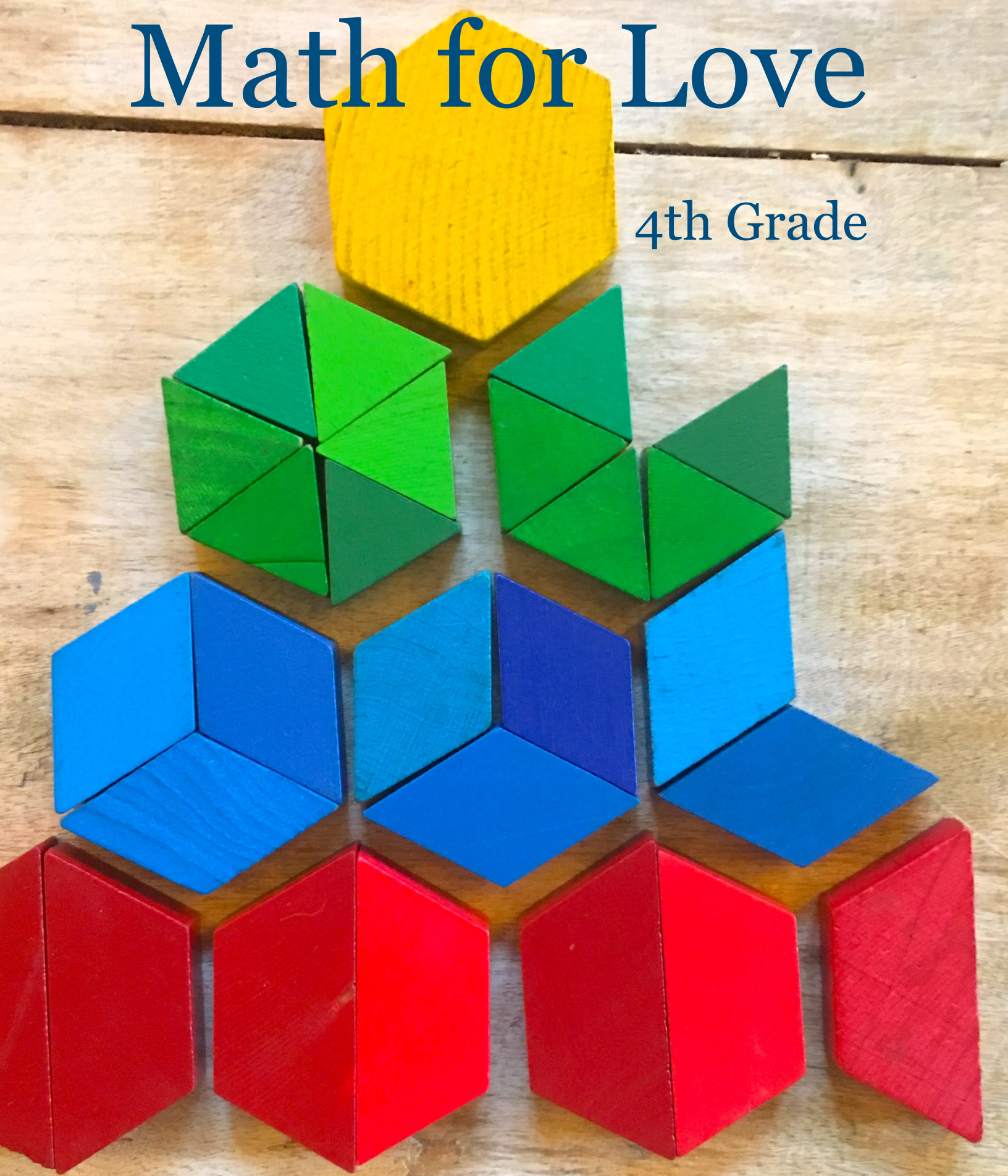
Math for Love

4th Grade



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A word about using this book

This book was designed to support a summer math program lasting sixteen 75 - 90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. Games can be played for 5 - 30 minutes. Deeper tasks might be good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this curriculum that should be helpful: mathforlove.com/video/math-for-love-video-pd.

Enjoy!

A word about the copyright

We want this book to be used by teachers to help students explore math in a positive way. Feel free to make photocopies, share ideas with parents and colleagues, and use this as a resource draw on. In general, we support this kind of fair use of our materials. Please don't post elements from this book online without citing the source, share large chunks of the book electronically, or sell parts of the book anyone.

Table of Activities

Where to find each activity the FIRST time it's used in this curriculum

Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

Goals of the Math For Love Curriculum

We wrote this program to be both *play-based* and *rigorous*. The goals of the program are two-fold:

- Improve conceptual understanding of and fluency in mathematics
- Give everyone an opportunity to have fun and enjoy math

Many students haven't had enough time working with conceptual models of mathematics before being pushed into abstraction. To remedy this, the curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorially while making connections to abstract reasoning.

Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement**. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation**. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.

3. **Thinking**. Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. **Positive Environment**. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Be ready with questions**
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value *their* thinking and contributions, not just the answer.

★ **Model how to play games, and teach how to win and lose**
Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Unit Chat, Fraction Talk, or opening game (see the Opener in the daily plan). Use the Math Games and Station Breaks for transitions between Activities. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Have a *growth mindset* classroom**
Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you *learn* to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Model learning from mistakes**
One important way to encourage a growth mindset is to understand mistakes as necessary. They are a natural part of learning, and even more than that, they actually

help us learn more and help us remember what we've already learned. Model for your students how to make mistakes, and how to use mistakes productively. This could be as simple as thanking students when they notice a mistake that you have made.

★ **Give your students *time* to think and explore**
Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**
We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

★ **Value play**
It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less, you can end sooner. If you need something longer, you should find many of the activities extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- *Goals*
- *Opener*
- *Activity*
- *Game*
- *Choice Time*
- *Closer*

Goals

These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students.

Opener

The Opener is the first activity of math class. The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

In general, the Opener should last about 5 - 10 minutes.

Activity, Game, Choice Time

Following the opener, there is a suggestion for an activity, a game, and Choice Time. This is where the bulk of class time will be spent. There are two recommended ways to approach these three elements.

1. Have students rotate between three stations. This is especially recommended when you have additional adults (instructional aides, parent volunteers, tutors) in the room aside from the teacher.
2. Take the whole class through the activities one by one. This is recommended when the teacher is the only adult in the classroom.

Either way you run your classroom, the elements are designed to give students the maximum opportunity to think & engage, practice skills, explore questions, and have fun.

Choice Time includes a suggestion of a small group of past games and activities for the students to try. This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

Closer

The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer should take 5 minutes or less.

Other Notes and Best Practices

★ **Math Breaks and Physical Games**
Check out the math-based movement breaks in **Appendix 2**. These are excellent as transitions.

★ **Folder for Worksheets**
Give each student a folder where they can keep their worksheets. If they finish an activity early, they can turn back to their unfinished worksheets and finish them. They can also work on them during Choice Time.

★ **Choice Time**
Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should both choose the activity that is right for them, and then stick with it for at least half of the time.

★ **Challenge Problems**
Challenge problems (see **Appendix 3**) are great options for Choice Time any day. Offering “spicy" variations of worksheets or unfinished activities as Choice Time activities can be another nice option.

★ **Station Transitions**
If you use stations, provide 1-2 minute warnings before station transitions, to apply a gentle transition, cleanup, and—especially at Activity 1—a brief reflection or wrap-up. If you use an alarm, make it a gentle sound (i.e., a gong) rather than an abrasive one (i.e., a clock radio alarm).

★ **Games to send home**
See **Appendix 1** for games to send home. These will help parents/guardians and students play math games at home. You can always send other favorite games home, or encourage students to share games they've learned with people at home. Note that there is no homework for this program otherwise.

Day 1

Goals

1. Establish class norms and community.
2. Collect data (briefly) with a preassessment.
3. Connect counting, addition, multiplication, and equations.

Opener

Penny Nickel Dime

Activities

1) Preassessment
2) Forty Faces

Make sure to let students know that this preassessment is not a “test," and not something they’re expected to know any or all of the answers to. It’s just a way for you, the teacher, to see what ideas they are familiar with, so you can make sure you keep them challenged and interested. They definitely shouldn’t worry if they can’t get all, or even most, of the answers. (We’ve included some very challenging questions!) So just tell them to do their best and not to sweat it.

Note: Try using Cuisenaire rods with students after you try it with pattern blocks.

Game/Puzzle

Pig

Choice Time

Pig
Penny Nickel Dime
Free play and exploration with pattern blocks, Cuisenaire rods, and other manipulatives.

Closer

Ask students to make a list (with a partner, a trio, or on their own) of traits that you need to do math. When they’re done, discuss their lists and their ideas.

In the discussion, ask them to consider the activities of the day. There was a lot of math in forty faces, but also a lot of choice and creativity. There was a lot of math in Penny Nickel Dime and Pig, and they're also both games, and hopefully are fun to play! How does this fit in with their conception of what math is, and what you need in order to do it well?

Close by letting them know that this program is designed to have them playing and exploring a lot, and also thinking deeply. The most important thing they'll need to know is that getting frustrated sometimes is part of the process, and if they can keep engaged and playing and thinking, they'll learn what they need to learn, and get better at what they're doing.

Penny, Nickel, Dime

Topics: Addition to 100, multiplication by 5 and 10, money, estimation
Materials: One 6-sided dice, pencil and paper, pennies, nickel, and dimes (optional)
Common Core: 3.OA.3, 3.OA.5, MP1, MP6, MP7

Roll the die 7 times. For every roll, you get to take that many pennies, nickels, or dimes. Whoever gets closest to $1 without going over wins.

Why We Love Penny Nickel Dime

This version of Don't Break the Bank is a hit with students, and also helps give a concrete meaning to place value by linking it to both money, addition, estimation, and multiplying by fives and tens. It takes very little time, so it can be used as a warmup, station activity, or in those five minutes before class ends. While kids may break the bank their first few games, they'll inevitably start estimating and choosing good strategies for themselves.

How to Play

Whole Class/Group Game: The teacher (or a student) rolls the die. Whatever number it lands on, each player can choose to take that many pennies, that many nickels, or that many dimes. More practiced players can just record the numbers in a T-chart like the one below. Repeat for six times total, with each player choosing whether each number goes in the dimes or pennies column as you go.

The winner is the person who comes as closest to $1 without going over.

Partner Game: Same as whole class game, except students take turns rolling the die, and everyone ends up entering different numbers into their grid.

Tips for the Classroom

1. Have students draw a chart (see following page) to track their choices.
2. When playing in stations or with a small group, you can demonstrate how to count your total after the game. Students can also check each other's work in pairs.
3. Let students bust (go over $1) as they refine their strategies. They'll catch on as they find they end up too high or too low.
4. To keep the game novel, ask students what they'd do differently if there were only 5 rolls per game, or six. Or eight! Try those variations and see what happens.
5. For students who need more help, you can play with physical coins, or have them draw the coins. Other students may only need to count how many of each coin they take on a turn.
6. Downlevel the game by removing nickels as an option. Uplevel the game by adding quarters.

Penny, Nickel, Dime

Roll	Dimes	Nickels	Pennies
1			
2			
3			
4			
5			
6			
7			
Totals			

Penny, Nickel, Dime
[Sample Game]

Roll	Dimes	Nickels	Pennies
1	4		
2		2	
3		3	
4			6
5			1
6	1		
7			5
Totals	5 dimes	5 nickels	11 pennies

50¢ + 25¢ + 11¢

= 86¢

Name_____

Preassessment

Fill in the blanks.

1) $6 \times$ _____ $= 96$

2) $84 \div 7 =$ _____

3) Cora gets eight pennies each day for sixteen days.
How many pennies does Cora get?

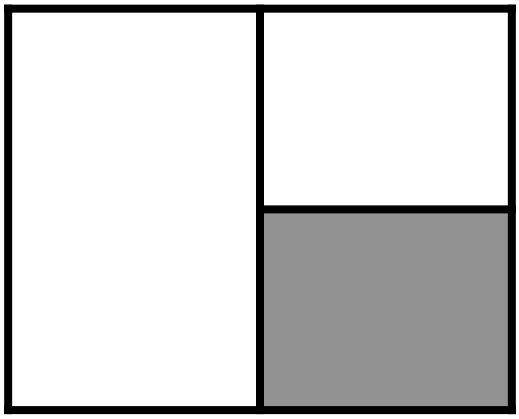
Explain with equations, words and/or pictures.

Name_____

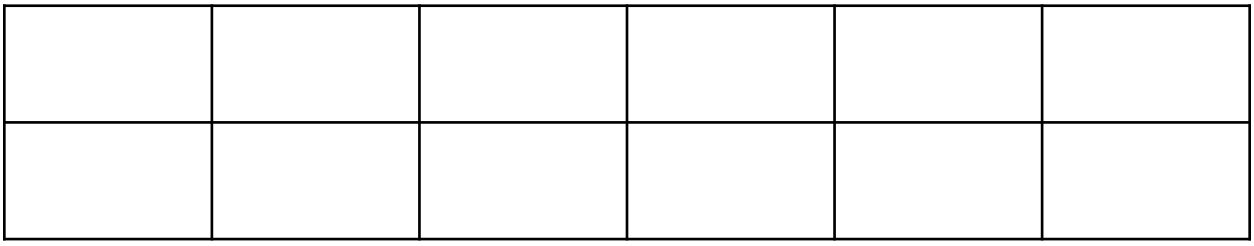
4) I had 6 cartons of eggs. Each carton had 12 eggs.
Then I cooked 15 eggs.
How many eggs were left?

Explain with equations, words and/or pictures.

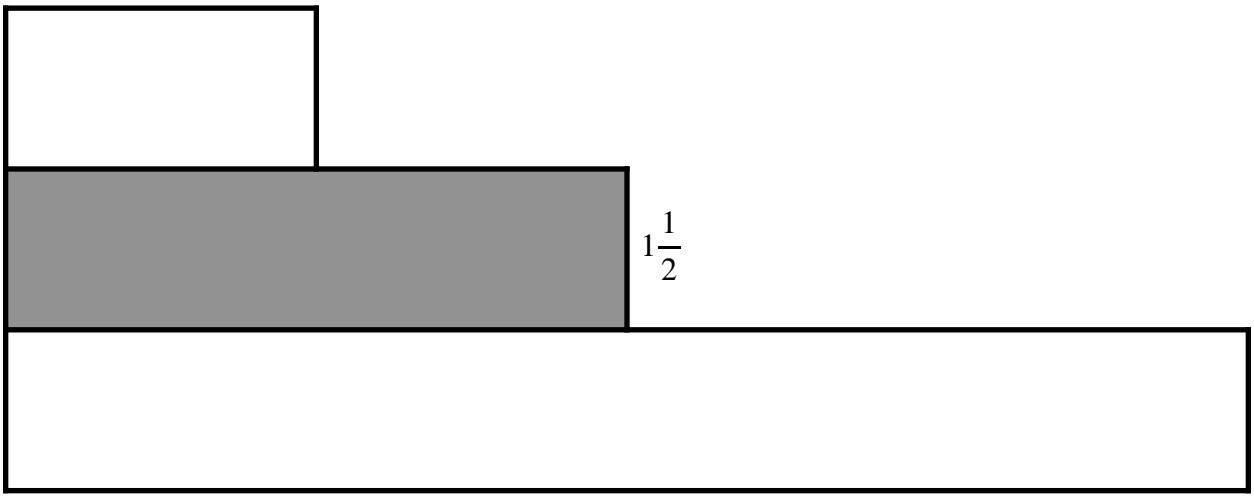
5) What fraction of this image is shaded? _____



6) Shade in $\frac{3}{4}$ of the rectangle.



7) The shaded area is $1\frac{1}{2}$. Label the other two regions.
One is half the shaded region. The other is double.



8) $12 \times$ _____ = _____ + 15

Fill in the blanks to make the equation true.

9) There are 41 adults and 35 children going to the festival.
Each van can hold 6 people.

How many vans do they need to get everyone to the festival?

Explain with equations, words and/or pictures.

10) Gum balls cost 4¢ each, and licorice costs 9¢ each.
I want to buy 23 gum balls and 12 pieces of licorice.
I have 5 quarters and 7 dimes.

Can I afford to buy the candy I want? Explain with equations, words and/or pictures

Name_____

Preassessment
Solutions and Rubric

Fill in the blanks.

1) $6 \times$ _____ $= 96$

10 points for correct answer: 16.

2) $84 \div 7 =$ _____

10 points for correct answer: 12.

3) Cora gets eight pennies each day for sixteen days.
How many pennies does Cora get?

Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing
5 points for correct answer: 128 pennies
Subtract 1 point if there are no units in final answer

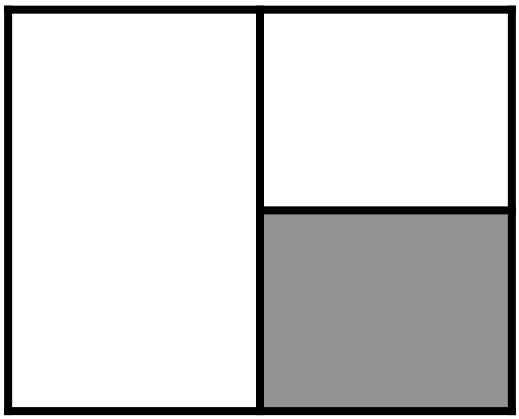
4) I had 6 cartons of eggs. Each carton had 12 eggs.
Then I cooked 15 eggs.
How many eggs were left?

Name_____

Explain with equations, words and/or pictures.

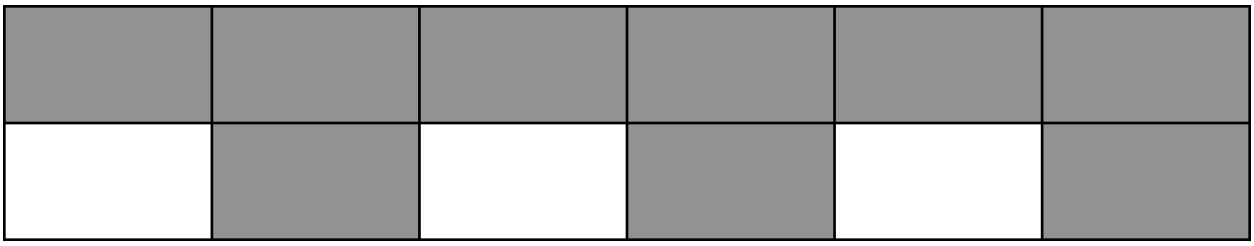
5 points for clear, correct explanation and/or drawing
5 points for correct answer: 57 eggs
Subtract 1 point if there are no units in final answer

5) What fraction of this image is shaded? _____



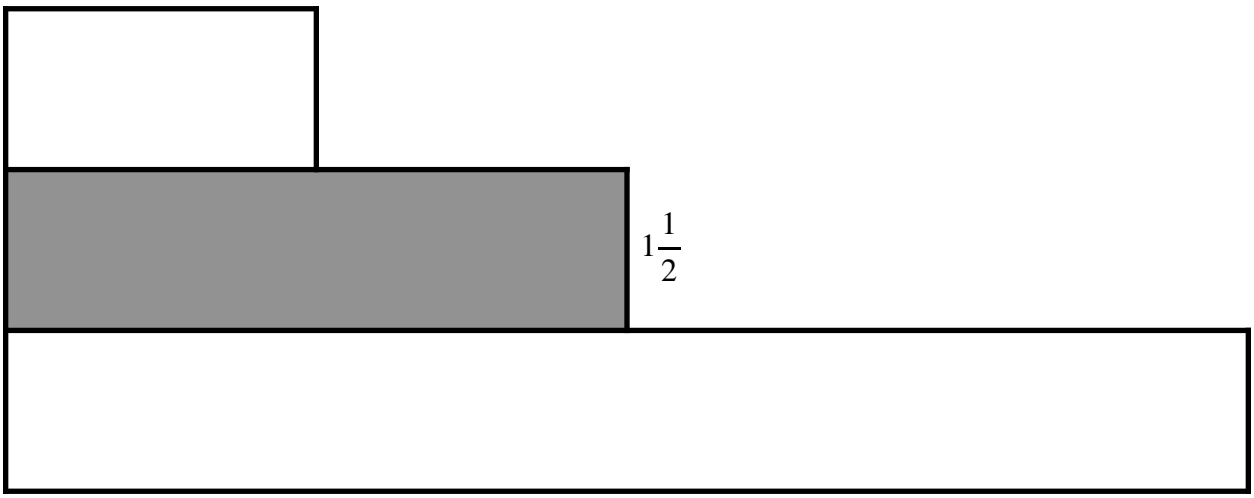
10 points for correct answer: 1/4.

6) Shade in $\frac{3}{4}$ of the rectangle.



10 points for any correct shading.

7) The shaded area is $1\frac{1}{2}$. Label the other two regions.
One is half the shaded region. The other is double.



5 points for each correct answer.
Top bar is 3/4. Bottom bar is 3.

8) 12 × _____ = _____ + 15

Fill in the blanks to make the equation true.

10 points for a correct answer. For example:

12 × 2 = 9 + 15

9) There are 41 adults and 35 children going to the festival.
Each van can hold 6 people.

How many vans do they need to get everyone to the festival?

Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing
5 points for correct answer: 13 vans
Subtract 1 point if there are no units in final answer

10) Gum balls cost 4¢, and licorice costs 9¢ per piece.
I want to buy 23 gum balls and 12 pieces of licorice.
I have 5 quarters and 7 dimes.
Can I afford to buy the candy I want?

Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing
5 points for correct answer: Total cost of candy is $2.00. Total money is $1.95. So I cannot buy all the candy I want.

Subtract 1 point if there are no units in final answer.

Forty Faces

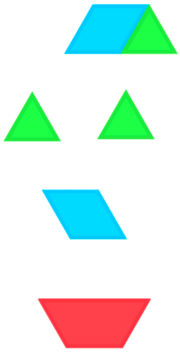
Topics: Addition, subtraction, skip counting, multiplication, logic
Materials: Pattern Blocks, Scratch paper and pencil, Cuisenaire rods (optional)
Common Core: 3.OA.3, 3.OA.8, 3.NBT.2, MP1, MP6, MP7

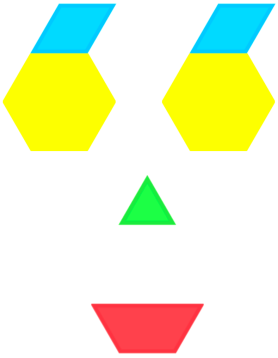
Why We Love Forty Faces

This delightful challenge provides an artistic exploration of ways to construct numbers by repeated addition or multiplication.

The Launch

Prepare the pattern blocks so they contain only green triangles, blue rhombuses, red trapezoids, and yellow hexagons. Ask students how many triangles it takes to build the blue rhombus (2), the red trapezoid (3), and the yellow hexagon (6). Then show them the faces below, either by building them or by projecting images of them.





Briefly discuss how these faces are made by putting together the equivalent of 10 or 20 triangles worth of area. For the second face, for example, there are 2 hexagons, 2 rhombuses, one trapezoid, and one triangle. In terms of triangle area, the total "value" would be 12 (in hexagons)+ 3 (in trapezoids) + 4 (in rhombuses) + 1 (in triangles) = 12 + 3 + 4 + 1 = 20 triangles worth of area.

Once students understand how to count the "value" of the face, challenge them to create their own faces from pattern blocks that have value (i.e., area) 10, 20, 30, or 40.

Prompts and Questions

- How much more area do you need to add to get to 30?
- Show me how you found the area.
- Let's count how much the hexagons are worth.
- The trapezoids came to 18 area? Let's write that down.
- Do you think the two of you could make a face with an area of 75?

The Wrap

Share a face that almost has area forty. Find its area/value with students, emphasizing the possibility of skip-counting or multiplying to find the value of specific blocks. Once everyone agrees on the area of the figure, take student suggestions for how it could be adjusted to come to forty exactly.

Tips for the Classroom

1. Remove the orange squares and tan rhombuses from the pattern blocks before the lesson begins. This is optional, but can be helpful.
2. Let students challenge themselves when they're ready. Can they make a "100 face"?
3. Encourage students to use pencil and paper to actually track the arithmetic. It gets difficult to find the answer without making a mistake once the faces get larger.
4. You can easily use Cuisenaire rods to make "forty faces" as well. Just use the white cube as the unit. Below is an example of a face with a value of 30.

Pig

Topics: Probability, strategy, addition, estimation
Materials: One 6-sided die, pencil and paper
Common Core: 2.OA.B.2, 2.NBT.B.5, 2.NBT.B.6, MP1, MP7

Roll the dice and collect points. You can go as long as you want, but roll the wrong number and you lose all your points from that turn!

Why We Love Pig

Pig is easy to learn and gives students lots of addition practice. Pig is also mathematically rich. Students get to articulate and defend strategies, and get practice with addition in a complex task.

The Launch

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students give you "thumbs up/down" if they think you should keep rolling. If students aren't comfortable adding up all the numbers they roll by hand, have them take tiles or other counters to one spot when it is their turn, and place them in another spot (with ten frames or a hundred chart) when they "bank" them.

How to Play

Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a 2, 3, 4, 5, or 6, the player adds that many points to their score for the turn. A player may choose to end their turn at any time and "bank" their points. If a player rolls a 1, they lose all their unbanked points and their turn is over.

Beginner Game: The first player to score 50 or more points wins.
Advanced Game: The first player to score 100 or more points wins.

Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?
- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?

The Wrap

The question of strategy is a fascinating one for Pig. What strategies are students using? Does strategy even matter? Let students share their ideas for strategies, and discuss which ones they think are better or worse, and why.

Tips for the classroom

1. Demonstrate the game a couple times with the whole group. Solicit advice about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.

2. For students who are less confident with addition, use ten frames and counters or a hundreds chart to keep track of the score. For example, students get to pick counters up as they roll. If they stop before they roll a one, then they transfer those counters to their ten frames. Fill up 5 ten frames to win.

3. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!

4. As kids play each other, circulate through the room and ask them about their strategies. It's okay for students simply to play, but there's an opportunity to probe deeper into the workings of chance and the strategy of the game too.

5. Remind students they can teach Pig to someone at home and play there.

Name_____

Pig

Rolls	Rolls
Bank	Bank

Day 2

Goals

1. Explore the connections between units and multiplication.

Opener

Unit Chat - see Appendix 4

Note: the lesson plan for unit chats is below. All unit chat images are in Appendix 4. You can go through unit chats in order, or skip around if you'd like to try different types earlier.

You can also find more unit chats at mathforlove.com/lesson/unit-chats.

Activity

Pattern Block Sunflowers

Note: Encourage students to use Cuisenaire rods when they make their own sunflower.

Game/Puzzle

Blockout

Choice Time

Pig
Blockout
Penny Nickel Dime
Challenge Problems - see Appendix 3

Closer

Take (or make) a simple Cuisenaire rod or pattern block sunflower and ask students how many of each block it would take to make 20 of those sunflowers. It would take a long time to extend a table that long. What else could help? Let students discuss their ideas and strategies.

For example, students might think about the blocks required to make 10 sunflowers and then double that number. Or they might just multiply their ingredient list by 20. A takeaway here is that once you figure out a pattern, you can sometimes jump forward without having to go through each step.

Unit Chats

Topics: Mental math, numerical fluency; argument & critique
Materials: White board or projector
Common Core: Variable, but generally good for NBT, NF, OA, MP1, MP3, MP6.

Counting with respect to different units.

Why We Love Unit Chats

Unit Chats are a kind of Number Talk that emphasizes not just how many, but also the unit involved. These are a fantastically productive, fun, differentiated, and delightful warm up for math. Perfect as a 5 - 10 minute opening or closing exercise.

The Launch

Post a Unit Chat image. It should have different kinds of objects to count in it, and be arranged in arrays or other structures as appropriate for the student level. Students get some time to look at what is in the picture, and how many of which object they see. After they've had 20 - 60 seconds to look, ask students what they see. You'll receive different answers about what they saw, and how many. You can ask students to explain different ways of counting what they saw, and also different things that they see to count in the picture.

Photo credit: Christopher Danielson

Example Unit Chat
Teacher: Take a look at this picture. Think about how many you see. [Waits for 30 seconds.] Quietly turn to the person next to you and tell them how many you see. [Students quietly discuss.] Who would like to share what they saw?
Student: I see avocados.
Teacher: How many avocados do you see?
Student: I see fifteen.
Teacher: Fifteen avocados. I don't see that at all.
Student: Look, there are five on the top, then another five, and then five on the bottom. So that's 15.
Teacher: Ah! You're talking about the *avocado halves*. In that case, I agree. That's 5, 10, 15 avocado halves. What else do you see?
Student: They're in a checkerboard pattern.
Teacher: That's true. The pitted avocado halves and the unpitted avocado halves form a checkerboard. Does that mean there are the same number of each?
Student: Yes! / No!
Student: There are 8 with pits.
Teacher: Let's count. 1, 2, 3, 4, 5, 6, 7, 8. That's right. Did you count one by one?

Student: No, I saw the 3 on the top, plus 2, plus 3.
Teacher: Ah, and 3 + 2 + 3 = 8.
Student: There are only 7 without pits.
Teacher: It's strange that it would be different if they were in a checkerboard pattern. I still don't see why there are more with pits than without.
Student: Because the first and last have pits. If there were one more row, it would be the same.
Teacher: I think I see. You're saying it goes "pit, no pit, pit, no pit,..." But it ends on "pit" and starts on "pit," so there's an extra.
Student: One "no pit" got thrown away.
Teacher: What do you mean?
Student: There were eight avocados that got cut in half, but one no-pit half isn't there.
Teacher: How do you know?
Student: Because if you put all the halves together, it would make wholes, and there would be eight wholes. But the last no-pit half is missing.
Teacher: So how many whole avocados are there?
Student: Seven and a half.
Teacher: I see. So we could see this as 15 half avocados, or we could see it as 7 and a half whole avocados. Very neat!

Prompts and Questions

- How did you see that?
- How did you count that?
- Does anyone else think they can explain what Therese is saying?
- Turn to the person next to you and see if you can see what Dwayne is describing.

Tips for the Classroom

1. Use images that are accessible to everyone. The best images have some easy things to count and some harder things to count.
2. You can emphasize how students counted, or shift the conversation to what they counted, depending on what will be the most engaging and enlightening. It can be okay if Unit Chats turn into something that resembles a Number Talk.
3. Remember: doing more short Unit Chats is better than doing just a few long ones. Aim for 5 - 10 minutes. You can use multiple images if they go super short, but often one image is plenty.

Pattern Block Sunflowers

Topics: Equality, substitution, addition, skip counting and multiplication
Materials: Pattern blocks, paper and pencil, worksheet with table
Common Core: 2.OA.1, 2.OA.4, MP1, MP2, MP6, MP7, MP8

How many blocks does it take to build the sunflowers?

Why We Love Pattern Block Sunflowers

This lesson is hands-on and intuitive. By increasing the number of flowers, it suggests the concept of multiplication, and the usefulness of counting by groups.

The Launch

Build or display the "sunflower" make of pattern blocks, and ask students how many of each type of block it uses. (1 hexagon, 2 trapezoids, 3 rhombuses, 6 triangles.) These are the ingredients it takes to build a sunflower.

Ask students how many of each block they'd need in total to build 2 sunflowers. Give them a bit of time with the blocks and a partner to work it out. Discuss students' approaches to solving this problem for each block. Some students may need to build each flower, while others may be able to work with the numbers alone.

Once students understand the idea, tell them their goal is to know how many of each blocks it takes to make anywhere from 1 - 5 sunflowers. There's a handout that includes a handy table where they can record their work. While they may be able to build sunflowers at first, it's likely they'll run out of the blocks they need, and have to come up with another way to track what they're doing.

The Work

The teacher circulates and helps the students as they complete the chart. Encourage students to need more concrete examples to build three sunflowers and count the blocks, and then record them clearly in the table.

For students ready for more challenge, ask them to continue the table for 6, 7, etc. sunflowers. Or they can jump ahead: how many of each block would it take to make 12 sunflowers? How do they know?

Prompts and Questions

- How do you know that's the number of rhombuses you need for three sunflowers?
- What patterns do you see in the table that might help you?
- Are you sure that pattern will still be true when you add another sunflower?
- (Challenge) How many total blocks would it take to build all 5 sunflowers?

The Wrap

Ask students to look a completed table and see what patterns they notice. Can they explain why any of these patterns are there? Do they make sense, or are they a mystery?

Can they use any of these patterns to predict how many of each block it would take to make 6 sunflowers? How? How much do they trust their own predictions?

Tips for the Classroom

1. A key element in the flow of the class will have to do with nudging students toward or any from building sunflowers when they need more concrete/abstract work. Encourage students to make predictions about what's coming next in the table. See if they can explain why their prediction makes sense to them, or if it's just a hunch.
2. Don't worry if you don't have enough pattern blocks for everyone to build 5 sunflowers! Students should be motivated by the very lack of blocks to come up with other ways of handling these problems, aside from counting one by one, say.

Pattern Block Sunflowers

I made a sunflower out of pattern blocks!
Now I want to make a lot of them.

Here is a chart to help me figure out how many of each kind of pattern block I will need.

Help me finish it!

Number of Flowers	Yellow Hexagons	Red Trapezoids	Blue Rhombuses	Green Triangles
 1	1	2	3	6
 2				
3				
4				
5				
10				
15				

Sunflower Challenges

Now it's your turn to make a sunflower.
Design it however you like!

Record how many hexagons, trapezoids, rhombuses, and triangles are in one flower using the chart below.

Then complete the chart by figuring out how many of each block it would take to build 2, 3, 4, 5 sunflowers. What about 10 sunflowers, or 15 sunflowers?

Number of Flowers	Yellow Hexagons	Red Trapezoids	Blue Rhombuses	Green Triangles
1				
2				
3				
4				
5				
10				
15				

Blockout

Topics: Multiplication, area, strategy, addition.
Materials: Crayons or colored pencils, Blockout game sheet
Common Core: 3.OA.A.1, 3.OA.C.7, 3.MD.C.6, 3.MD.C.7

Roll the dice and shade in a rectangle. How can you claim the most space on the board?

Why We Love Blockout

This is one of those rare games that reinforces both the skill of multiplication and the visual model that makes sense of it. Blockout can be played competitively or collaboratively, and is a wonderful game to introduce or reinforce the concepts behind multiplication.

The Launch

Take a volunteer and demonstrate the first several turns of a game of Blockout. Players choose colors, then take turns rolling the dice, and shading in a rectangle given by the dice rolls. If you roll a 2 and a 5, you can shade in a 2 by 5 (or 5 by 2) rectangle. No one can shade in a square that has already been colored. If there is no room to fit the rectangle you rolled on the board, you pass. If all players pass in a row, the game is over. Players get a point for each square they have colored in at the end of the game.

Students can play in groups of 2-4, though 2 is preferable. It is also possible to play individually or collaboratively. For a collaborative or solitaire game, players roll and try to cooperatively fill up as much of the board as possible. If every player must pass in a row, the game is over. The fewer the number of leftover squares, the better the game.

Prompts and Questions

- How many points does that roll give you?
- Who's ahead?
- What roll are you hoping to get this turn?

The Wrap

Discuss how students counted up their rolls. With a roll of 5 and 4, how would they have counted up the number of squares in their rectangle? (I.e., counting by 5s? counting by 4? Other strategies?) Discuss other possible rolls, and how they're counted. How many points to you get for rolling 6 and 6?

Tips for the Classroom

1. For the first time playing, students can play as above. For subsequent games, show students how to track their points as they go. For example, they can write $2 \times 5 = 10$ inside the 2 by 5 rectangle, and know that they have 10 points for that turn. This connects the game to multiplication without feeling to academic right away.

2. Once students are comfortable writing equations in the rectangles, you can abstract one step further and introduce the scoring sheet.

3. Blockout can also be played as a solo or collaborative game by trying to fill up as much of the board as possible. Play ends when a roll is made that cannot be placed.

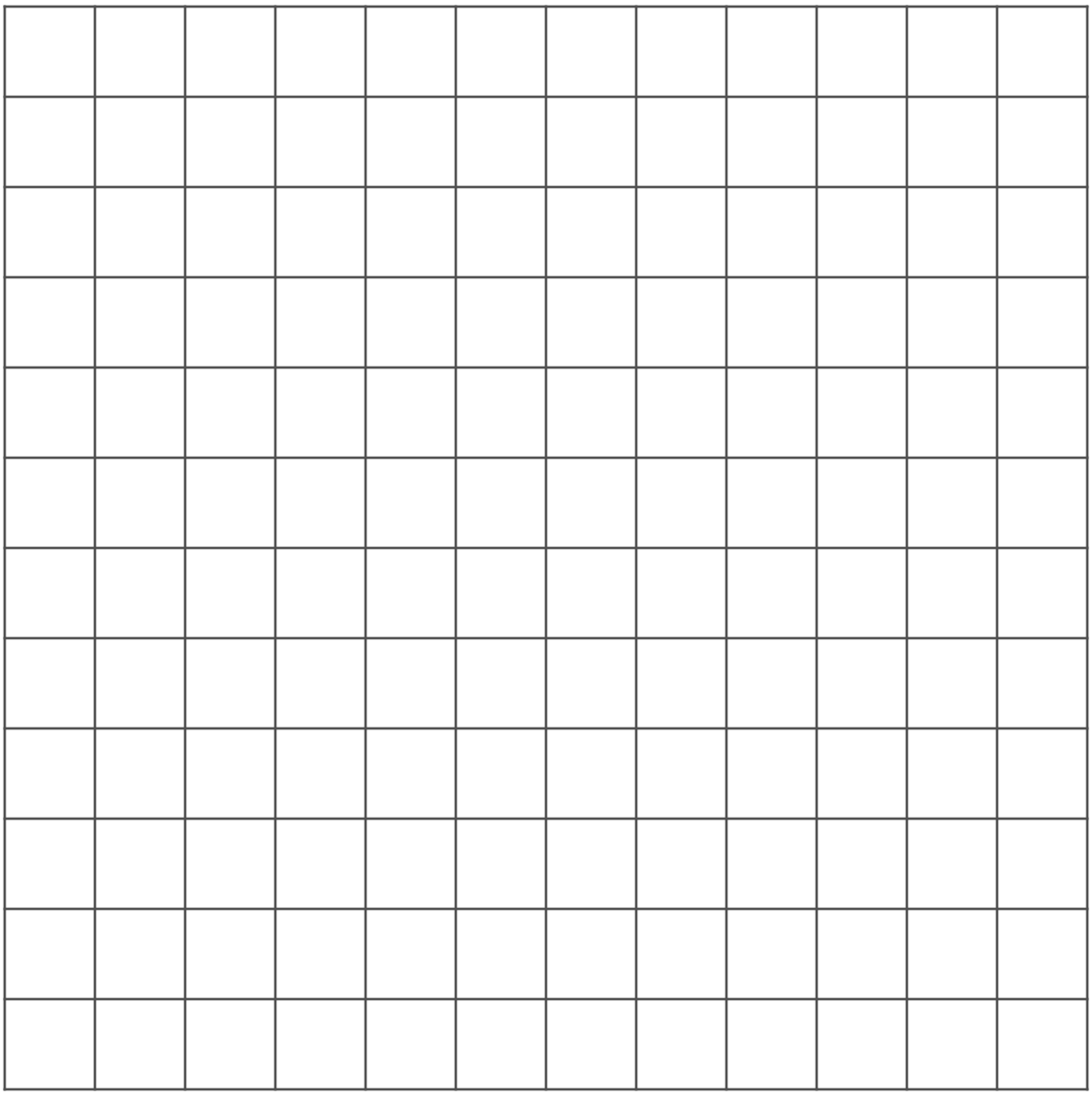
Blockout

For 2 players.

Rules. Players take turns rolling two dice, and drawing a rectangle on the game board with side lengths given by the two numbers they rolled. For example, if you rolled a 3 and a 6, you would draw a 3 by 6 rectangle, placed horizontally or vertically on the board.

Your rectangle cannot intersect or be contained in any previously drawn rectangles. If you cannot add a rectangle to the board on your turn, pass the dice to the next player. If all players pass in a row, the game is over.

Players get a point for each square they've drawn a rectangle around. For example, a 3 by 4 rectangle is worth 12 points. Whoever boxes the most squares wins.



Blockout Scoring Sheet

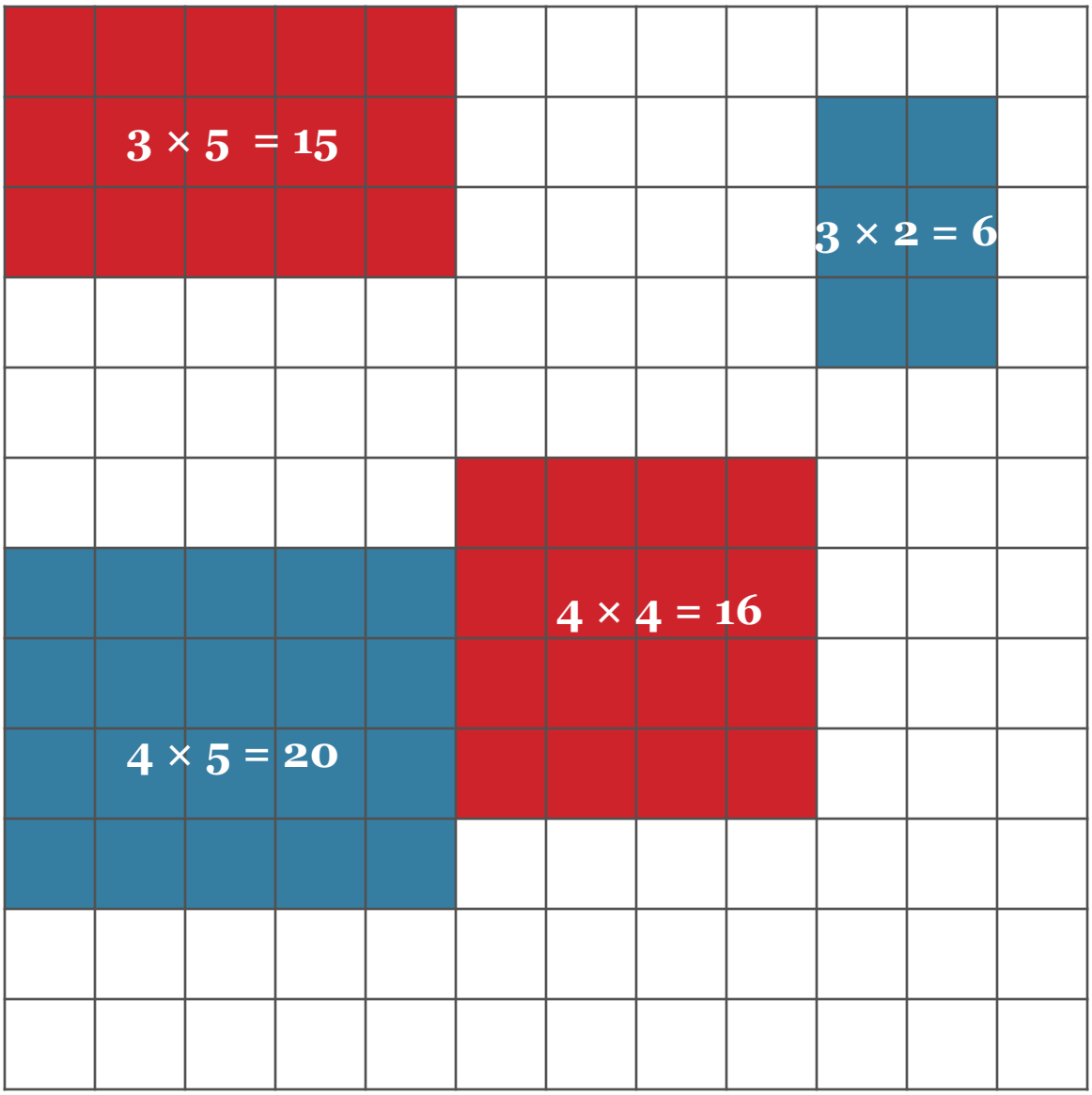
Turn	Player 1 Equation	Player 1 Score	Player 2 Equation	Player 2 Score
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Example Game

Rules. Players take turns rolling two dice, and drawing a rectangle on the game board with side lengths given by the two numbers they rolled. For example, if you rolled a 3 and a 6, you would draw a 3 by 6 rectangle, placed horizontally or vertically on the board.

Your rectangle cannot intersect or be contained in any previously drawn rectangles. If you cannot add a rectangle to the board on your turn, pass the dice to the next player. If all players pass in a row, the game is over. So Player 1 doesn't get too great an advantage, their first rectangle must be drawn in the corner. After that, rectangles may be drawn in any open spot.

Players get a point for each square they've drawn a rectangle around. For example, a 3 by 4 rectangle is worth 12 points. Whoever boxes the most squares wins.



Day 3

Goals

1. Continue exploring the connections between unit choice and multiplication.
2. Practice multiplication facts with Odd Pig Out.

Opener

Don't Break the Bank

Activity

Cuisenaire Rod Multiplication

Game/Puzzle

Odd Pig Out

Choice Time

Odd Pig Out
Blockout
Don't Break the Bank
Challenge Problems - see Appendix 3

Closer

If you knew that the *red* Cuisenaire rod equals 10, could you figure out what all the rest of the rods equal? Pose the question to students and let them try to solve and discuss in pairs or small groups. Then discuss methods and strategies as a class.

Don’t Break the Bank!

Topics: Triple-digit Addition, Estimation, Probability
Materials: One 6-sided dice, pencil and paper
Common Core: 3.NBT.2, 4.NBT.2, 4.NBT.4

How close can you get to 999 without going over?

Why We Love Don’t Break the Bank

Don’t Break the Bank is a place value powerhouse. It takes very little time, so it can be used as a warmup or in those five minutes before class ends. It’s fun, and kids *love* it, even though it involves addition practice. And, while kids will usually break the bank (that is, go over 999) their first few games, they’ll inevitably start estimating and choosing good strategies for themselves. Should the digits in the hundreds column add up to 9 or 8? How common is it to carry? The deeper thinking is almost inevitable.

The Launch

Everyone makes a diagram like this on their paper:

Whole Class Game: The teacher (or a student) rolls the die. Whatever number it lands on, everyone enters it in one of the nine spots on the board. After nine turns, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum **without going over 999**. (See next page for example game.)

Small Group Game: Same as whole class game, except that you take turns rolling the die, and everyone ends up entering different numbers into their grid.

Prompts and Questions

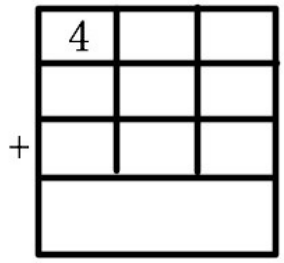
- What’s a good strategy for this game?
- Where would you put this 5?
- Have you already “broken the bank?” How can you tell?

Tips for the Classroom

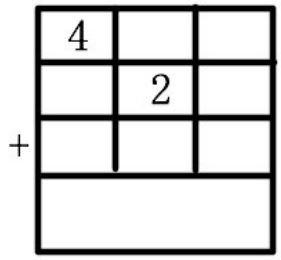
1. When you are playing a game with the full class, let students take turns rolling.
2. You can narrate your own thoughts when placing digits in the grid. Remember to be clear that you are placing ones, tens, and hundreds.
3. Students may not entirely understand the game the first time through, but they should get the hang by the second game.
4. Extend the game to decimals by adding decimal points up and down one column.
5. Play virtually at mathforlove.com/lesson/dont-break-the-bank

Example Game.

Turn 1: I roll a 4, and place it in my grid. So does the rest of the class.

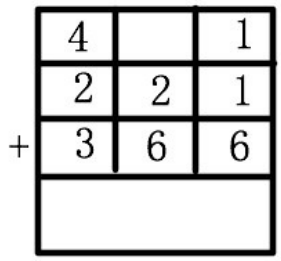


Turn 2: I roll a 2, and place it in the middle.



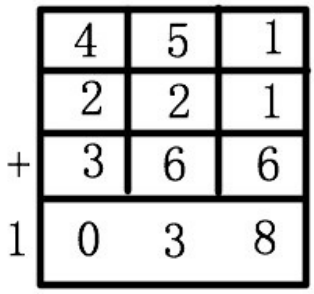
Turns 3 - 8 pass in the same way. Perhaps I have a grid like this:

At this point, I see that I'll be in trouble if anything except a 1 is rolled, since I'll have broken the bank by going over 999.



Turn 9: A 5 is rolled, and I broke the bank! When I enter the 5 and add up my numbers, I'm over 999, and I'm out this game.

Now it's time to play again!



Cuisenaire Rod Multiplication

Topics: Multiplication, changing units
Materials: Cuisenaire rods, paper and pencil
Common Core: 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.6, 3.OA.7, 4.OA.2, MP1, MP2, MP3, MP7

You know the value of the white rod… how can you figure out the other pieces?

Why We Love Cuisenaire Rod Multiplication

This lesson combines the fundamentals of multiplication with deeper problem solving in a context that's natural and hands-on.

The Launch

This lesson is designed to alternate between the teacher posing problems by assembling groups of Cuisenaire rods physically and saying their value, and students solving the question on their own, and writing up their solutions. Give students time as needed —at least a few minute for the early problems, and more as they get harder.

Problem 1. If the white Cuisenaire rod equals 1, what are the other rods worth?
Note: You can pose problems with almost no words by placing the Cuisenaire rods on a white board, and writing the numbers underneath or beside them.

If student haven't thought through this kind of problem before, this is a good warmup problem. Students will likely build a staircase from the rods, and see that red = 2, light green = 3, and so on, up to orange = 10. Challenge them to determine what orange + blue + brown is (orange + blue + brown = 10 + 9 + 8 = 27).

Once students have found what all the rods are worth, you can ask them to prove how they know that the blue rod is 9. There are many ways to prove it using what you know about the smaller. For example, the blue rod is 4 reds (i.e., 4 twos) plus 1 white (one). That's 9. It's also a yellow plus a purple, which is 5 + 4 = 9. It's also one white less than an orange rod, which gives 10 - 1 = 9. And so on.

Problem 2. If white equals 2, what are the other rods worth?

In this case, every rod will be equal to a multiple of 2. Note that some students may mistakenly mistakenly believe that red = 3, light green = 4, etc. This can be proved wrong by noting that white + white = red, which would mean 2 + 2 = 3. Clearly a mistake!

Once students have show their solutions to this problem, you may want to pose several questions at once, so students can work through to harder problems when they're ready.

Problem 3. If white equals 5, what are the other rods?

Problem 4. If white equals 4, what are the other rods?

Problem 5. If white equals 6, what are the other rods?

Problem 6. If white equals 8, what are the other rods?

Problem 7. If white equals 12, what are the other rods?

*Problem 8. If **red** equals 14, what are the other rods?*

If more problems are needed, let students make up their own.

Prompts and Questions

- What if the red rod equaled 10? Is that too big or too small?
- How do you know that the brown rod has that value?

Wrap Up

Take the last problem all students have attempted and spend a few minutes letting students share their answers with each other. You can have them share their methods with a partner, and then take one or two volunteers to share their method with everyone.

Tips for the Classroom

1. Make sure students can build their own version of the problem and solve physically.
2. Adjust the difficulty of the problems as necessary.
3. Students can always guess and check. This is a good strategy to encourage, since it makes the connection between division and multiplication more explicit.

Odd Pig Out

Topics: probability, strategy, multiplication, addition
Materials: Two 6-sided dice, pencil and paper
Common Core: 3.OA.7, 3.NBT.2, MP1, MP5, MP6, MP7

Roll the dice and multiply. You can go as long as you want, but roll an odd number and you lose all your points from that turn!

Why We Love Odd Pig Out

Odd Pig Out is a natural extension of Pig to multiplication. It is great practice for multiplication and addition in a fast-moving, fun game.

The Launch

The teacher chooses a volunteer, explains the rules, and plays a demonstration game. Because students already know Pig, this game should be relatively intuitive to learn.

Players take turns rolling the dice as many times as they like. After each roll, they multiply the numbers they rolled together. If the product is even, they add that number to their current points for the turn. If the product is odd, players lose all their points from that turn and their turn is over. A player may choose to end their turn at any time and "bank" their points.

Play to 300.

Prompts and Questions

- Is there an easier way to add up all those numbers?
- How many points to you have for this turn so far?
- Who's ahead?
- Are you sure that's the product of those two numbers? What does your multiplication table say?
- What strategy are you using?

The Wrap

Ask students whether they're more likely to roll odd products or even products. How many odd numbers are there on the multiplication table (up to 6 by 6)? How many even numbers? How are they distributed? Do students see any patterns?

Tips for the classroom

1. Demonstrate the game a couple times with the whole class (or in a station). Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.

2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing.

3. Make sure students have a copy of the dot array multiplication table, or the multiplication tables that they have made, handy to help them if they need them.

Odd Pig Out

Roll two dice and write down their product. You may choose to continue rolling as long as the products are even. End your turn to bank your points.
If you roll an odd product, end your turn and lose all unbanked points.

Products

Products

Bank

Bank

Day 4

Goals

1. Continue exploring the connections between unit choice and multiplication.
2. Practice arithmetic and strategic thinking with Prime Climb.

Opener

Unit Chat - see Appendix 4

Activity

Pattern Block Multiplication

Game/Puzzle

Prime Climb

Note: Prime Climb is an excellent option to make available at Choice Time every day.

Choice Time

Prime Climb
Odd Pig Out
Blockout
Challenge Problems - see Appendix 3

Closer

Pose the question to students:

If a blue rhombus is worth 4, what does 1 hexagon, 2 hexagons, and 3 hexagons equal?

Let students discuss in pairs or small groups, then discuss as a class to see how students approached the problem.

[Note: it takes three rhombuses to make a hexagon, so:
1 hexagon = 3 rhombuses = 3 fours = 12
2 hexagons = 6 rhombuses = 6 fours = 24
3 hexagons = 9 rhombuses = 9 fours = 36

Those are the answers, and one possible approach. There will be others from students.]

Pattern Block Multiplication

Topics: Multiplication, Multi-Step Problems
Materials: Pattern Blocks, scratch paper and pencil
Common Core: 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8, 3.MD.7.d, MP1, MP3, MP7

If you know what one block equals, can you figure out the value of all the shapes?

Why We Love Pattern Block Multiplication

This lesson involves fundamental ideas like changing the unit and multiplication in a hands-on context that prepares students for subtle concepts like division and fractions. Highly accessible, and easily differentiable.

Launch

The game in this activity is to change the value of the triangle and see what the other blocks—and larger collections of blocks—are worth. Start by posing a simple series of questions:

- If the triangle equals 1…
  What does the rhombus equal? (2)
  What does the trapezoid equal? (3)
  What does the hexagon equal? (6)

Let students prove these values are correct by covering the shapes with triangles, or making equivalent arguments (3 triangles in a trapezoid and two trapezoids make a hexagon, so $2 \times 3 = 6$ triangles in a hexagon).

Once these values are established, move on to some harder questions:

- If the triangle equals 1…
  What is the value of 4 trapezoids? (12)
  What is the value of 4 hexagons? (24)

Let students share their thinking on these questions as well. You can write out the arguments on the board or on scratch paper to demonstrate the kind of recording you'll expect from students.

Now we move in to the main part of the activity. Let students build a shape of their choosing, giving them a minute to build. When a minute is up, ask them to determine the value of their shape (given that the triangle is equal to 1), and the value of their neighbor's shape. When they have written up their answer with a clear explanation, they can build a bigger, more complicated shape and solve that too. Repeat as time permits.

Extension: if students are showing a general proficiency for the activity, deepen the challenge by changing value of the blocks. For example: what if the triangle were worth

5 instead of 1? What if the blue rhombus were worth 4? What if the trapezoid equals 9? In each scenario, what is the value of your shape?

Prompts and Questions

- How did you find that answer?
- What's the value of just your hexagons?
- Show me what you've written down so far.

The Wrap

Find a design that's easy enough to be accessible to everyone, and pose it as a final problem. Let students attempt it on their own, writing down their work as clearly as they can. Then share some different student attempts to solve the problem.

For example, say your final problem was to find the value of 2 hexagons and 6 trapezoids. Students may have many different methods:

Method 1
Hexagon = 6, so the value of the hexagons is $2 \times 6 = 12$.
Trapezoid = 3, so the value of the trapezoids is $6 \times 3 = 18$.
Total value is $12 + 18 = 30$.

Method 2
Put together the 6 trapezoids to make 3 more hexagons, for a total of 5.
That gives us a total value of 5 hexagons = $5 \times 6 = 30$.

Method 3
Count each piece and add.
Hex + hex + trap + trap + trap + trap + trap + trap = $6 + 6 + 3 + 3 + 3 + 3 + 3 + 3 = 30$.

Tips for the Classroom

1. An excellent uplevel for this activity is to ask a pair of students to find the sum of and difference between the shapes they built.
2. Don't try to keep all the students together and working on the same problem. Rather, let students work at the appropriate level of difficulty. Just make sure that everyone has attempted (or can do) the problem you discuss at the end.
3. Encourage students to write down their work with simple pictures and equations. A helpful way to encourage recording is to count all the hexagons, record that number, and use a multiplication equation to determine how many triangles that is, then repeat for other shapes, and find the sum. (Other methods work as well, of course.)
4. Some students may not be comfortable with multiplication. They can use addition to solve their problems.

Prime Climb

Topics: Multiplication, division, addition, subtraction, multi-step problems, factoring
Materials: Prime Climb board game
Common Core: 3.OA.C.7, 3.OA.D.8, 3.OA.D.9, 4.OA.A.3, 4.OA.B.4, MP1, MP7

How can you get your pawns to 101?

Why We Love Prime Climb

We invented Prime Climb to give students a more playful way to explore complex arithmetic problems and understand factoring.

Launch

Show students the color scheme of the board and multiplication table, and ask them what they notice. In particular, what's happening with the color scheme?

Let students discuss their thoughts. A specific point to underline, especially with respect to the multiplication table: if you look at two numbers that multiply together, (i.e. 7×8), the answer has exactly the same colors of each of the factors, just put together. (7 is purple, 8 is three orange; 56 is purple and three orange.)

Divide your small group into teams (individuals or pairs). Each team chooses a color to play. Each team gets two pawns, and place them on 0. The goal is to get a pawn to 101. (This is a quick version of the game. In the full game, the goal is to get both pawns to 101.) Explain the rules to students by demonstrating a few example moves.

Quick Start Rules
During a turn, there are four phases.

1. *Roll*. Roll the dice. You get two numbers from 1 to 10 to use for moving. In you roll doubles, you get that number four times instead of two. (The 0 on the die stands for 10.)

2. *Move*. Move your pawn(s). Apply your dice rolls one at a time to the number your pawn(s) is on, using your choice of +, -, x, or ÷. You can also use Keeper cards if you have them.

3. *Bump*. If you end your Move phase on the same space as another pawn, send it back to start. You may bump your own pawn.

4. *Draw*. If you end your Move Phase on an entirely red space (i.e., a prime greater than 10), draw a Prime card. If it is a Keeper card, save it for a future turn. Otherwise, apply the card now.

When someone lands a pawn exactly on 101, they win the game. You're never allowed to move to numbers off the board.

Example
With pawns on 4 and 26, you roll a 3 and a 9. You could:
- Add 3 to 4 to move your pawn to 7, then multiply by 9 to move your pawn to 63.
- Multiply 26 by 3 to move your pawn to 78, then add 9 to move it to 87.
- Add 9 to 4 to move one pawn to 13, and multiply 26 by 3 to add the other to 78. Since 13 is completely red, you would them draw a card.

You CANNOT add the 3 and 9 first and use a 12 for anything. You have to apply the numbers on the dice one by one.

The Work
Once they understand the rules, let students play the game. They may have questions that come up during the course of play. You can consult the full rules of the game, or just have students respond by deciding on what seems like the best way to settle the question and keep play going.

Prompts and Questions
- Can you get either pawn to a red circle with that roll?
- Can you bump anyone with that roll?
- You rolled a 3 and a 5. What if you added the 3 to your pawn first, then multiplied by 5?
- If you subtract, you could land on a red circle and draw a Prime card!

The Wrap
Settle any remaining questions about the rules, if there are any. Ask what strategy students have found to be useful in the game. For example, does it make sense to add and go past fifty? If you do, you won't be able to multiply again. How important are the cards for your strategy? How quickly can you reach 101 if you get a good roll?

Tips for the Classroom
1. Have students roll in the box lid to prevent them from knocking over pawns during the game.
2. Students can use the multiplication table or scratch paper to help themselves with hard multiplication problems. The board's color scheme can help too.
3. Students may dislike getting knocked back to start. However, they'll quickly learn that they can make fast progress if they get a good roll, especially when they roll doubles.
4. Encourage students to try to get cards on their turn by landing on red circles. That's a good hint for success in the game.

Day 5

Goals

1. Connect division to multiplication with Cuisenaire Rod Division.
2. Play logical games, and games involving all arithmetic operations.

Opener

Pico Fermi Bagels

Activity

Cuisenaire Rod Division

Game/Puzzle

Bowling

Choice Time

Prime Climb
Odd Pig Out - use ten-sided dice if students want a challenge
Bowling
Challenge Problems - see Appendix 3

Closer

Pose this problem for students to solve on their own, or in a pair or trio: 13 kids are taking a river trip in canoes. Each canoe holds at most 4, and it's not safe to take a canoe out alone - you need at least one other person with you.

How many canoes do they need for their trip, and how many kids should sit in each canoe?

[Note that there are several answers. Try to help the discussion by involving drawings of the canoes (4 is the minimum required, and 6 is the maximum they could take), along with passengers. For example, here's one way you could put the 13 people in 4 canoes.