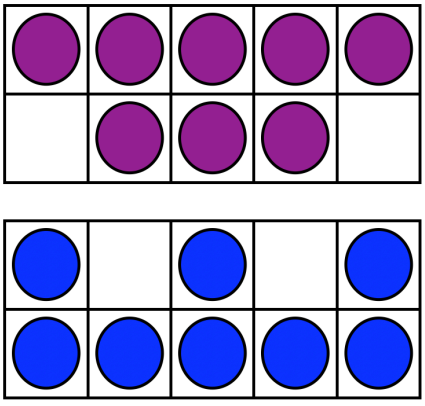
A common approach is to give students some time to think about the questions on their own first, and then share with a partner. Once students have had time to think the question through, lead a class discussion where students share their answers and approaches.

The key elements to these talks are a de-emphasis on speed and right answers and an added emphasis on process and communication. Expect some disagreements over the answers, and try to use those disagreements as a motivation for students to articulate their ideas to their classmates.

The Work

Students may have all kinds of approaches to count the dots, from counting by ones, twos, fives, or tens, to using the ten frames, to using the colors of the dots. Their conversations and arguments are the key, and the more you communicate that you're most interested in understanding what they're thinking (and writing it down so others can follow), the more they'll step up with more ideas to share.

In the image to the right, for example, one student might see "8 + 8," while another sees that if one dot is removed, the remaining make 3 rows of 5, meaning the total is one more than 15. Another students might imagine moving 2 dots from the bottom frame to the top to complete a set of ten, and thus see 16 total. Another might notice that the total is 4 less than 20.



Prompts and Questions

- Who would like to defend this answer?
- I don't quite follow. Do you mean I should count this group first?
- How did you do that/know that?
- Does anyone else think they can explain what Shawn is saying?
- Turn to the person next to you and explain how you counted.

Tips for the Classroom

1. Students will be looking to see if you indicate what the right answer is. Don't favor right answers over wrong ones. Make sure that the explanations are what matters.
2. A smart protocol is to have students put a thumb up at their chest rather than waving a hand in the in the air.
3. Give students constructive language to use in the discussion, like, "I respectfully disagree, because…" and "I agree with _____, because…"
4. Always keep the environment safe and positive.
5. Don't worry if you don't reach total consensus on every problem. Sometimes a student will need more time to process. You can move on when it feels like it is time.
6. Doing short (5 - 10 minute) Dot Talks regularly is more powerful than long ones infrequently. Do a maximum of two talks per day.

Counting Collections

Topics: Counting, skip counting, addition, multiplication (optional)
Materials: Paper, pencil, objects of many types (buttons, beans, stones, pencils, markers, blocks, etc.)
Common Core: 1.NBT.1, 1.NBT.2, 1.NBT.2.a, 1.NBT.2.b, 1.NBT.2.c, 2.OA.4, MP1, MP6

Kids love to count things. This is a simple exercise, but a great way to get kids excited about arithmetic. Counting Collections also seeds ideas about how the place value system helps make counting work.

Why We Love Counting Collections

So much of the work of teaching math in elementary school is about sharing arguments that make counting faster and more efficient. But why should the students care if they don't need to count anything? By making counting explicit, hands-on, and fun, this activity actually provides a motivation for the mathematics of place value (counting by ones, tens, and hundreds), addition (counting two or more smaller groups and adding them together), and multiplication (skip counting, arranging objects in arrays). Not only that, each new method and algorithm can be immediately tested in a concrete setting. Counting Collections are an indispensable tool for young children, and a foundational structure to return to again and again.

The Launch

Set collections of objects in different places in the room. Examples might include cups of legos, straws, blocks, pencils, buttons, markers, etc. Make sure that there are enough collections with a small numbers of objects (i.e., 11 - 30), as well as some with larger numbers.

Point out the collections around the room. Let students know that their job will be to take an inventory of the objects in the room—that is, to count how many objects are in each collection. In addition to recording how many objects are in a collection, they can make a sketch to remember how they got the answer. Demonstrate counting a small collection to show them how one might do it.

Students will work in pairs. Each pair will get to choose a collection and count it. For each collection, they will record what they counted, and how many objects were in that collection.

Main Activity

The students count and record their numbers.
The teacher can use this time in many ways. She can observe how students are counting, and take notes on the strategies they're using, and where they are in their developing understanding of numbers. She might distribute ten frames, rubber bands, cups, or other devices to help kids count or bundle objects.

Prompts and Questions

- What strategies are you using to count?
- What number do you find is easiest to count by?
- Can you tell how your partner is counting by looking at their picture?

The Wrap

You don't have to wrap up this activity after the first day; Counting Collections can be something to return to, to let students test their methods on larger and more difficult groupings of objects.

However, it can be nice to discuss different strategies along the way. After students have done their counting for the day, ask for reports on counts, and discuss strategies kids used for counting. Was it easier to count by 2s? By 10s? What other strategies did people have? Did all the groups who counted the same thing get the same answer? Which counting methods are most accurate? Which are easiest?

Tips for the Classroom

1. The recording forms for Counting Collections can get gradually more complicated. At first, the form might just contain a place for what and how many objects students counted. A more advanced form might contain an estimation (made before the count), space to record a drawing of how the students counted, or groupings of the number of tens and ones (or hundreds, tens, and ones) included in the final count. Best of all is if students add these elements themselves. However, some ideas for variants on a handout are at: mathforlove.com/lesson/counting-collections.
2. You can differentiate Counting Collections most easily by including collections with many or fewer objects to count.
3. For a more advanced (but less hands-on) launch, you can try asking students what else they see in the room that they might count. They might list things like windows, chairs, tables, ceiling tiles, as well as blocks, markers, etc.
4. Don't take too long with the launch of Counting Collections. The students will catch on quickly, and too much instruction on how to do the counting may get in their way.

Name_____

Counting Collections

I am counting: _____

Picture of how I counted

Match the Dots

Topics: Counting and cardinality, subitizing, recognizing numbers
Materials: Tiny Polka Dot cards
Common Core: K.CC.4, K.CC.5, MP6, MP7

The numbers look different. Which ones are the same?

Why We Love Match the Dots

This simple game of counting and matching helps students see how different-looking collections of dots may actually represent the same number.

The Launch

Choose a student volunteer to demonstrate the game, or put the cards where everyone can see them (center of a circle, document camera) and play with the entire class, where you find a match, and then the students raise their hands if they see a match.

The rules are simple: deal a collection of cards face up. On your turn, simply find two cards of the same number, and remove that pair from the board. There aren't really winners or losers in this game—you just play taking turns until all the cards are gone. For early games, pick just two suits at a time, and start with numbers the students know. For a first game, you might pick Teal 0-4 and Blue 0-4. In subsequent games, students can play with larger numbers and additional suits.

Prompts and Questions

- How do you know those two cards have the same number of dots?
- I'm going to pick this card. It has two dots. Do you see a card that matches it?
- Are you sure those two cards are the same? Let's count them together.

The Wrap

Ask students which cards were easiest and hardest to count.

Tips for the classroom

1. Start students with very easy numbers, and slowly let them increase the difficulty. Every step up should feel like a fun new challenge. "Do you think you're ready to add in some sixes?"
2. The game is more fun if you're using an even number of suits, so every card gets taken in the end.
3. This game evolves into Dot Memory and Dot Fives. If students are ready for a greater challenge, you can show them those games.
4. If students aren't ready for Match the Dot, you can let them play Hungry Numbers, or just play with the cards, or organize them into color piles.

Day 3

Goals

1. Practice addition and explore counting strategies.

Warm Up (optional)

Block Free Play

Opener

Counterexamples

Activity

Pattern Block Triangles

Game

1-2 Nim

Choice Time

Block Free Play
Match the Dots
1-2 Nim
Counting Collections

Closer

Ask students to share a conjecture that they had today that got broken by a counterexample. It could involve pattern blocks (you can't built a triangle with 7 blocks) or 1-2 Nim (You can win if you start with an odd number of counters). Underline the point that by making conjectures and having them disproved, they've made a major leap forward in their learning and understanding!

Counterexamples

Topics: logic, deduction, mathematical argument, communication
Materials: None
Common Core: Variable, but especially MP3

Prove the teacher wrong. Rigorously.

Why We Love Counterexamples

Every kid loves to prove the teacher wrong. With Counterexamples, they get to do this in a productive way, and learn appropriate mathematical skepticism and communication skills at the same time.

It is possible to play Counterexamples with kids as young as kindergarteners as a kind of reverse "I Spy" ("I claim are no squares in this classroom. Who can find a counterexample?"). What's great, though, is that you can transition to substantial math concepts, and address common misconceptions. Counterexamples is a perfect way to disprove claims like "doubling a number always makes it larger" (not true for negative number or 0) or sorting out why every square is a rectangle, but not every rectangle is a square. For older kids, you can even go into much deeper topics, like: "every point on the number line is a rational number."

The language of counterexamples is crucial to distinguish true and false claims in mathematics; this game makes it natural, fun, and plants the skills to be used later. Counterexamples is also a great way to practice constructing viable arguments and critiquing the reasoning of others.

The Launch

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher, though it can be a student) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All _____s are _____" or "No _____s are _____." You can also play around with statements like "If it has _____, then it can _____." For instance:

It's often best to start with non-mathematical examples.

- All birds can fly. (Counterexample: penguins)
- No books have pictures in them.
- All books have pictures in them.
- If something produces light, then it is a light bulb.
- If something has stripes, then it is a zebra.

Once students have the hang of it, you can make the examples more mathematical.

- There are no squares in the room.
- Odd numbers have to have only odd digits (counterexample: 21)
- Even numbers have to have have only even digits (counterexample: 12)
- If a shape has all sides the same length, then it is a square (counterexamples: equilateral triangle; rhombus; many more)
- Adding makes number larger (counterexample: adding 0. Also, -1, etc.)
- You can't make a triangle using 2 or more pattern blocks. (This is a great transition into Pattern Block Triangles.)

Example
Teacher: I claim all animals have four legs. Who can think of a counterexample?
Student 1: A chicken!
Teacher: Why is a chicken a counterexample?
Student 2: Because it has two legs.
Teacher: Right. I said every animal has four legs, but a chicken is an animal with just two legs. So I must have been wrong. Let me try to refine my conjecture then. I should have said that animal must have 2 or 4 legs. That feels right.
Student 3: What about a fish?
Teacher: Aha. A fish is an animal with no legs. Thank you for showing me the error of my ways. What I should have said is that animals have *at most* four legs.
Student: 4: What about insects?
And so on.

When the moment feels right, you can transition into Pattern Block Triangles (see next lesson) by posing the conjecture:

"All triangles you can make from pattern blocks are green."

One counterexample is a triangle made from a trapezoid and a green triangle. From there, you can refine the counterexample to include numbers, e.g.:

"You can make a triangle from one or two pattern blocks, but no more than that."

This should provide some good momentum for Pattern Block Triangles! Keep the language of conjectures and counterexamples in mind for the rest of today, and throughout this program.

Tips for the Classroom

1. It's good to make up false conjectures that are right for your students. Start simple.
2. Kids can think of their own false claims, but sometimes these aren't the right kind, and they often have to be vetted.

3. Once you introduce the language of counterexamples, look for places to use it in the rest of your math discussions!

References: http://kuow.org/post/getting-kids-interested-math-without-their-knowing

Pattern Block Triangles

Topics: Addition, counting, geometry
Materials: Pattern blocks, scratch paper and pencil
Common Core: K.CC.4, K.CC.5, K.OA.1, K.OA.2, 1.OA.1, 1.OA.2, , 1.OA.4, 1.OA.5, 1.G.2, 2.OA.1, MP1, MP2, MP6, MP7, MP8

How many blocks can you use to build a triangle from pattern blocks?

Why We Love Pattern Block Triangles

This beautiful lesson combines student creative work, counting and addition practice, combining geometric shapes, and a slow build from easier to more challenging work.

The Launch

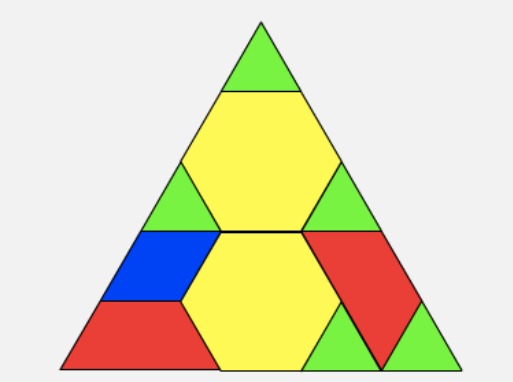
Prep the pattern blocks beforehand by removing the tan rhombuses and orange squares, or else tell students not to use those blocks.

Make sure every student has access to pattern blocks, and start with two warmup challenges: can students build a (filled in) triangle with pattern blocks using

1) Exactly 5 blocks?
2) Exactly 10 blocks?

Choose one student's work and show how to double-check the count by counting each type of block, and then adding those together, using an equation or a ten frame as necessary. For example, if you were using the triangle below as an example, you might write:

2 hexagons + 2 trapezoids + 1 rhombus + 5 triangles = 10 blocks



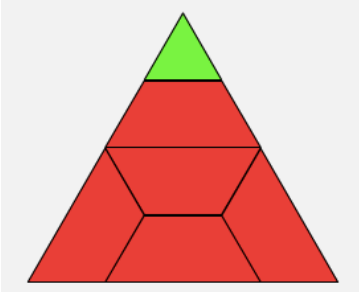
Or simply 2 + 2 + 1 + 5 = 10.

Once students are done with the warmups, pose a greater question: is it possible to build a triangle with whatever number of blocks you want? Can you build one with 2 blocks? With 3 blocks? 4 blocks? How far can you go?

Write a list of numbers from 1 to 20, and have students make their own list of numbers. Leave enough space so students can write an equation for each number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

If students can make a triangle using some number of blocks, they can write an equation for that number. For example, when they make a triangle using exactly 6 blocks, they can write an equation describing that triangle.
For the triangle to the right, for example, the equation might be 6 = 5 + 1.



The big challenge for students is: can they build a triangle for each number from 1 to 20? Or is there any number that they won't be able to build?

Students can work alone or with partners to build different triangles.

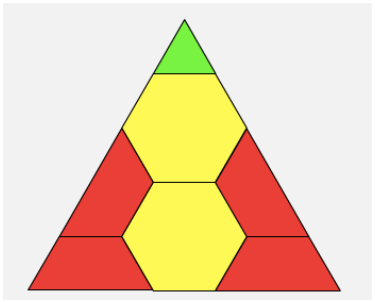
Prompts and Questions

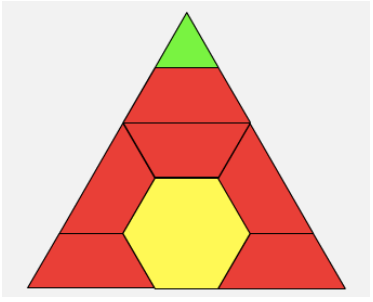
- How many blocks did you use in that triangle?
- Did you record it yet? Show me the equation for that triangle.
- Have you built a triangle with 11 blocks yet? How did you do it?

The Wrap

There is a powerful idea to underline as you wrap up this activity, which is that you can substitute smaller blocks—say, two trapezoids in the place of one hexagon—to raise the number of blocks you used in a triangle without changing anything else. Demonstrate this kind of substitution on a specific triangle, and then ask students if they can predict how making this substitution will change the number of blocks. For example:

2 hexagons + 4 trapezoids + 1 triangle = 7 blocks



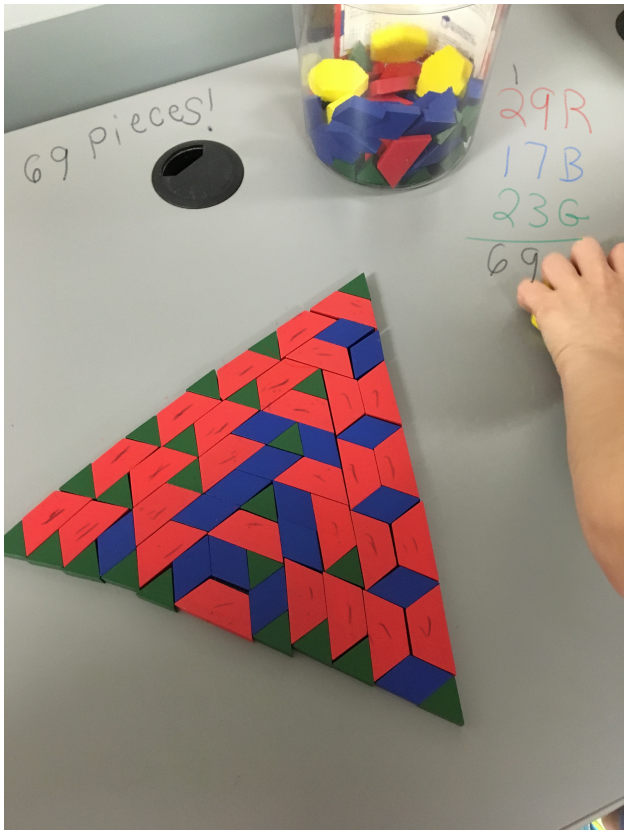


1 hexagon + **6** trapezoids + 1 triangle = **8** blocks

Can students use this idea to make triangles using any of the missing numbers from their list?

Tips for the classroom

1. The idea in using the list is that every triangle is a success to start, and then certain holes in the list become more challenging to make.
2. There are two excellent ways to challenge students who successfully make all the triangles from 1 to 20. First, ask them if they can make a larger number, say, 31.
3. Second, challenge them to make a much larger triangle, and show you how they can correctly add all the pieces inside it.
4. For today, “triangle” means triangles made out of pattern blocks with no empty spaces inside. Alternative definitions may pop up from students. Let them know that for today, we just mean triangles of this type.
5. For young students, you can just go from 1 to 10 instead of 1 to 20, and not require writing equations for each triangle.



1-2 Nim

Topics: logic, patterns, addition, counting, subtraction
Materials: Counters (tiles, beans, pennies, etc.) or paper and pencil
Common Core: 1.OA.4, 1.OA.5, 2.OA.2, MP1, MP2, MP3, MP5, MP7, MP8

You can take one or two counters from the pile. How do you get the last one?

Why We Love 1-2 Nim

Nim is fun, challenging, and rewarding for a wide range of kids. Done right, it can engage everyone from Kindergarten to upper elementary kids, and connect to basic counting and arithmetic up to division. Completely unlocking the game is an exciting and powerful achievement for a student.

The Launch

Take a volunteer to play a demonstration version of the game. You can explain the game to them very quickly and get to playing (which is how the other students will learn the game, as they watch). It's a good idea to get students to suggest moves to you as you play —quietly, by holding up one or two fingers. Ideally, you should win the initial games. See the video at mathforlove.com/video/rich-task-1-2-nim-lesson-plan-with-dan-finkel for more ideas.

How to Play

Nim is a two-player game. You start with a pile of 10 counters. On your turn, remove one or two counters from the pile. You must take at least one counter on your turn, but you may not take more than two. Whoever takes the last counter is the winner.

Example Game

Player 1 takes one counter, leaving 9.
Player 2 takes one counter, leaving 8
Player 1 takes one counter, leaving 7.
Player 2 takes two counters, leaving 5.
Player 1 takes one counter, leaving 4.
Player 2 takes one counter, leaving 3.
Player 1 takes one counter, leaving 2.
Player 2 takes two counters, leaving 0 and winning the game.

The Work

The central question here is: how can you win 1-2 Nim? What would a perfect strategy look like? Encourage students to make conjectures as to how they think they can win every time. Then put the strategies to the test! Can they truly win no matter what their opponent does?

Playing in trios is a nice structure here. Two players can play while the third observes. Then they switch roles. That prevents a pairing from feeling stale after a few minutes.

Possible student conjectures, true and false, that may arise:
- Whoever goes first wins.
- Whoever goes second wins.
- Odd vs. Even determines your strategy.
- It matters/doesn't matter what you do until there are less than six counters in the pile.
- Whoever can give their opponent four open spaces wins.

A key idea is that you can play with fewer counters in the pile, and that actually makes it easier to think about what to do. Gathering the students and challenging them to games with fewer counters (they can request how many fewer) is a nice way to reengage everyone.

For students who figure out the game, some challenge questions:
- How would you win 1-2 Nim if you start with a pile of 20? 30? 100? 217?
- These challenges can also apply to 1-2-3 Nim and Poison variations.

1-2-3 Nim: players may take one, two, or three counters per turn.
Poison: Whoever takes the last counter loses.

Prompts and Questions
- What move should I (the teacher) make?
- How did you/they/I win that game?
- What do you think your/my opponent will do if you/I take two counters?
- Would you like to take back your move?
- What have you noticed about this game?

The Wrap
Students will likely be in different places with respect to their strategy by the end of the lesson. You can close by asking for another challenger to play you and see if they can beat you. If they can, the class will be elated. If they can't, that means there's still more for them to figure out before they become Nim Masters.

Tips for the Classroom
1. Demonstrate the game with volunteers for at least three games (or many more!), until you are certain everyone understands it and is excited to play.

2. When demonstrating 1-2 Nim, narrate the game out loud, using mathematical language, and leaving empty space for students to chime in: "My opponent just took 2 leaving... [wait for students] 5 in the pile. Who has advice for what I should do next?"

3. Remind students that they will lose many games as they play, and that every loss is an opportunity to learn. Can they steal the strategy of the person who just beat them? Point out how students are trying out new strategies as they play you in demonstration games.

4. As kids play each other, circulate to see what strategies they are developing. Challenge them to play you, and see if they can beat you.

5. Encourage student conjectures, but do not call them as true or false. Challenge students to break their own conjectures.

6. This game is great for station work once students already know how to play it.

7. We use the term "the 3 trap" to describe what happens when you give your opponent a pile of three counters. Understanding how to win boils down to understanding what pile sizes you want to leave your opponent with.

8. There are two incredibly powerful approaches to solving Nim. We'll discuss them here, BUT don't be in a rush to push the kids to find them. Help students only as far as they seem ready to go, and if for many weeks, or even the entire summer session, they never articulate a correct answer to the question of how to win at Nim, they'll still get lots of beneficial mathematical practice by simply playing the game.
   The first approach is to simplify. How could the game be easier? What if the pile had only one counter? From this place of almost absurd simplicity, we slowly raise the difficulty. What about two counters? Three counters?

   The second approach is to organize the data in a coherent way. A table does this very nicely.

Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second.
4	?
5	?

9. We discourage a three-player game. Normally trying out different numbers of players is a great impulse. In Nim, it leads to spoilers, who can't win, but can choose who does win.

10. Optional Homework: teach 1-2 Nim to a friend or family member.

Day 4

Goals

1. Play games and activities to count, sort, and compare numbers.

Warm Up (optional)

Block Free Play

Opener

Target Number

Activity

Handfuls of Snap Cubes

Game

Dot Memory

Note: Dot Memory is exactly like Match the Dots, except that instead of playing with the cards face up, we deal them face down. Take turns turning up two cards, and keep them if you turn up two that are the same number.

Choice Time

Block Free Play
Match the Dots and/or Dot Memory
1-2 Nim

Closer

Ask students how many snap cubes they guess you can grab if you take three handfuls. Let them make their guesses and discuss why they made them with peers and then with the class. Then take three handfuls, one after another, counting how many snap cubes you get each time. It might be 9, then 8, then 11. Record what you get on the board, and have volunteers arrange the snap cubes in groups of 5 or 10. Then find out what number you actually grabbed. Did anyone get close?

Repeat if time permits.

Target Number

Math concepts: Arithmetic, equivalencies
Equipment: pencil & paper
Common Core: Variable, but especially OA, i.e., 1.OA.C.5

You know the answer. What's the question?

Why We Love Target Number

This is a quick check-in that adjusts to the abilities of each student, allows for creativity and arithmetic practice together, and is a lot of fun. Target Number is a perfect warm-up.

Launch

The teacher writes a "target" number on the board. The students try to write down as many different equations that have that the target number as the answer. Then students share their favorite answers. For younger students, drawing different pictures or arrangements of ways to see/understand that number is an okay alternative.

Example
The teacher writes 7 on the board, and lets kids write on their own paper for about a minute, then asks students to share what they found. Students raise their hands to volunteer solutions while the teacher writes them on the board. These equations may go from simple equations like 6 + 1 = 7 to the more complex (4 x 3) - 5 = 7. The great thing is, anyone can start, but the sky is the limit!

Prompts and Questions

Don't do these the first time you do Target Number, but if you want to go deeper with this activity in the future, here are some interesting questions to pursue.

- If we only add 2 numbers, how many answers can we find?
- What if we add 3 numbers, or 4 numbers?
- What about any number of numbers?
- What if we only subtract, or only multiply, or only divide?
- What's the longest number sequence you can find that hits the target number?
- Can you hit the target number if you only use a single number, such as the number 4, in your equation?

Tips for the Classroom

1. Don't praise answers with many steps as "smart." This activity gives everyone a chance to contribute and be valued. You can describe those answers as "long," or as having many parts.
2. If answers are wrong or unclear, you can take the opportunity to do the arithmetic with the class. On the other hand, if a student uses terms (like square root) that the

class isn't ready for yet, you can write down their answer but move on to other solutions.

3. One opportunity this lesson gives you is the chance to emphasize equivalency. If one student knows that 6+1 = 7, and someone else knows that 3 + 3 +3 - 2 = 7, then that means that 6 + 1 = 3 + 3 + 3 - 2. It's nice to underline the point that there are many ways to equal 7, and that these ways are all equal to each other.
4. Let's say someone says that 7 = 5 + 3. Rather than just saying "wrong," say that 5 + 3 gets us close to 7, but we need to do something else to get all the way there, then challenge students to find what still needs to be done. If someone can explain that 5 + 3 is 8, and so you need to take 1 away, you have the equation 7 = 5 + 3 - 1. This is both more sophisticated and accepts the original students wrong answer as a path toward a better, accurate answer, rather than a dead end.
5. Sometimes kids offer new ways of making the number with units, i.e. if the target number is 7, a student might say "3 bunnies plus 4 bunnies is 7 bunnies". Depending on the class, once this has begun it is very seductive to the other kids, and can sometimes derail the mathematics. Be open at first, but if the game becomes more about picking different units than finding different expressions, you can say, "Okay, for the next 3 solutions, let's use numbers by themselves."

Handfuls of Snap Cubes

Math Topics: Geometry, addition, counting, estimation, symmetry
Materials: Snap cubes, ten frames, square tiles, pencil and paper
Common Core: 1.OA.1, 1.OA.5, 1.NBT.2, MP1, MP6, MP7

How many cubes can you hold in your hand?

Why We Love Handful of Snap Cubes

Snap cubes give a powerful and concrete way to understand counting, grouping, addition and subtraction. This lesson is a good introduction to snap cubes and a fun & engaging first activity for estimating and counting.

The Launch

Gather the students together, and ask for a volunteer. Their goal is to predict how many snap cubes they can grab in one hand. Once they make a guess, write it down together, then let them reach into a box of snap cubes with one hand and pull out as large a handful as they can. (The snap cubes in the box should be disconnected ahead of time.) Did they guess above or below what they grabbed? Or did they get it exactly? Use a ten frame to make the counting clearer. Count the number they actually got, and write that number down too.

Repeat for several more students. Then let students try on their own. They should write down their guess first, then count and write down how many they actually pulled out. Which number is bigger?

As the activity progresses, move to harder challenges as the students are ready for them.
Challenge 1: How many snap cubes can you grab in one hand?
Challenge 2: How many snap cubes can you grab in two hands?
Challenge 3: How many square tiles can you grab in one hand?
Challenge 4: How many square tiles can you grab in two hands?

Prompts and Questions

- How many did you get last time? How many do you think you'll get this time?
- How are you counting the cubes?
- Let's count them together.
- I bet they would be easier to count if they were straightened up. Let's organize them in a row/on a ten frame.

The Wrap

(If developmentally appropriate for your students.) Gather the students together again and show them the mathematical symbol ">" and how to use it to compare numbers that are greater or smaller than each other. Write down a guess (i.e., 12) and then how many

you actually picked (8), and show how to write a number sentence 12 > 8, to show that 12 is bigger than 8.

Tips for the classroom

1. Once students are accustomed to the activity, you can let everyone do the activity at once and compare their results.
2. If students are ready for greater challenges, have them pair up: how many cubes/ tiles can two students grab together?
3. Another excellent way to extend this activity is to try grabbing other materials. How many pattern blocks/erasers/etc. can students grab in one or two hands?

Day 5

Goals

1. Explore addition and subtraction.
2. Solve story problems.

Warm Up (optional)

Block Free Play

Opener

Dot Talk - see Appendix 4 for Dot Talk images

Activities

Mini-lesson on Story Problems
Story Problems—In the Garden

Game

Dots and Boxes

Choice Time

Block Free Play
Match the Dots and/or Dot Memory
1-2 Nim
Dots and Boxes
Counting Collections

Closer

Ask students to make up a story problem to match the equation 14 - 5 = 9.
[Example: there were 14 crows. 5 flew away. How many were left?]

Let them share their stories with each other, and take a few volunteers to share theirs with the group.