## Math for Love

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## A word about using this book

This book was designed to support a summer math program lasting sixteen 75-90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. Games can be played for 5-30 minutes. Deeper tasks might be good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this curriculum that should be helpful: mathforlove.com/video/ math-for-love-video-pd.

Enjoy!

## A word about the copyright

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## Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

## Goals of the Math For Love Curriculum

We wrote this program to be both play-based and rigorous. The goals of the program are two-fold:

- Improve conceptual understanding of and fluency in mathematics
- Give everyone an opportunity to have fun and enjoy math

Many students haven't had enough time working with conceptual models of mathematics before being pushed into abstraction. To remedy this, the curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorially while making connections to abstract reasoning.

## Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are doing math and thinking math.

## Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. Engagement. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. Differentiation. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.
3. Thinking. Get students thinking as soon as possible every day, and help keep them productively stuck, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. Positive Environment. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.
$\star$ Be ready with questions
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value their thinking and contributions, not just the answer.

* Model how to play games, and teach how to win and lose

Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.
$\star$ Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Unit Chat or opening game (see the Opener in the daily plan). Use the Math Games and Station Breaks for transitions between Activities. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

## $\star$ Have a growth mindset classroom

Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you learn to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

## $\star$ Encourage conjectures and counterexamples

Establish a habit of supporting students' conjectures, hypotheses and predictions, and students will learn more and commit to the thinking process. Help them use
counterexamples as a tool to break and improve conjectures (see the
Counterexamples lesson plan on Day 2), and they'll begin behaving like true mathematicians. Making conjectures and counterexamples normalizes mistakes as part of the learning process, and gives students a practical way to learn from them. It also makes doing and thinking mathematics the central activity of your class.

## $\star$ Give your students time to think and explore

Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

## $\star$ Give your students the right amount of struggle

We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

## Value play

It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

## Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less, you can end sooner. If you need something longer, you should find many of the activities extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

## Note on materials substitutions

This curriculum was designed with Upscale Pattern Blocks and Number Rods in mind. It is possible, however, to make the following substitutions:

## Upscale Pattern Blocks

We recommend Upscale Pattern Blocks (available at mathforlove.com/upscale), which include 3 sizes of the 4 basic pattern block shapes. You can use generic pattern blocks (which only come in 1 size) in place of them. If using generic pattern blocks, make sure to remove the orange squares and tan rhombuses before starting the activities in this curriculum. You will be left with the yellow hexagons, red trapezoids, blue rhombuses, and green triangles.

## Number Rods

You can use Cuisenaire rods in place of Number Rods. As Cuisenaire rods are colored differently from Number Rods, you will need to disregard the color identifications included in this curriculum's instructions, and instead focus on rod lengths.

| Turquoise | 1 cm |
| :--- | :--- |
| Orange | 2 cm |
| Dark Green | 3 cm |
| Red | 4 cm |
| Blue | 5 cm |
| Pink | 6 cm |
| Purple | 7 cm |
| Yellow | 8 cm |
| Light Green | 9 cm |
| Gray | 10 cm |



## Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- Goals
- Opener
- Activity
- Game
- Choice Time
- Closer


## Goals

These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students.

## Opener

The Opener is the first activity of math class. The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

In general, the Opener should last about 5-10 minutes.

## Activity, Game, Choice Time

Following the opener, there is a suggestion for an activity, a game, and Choice Time. This is where the bulk of class time will be spent. There are two recommended ways to approach these three elements.

1. Have students rotate between three stations. This is especially recommended when you have additional adults (instructional aides, parent volunteers, tutors) in the room aside from the teacher.
2. Take the whole class through the activities one by one. This is recommended when the teacher is the only adult in the classroom.

Either way you run your classroom, the elements are designed to give students the maximum opportunity to think \& engage, practice skills, explore questions, and have fun.

Choice Time includes a suggestion of a small group of past games and activities for the students to try. This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

## Closer

The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer should take 5 minutes or less.

## Other Notes and Best Practices

$\star$ Math Breaks and Physical Games
Check out the math-based movement breaks in Appendix 2. These are excellent as transitions.

## $\star$ Folder for Worksheets

Give each student a folder where they can keep their worksheets. If they finish an activity early, they can turn back to their unfinished worksheets and finish them. They can also work on them during Choice Time.

## $\star$ Choice Time

Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should both choose the activity that is right for them, and then stick with it for at least half of the time.

## $\star$ Challenge Problems

Challenge problems (see Appendix 3) are great options for Choice Time any day. Offering "spicy" variations of worksheets or unfinished activities as Choice Time activities can be another nice option.

## $\star$ Station Transitions

If you use stations, provide 1-2 minute warnings before station transitions, to apply a gentle transition, cleanup, and-especially at Activity 1-a brief reflection or wrapup. If you use an alarm, make it a gentle sound (i.e., a gong) rather than an abrasive one (i.e., a clock radio alarm).

## $\star$ Games to send home

See Appendix 1 for games to send home. These will help parents/guardians and students play math games at home. You can always send other favorite games home, or encourage students to share games they've learned with people at home. Note that there is no homework for this program otherwise.

## Day 1

## Goals

1. Establish class norms and community.
2. Take preassessment.
3. Connect counting, addition, multiplication, and equations.
4. Play arithmetic and logic games.

## Openers

Mingle and/or Bullseyes and Close Calls (2-digit, then 3-digit)
Note: feel free to substitute your favorite name game instead of using Mingle.

## Activities

1) Preassessment
2) Forty Faces

Make sure to let students know that this preassessment is not a "test," and not something they're expected to know any or all of the answers to. It's just a way for you, the teacher, to see what ideas they are familiar with, so you can make sure you keep them challenged and interested. They definitely shouldn't worry if they can't get all, or even most, of the answers. (We've included some very challenging questions!) So just tell them to do their best and not to sweat it.

## Game/Puzzle

Fill the Stairs

## Closer

Pose the question:
We know forty is bigger than thirty. Does this mean a forty face (made out of pattern blocks) uses more blocks than a thirty face? Or is it possible for a thirty face to use more blocks than a forty face?

Let students discuss with a partner or small group, and then discuss with the class. The key takeaway is that while a forty face would take forty green triangles to build (vs. thirty green triangles for a thirty face), the number of blocks are a different unit. So a forty face could be made out of six hexagons and two blue rhombuses ( 8 blocks total) vs. a thirty face made out of 30 green triangles.

## Mingle

The teacher calls out a number (e.g., 3), and the students get themselves into groups of that size as quickly as they can. It might be impossible for everyone to get in a group every time, but each new number gives everyone another chance.

Once students get in groups, they can learn each other's names.
In the basic game, just call out single numbers. Once students get the gist, you can call out addition or subtraction problems (i.e., "get into groups of 7-4").

## Tips for the Classroom

1. The teacher can get into or out of the groups in order to make sure no student is by themselves.
2. Don't forget to call out a group of 1 and a group of however many students are in the entire class at some point in the game! Both make for a fun surprise.

## Bullseyes and Close Calls

Topics: Logic and deduction, place value
Materials: Paper or blackboard/whiteboard to record guesses
Common Core: 1.NBT.B.2, 2.NBT.A.1, 2.NBT.A.3, MP1, MP3
Can you use the clues to get the number with the fewest possible guesses?

## Why We Love Bullseyes and Close Calls

This game is a wonderful exercise in logic, and a nice way to get kids playing with the ideas of digits and place value.

## How to Play

The teacher secretly chooses a number with no repeated digits and writes it down where no one can see it. (Play with 2-digit numbers when you're beginning.) Students attempt to guess the number. After each guess, the teacher gives feedback.

| Outcome of Guess | Feedback |
| :---: | :---: |
| Correct digit in the wrong place | "Close Call" |
| Correct digit in the correct place | "Bullseye" |
| No correct digits | "Nothing" |

Note: you might need to say " 2 Close Calls" or " Bullseye and 2 Close Calls" some other combination.

DON’T say "first digit Close Call, second digit Bullseye." Your responses apply to the entire numbers, not individual digits. That's what makes the logic interesting.

## Example Game

There's a secret number that gives the following feedback for these guesses. Can you figure out what it is? (See next page for the answer.)

| Guess | Feedback |
| :---: | :---: |
| 139 | Nothing |
| 820 | Close Call |
| 468 | 1 Close Call, 1 Bullseye |
| 568 | Close Call |
| 482 | 2 Bullseyes |


| Guess | Feedback |
| :---: | :---: |
| 487 | 3 Bullseyes! |

The secret number is 487 !
The guessers got it in six guesses. Can they do it in even fewer next time?

## Tips for the Classroom

1. Note that students DON'T get a Bullseye or Close Call for each digit. The clue applies to the entire 2 - or 3 -digit number.
2. Start with 2-digit numbers. Go to three-digit numbers only when the 2-digit numbers have become straightforward.
3. Write the guesses and the responses somewhere that everyone can see it.
4. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses. For example, after guessing 139 and finding that none of those digits are in the number, cross off the 1,3 , and 9 from the list of possible digits.
5. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

Reference: Ben Orlin's Math Games with Bad Drawings
$\qquad$

## Preassessment

1) How many dots? Write an equation and solve.


Equation: $\qquad$
2) How many dots? Write an equation and solve.


Equation: $\qquad$

Fill in the blanks to make the equations true.
3) $6 \times \square=24$
4) $42 \div 7=$

Use the area model to solve.
5) $13 \times 8=$
$\qquad$
6) Cora gets eight pennies each day for six days. How many pennies does Cora get?

Explain with equations, words and/or pictures.
7) I had 6 cartons of eggs. There are 12 eggs in each carton. I cooked 15 eggs. How many eggs were left?

Explain with equations, words and/or pictures.
8) I have 36 books. I put them into four equal groups. How many books are in each group?

Explain with equations, words and/or pictures.
9) There are 45 people going to the festival. Each van can hold 6 people. How many vans do they need?

Explain with equations, words and/or pictures.
10) Comic books cost $\$ 4$, and paperback books cost $\$ 9$. If I want to buy 6 comic books and 3 paperback books, how much money do I need?

Explain with equations, words and/or pictures.
$\qquad$

## Summer Staircase 3rd Grade Preassessment

1) How many dots? Write an equation and solve.


Equation: $\qquad$ $7 \times 9=63$ dots
5 points for correct expression ( $7 \times 9$ or $9 \times 7$ or other option)
5 points for correct answer ( 63 or 63 dots)
2) How many dots? Write an equation and solve.


Equation: $\qquad$ $12 \times 4=48$ dots
5 points for correct expression ( $12 \times 4$ or $8 \times 6$ or other option)
5 points for correct answer ( 48 or 48 dots)

Fill in the blanks to make the equations true.
3) $6 \times$ $\qquad$ $=24$

## 10 points for correct answer: 4

4) $42 \div 7=$ $\qquad$

10 points for correct answer: 6

Use the area model to solve.
5) $13 \times 8=$ $\qquad$

5 points for correct use of area model 5 points for correct answer: 104
6) Cora gets eight pennies each day for six days. How many pennies does Cora get?

Explain with equations, words and/or pictures.
5 points for clear, correct explanation and/or drawing 5 points for correct answer: 48 pennies Subtract 1 point if there are no units in final answer
7) I had 6 cartons of eggs. There are 12 eggs in each carton. I cooked 15 eggs.
How many eggs were left?
Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing 5 points for correct answer: 57 eggs
Subtract 1 point if there are no units in final answer
8) I have 36 books. I put them into four equal groups. How many books are in each group?

Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing 5 points for correct answer: 9 books Subtract 1 point if there are no units in final answer
9) There are 45 people going to the festival. Each van can hold 6 people. How many vans do they need?

Explain with equations, words and/or pictures.
5 points for clear, correct explanation and/or drawing 5 points for correct answer: 8 vans Subtract 1 point if there are no units in final answer
10) Comic books cost $\$ 4$, and paperback books cost $\$ 9$. If I want to buy 6 comic books and 3 paperback books, how much money do I need?

Explain with equations, words and/or pictures.
5 points for clear, correct explanation and/or drawing
e.g.
$\mathbf{6} \times \$ 4=\$ 24$ to buy the comic books
$3 \times \$ 9=\$ 27$ to buy the paperbacks
$\$ 24+\$ 27=\$ 51$ to buy everything
$\mathbf{5}$ points for correct answer: \$51 or $\mathbf{5 1}$ dollars Subtract 1 point if there are no units in final answer

## Forty Faces

Topics: Addition, subtraction, skip counting, multiplication, logic
Materials: Upscale Pattern Blocks, scratch paper and pencil, Number Rods (optional)
Common Core: 3.OA.3, 3.OA.8, 3.NBT.2, MP1, MP6, MP7
Note: Alternatively, it's possible to use generic pattern blocks instead of Upscale Pattern Blocks. If doing so, be sure to remove the orange squares and tan rhombuses before starting this activity, or tell students not to use them.

## Why We Love Forty Faces

This delightful challenge provides an artistic exploration of ways to construct numbers by repeated addition or multiplication.

## The Launch

Ask students how many triangles it takes to build the blue rhombus (2), the red trapezoid (3), and the yellow hexagon (6). Then show them the faces below, either by building them or by projecting images of them.


Briefly discuss how these faces are made by putting together the equivalent of 10 or 20 triangles worth of area. For the second face, for example, there are 2 hexagons, 2 rhombuses, one trapezoid, and one triangle. In terms of triangle area, the total "value" would be 12 (in hexagons)+ 3 (in trapezoids) +4 (in rhombuses) +1 (in triangles) $=12+3+4+1=20$ triangles worth of area.

Once students understand how to count the "value" of the face, challenge them to create their own faces from pattern blocks that have value (i.e., area) $10,20,30$, or 40 .

## Prompts and Questions

- How much more area do you need to add to get to 30 ?
- Show me how you found the area.
- Let's count how much the hexagons are worth.
- The trapezoids came to 18 area? Let's write that down.
- Do you think the two of you could make a face with an area of 75 ?


## The Wrap

Share a face that almost has area forty. Find its area/value with students, emphasizing the possibility of skip-counting or multiplying to find the value of specific blocks. Once everyone agrees on the area of the figure, take student suggestions for how it could be adjusted to come to forty exactly.

## Tips for the Classroom

1. Let students challenge themselves when they're ready. Can they make a " 100 face"?
2. Encourage students to use pencil and paper to actually track the arithmetic. It gets difficult to find the answer without making a mistake once the faces get larger.
3. You can easily use Number Rods to make faces as well. Just use the 1 cm rod as the unit.
4. Using the medium and large Upscale Pattern Blocks creates an additional challenge: what are those blocks worth? Figuring out the answer requires filling in the larger blocks with triangles or other blocks. The medium size hexagon below is actually the same area as 24 small triangles!


## Fill the Stairs

Topics: Comparison of two-digit numbers, estimation
Materials: Fill the Stairs sheet, 2 ten-sided dice per game (different colors)
Common Core: MP1, MP6, MP7
The numbers have to increase as they go up the stairs. Where should each number go?

## Why We Love Fill the Stairs

This clever game requires the thoughtful placement of two-digit numbers in order, before all the numbers are known. Fill the Stairs is a compelling and fun game that holds up after repeated playing.

## The Launch

For the first game, you can play at a station with everyone together. Everyone gets a Fill the Stairs worksheet for each game. Choose one 10 -sided die to be the "tens" die, and the other ten-sided die to be the "ones" die. When you roll them both, you get a one- or twodigit number. After every roll, everyone places the number that was rolled where they like on the stairs. The only rule is that numbers higher up on the stairs must be greater than all the numbers below them. If a player can't use a number, it gets written under the stairs as a "discard." Whoever fills up their stairs first is the winner.

Once you've played a game altogether, students can play on their own in pairs or groups of three. The best way to play is with everyone in one game using the same rolls, so everyone has something to do on every turn.

## Prompts and Questions

- Where are you going to put that number? Why there?
- What number are you hoping for on the next roll?
- How do you know that number is bigger than that one?


## The Wrap

Discuss strategies with the students. What's the best way to win the game? Can students find any potential problems with the proposed strategies of their peers?

## Tips for the Classroom

1. If the dice roll off the table, they should re-roll.
2. A quick chant like "shake, shake, roll!" can help the game move, and avoid students spending too long shaking the dice.
3. Fill the Stairs can easily be played collaboratively rather than competitively. Students try to fill the stairs as a team with the fewest number of "wasted" moves.
4. Let students decide after each roll which digit gives the ten and which gives the one.

## Fill the Stairs

Roll the dice to make a two-digit number, and write it in on one of the stairs. Each number you write in must be bigger than all the numbers below it, and smaller than all the numbers above it. If you can't use a number, write down the number under the stairs, and skip your turn.

The game is over if someone fills in all the steps in their staircase.


## Day 2

## Goals

1. Learn the key ideas of conjectures and counterexamples.
2. Explore solving rich problems with multiple steps, and connect to multiplication.

## Opener

Counterexamples

## Activities

1) Square Building
2) Mini-Lesson - What is Multiplication?

Note: Square Building, when it catches the interest of class, can last the entire period, with some students making their own list of square numbers going far beyond the number of square tiles available.

## Game

Blockout

## Choice Time

Blockout
Bullseyes and Close Calls
Square Building
Fill the Stairs

## Closer

Ask students to reflect on a conjecture they had or heard that ended up being disproven by a counterexample today. Have students discuss in partners or small groups, then ask for a few examples.

The moral to highlight is that it's natural for conjectures to be wrong, but by making them and breaking them, you start the process of learning what's actually true. If you never wanted to make a conjecture because you never wanted to be wrong, you wouldn't get to figuring out what's right!

## Counterexamples

Topics: Logic, deduction, mathematical argument, communication
Materials: None
Common Core: Variable, but especially MP3
Prove the teacher wrong. Rigorously.

## Why We Love Counterexamples

Every kid loves to prove the teacher wrong. With Counterexamples, they get to do this in a productive way, and learn appropriate mathematical skepticism and communication skills at the same time.

It is possible to play Counterexamples with kids as young as kindergarteners as a kind of reverse "I Spy" ("I claim are no squares in this classroom. Who can find a counterexample?"). What's great, though, is that you can transition to substantial math concepts, and address common misconceptions. Counterexamples is a perfect way to disprove claims like "doubling a number always makes it larger" (not true for negative number or 0 ) or sorting out why every square is a rectangle, but not every rectangle is a square. For older kids, you can even go into much deeper topics, like: "every point on the number line is a rational number."

The language of counterexamples is crucial to distinguish true and false claims in mathematics; this game makes it natural, fun, and plants the skills to be used later. Counterexamples is also a great way to practice constructing viable arguments and critiquing the reasoning of others.

## The Launch

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher, though it can be a student) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All $\qquad$ s are $\qquad$ " or "No
$\qquad$ s are $\qquad$ ." You can also play around with statements like "If it has
$\qquad$ , then it can $\qquad$ ." For instance:

It's often best to start with non-mathematical examples.

- All birds can fly. (Counterexample: penguins)
- No books have pictures in them.
- All books have pictures in them.
- If something produces light, then it is a light bulb.
- If something has stripes, then it is a zebra.

Once students have the hang of it, make the examples more mathematical.

- Doubling any number makes it bigger. (Counterexample: o doubled is 0 , which isn't bigger)
- Multiplying two numbers never gives the same answer as adding them. (Counterexample: $2 \times 2=2+2.0 \times 0=0+0$. Also, $3+1.5=3 \times 1.5$.)
- Fractions are always between 0 and 1 .
- If a shape has all its sides the same, then it's a square. (Counterexample: a rhombus. Squares need four equal sides AND four equal angles.)


## Example

Teacher: I claim all animals have four legs. Who can think of a counterexample?
Student 1: A chicken!
Teacher: Why is a chicken a counterexample?
Student 2: Because it has two legs.
Teacher: Right. I said every animal has four legs, but a chicken is an animal with just two legs. So I must have been wrong. Let me try to refine my conjecture then. I should have said that animal must have 2 or 4 legs. That feels right.
Student 3: What about a fish?
Teacher: Aha. A fish is an animal with no legs. Thank you for showing me the error of my ways. What I should have said is that animals have at most four legs.
Student: 4: What about insects?
And so on.

A nice transition into Square Building from here is with the conjecture: "there's no way to make a square using smaller squares." From there, students can disprove the conjecture by putting four square tiles together. Then ask for their conjectures on the next number of square tiles you could put together to make a larger square, and you've launched Square Building.

## Tips for the Classroom

1. It's good to make up false conjectures that are right for your students. But start simple.
2. Kids can think of their own false claims, but sometimes these aren't the right kind, and they often have to be vetted.
3. Once you introduce the language of counterexamples, look for places to use it in the rest of your math discussions.
4. You can also use Counterexamples to motivate a normal math question. Instead of saying, "draw a triangle with the same area as this square," you can say, "I claim there is no triangle with the same area as this square." If students know to look for counterexamples, this will set them to work trying to disprove the claim right away.

References: http://kuow.org/post/getting-kids-interested-math-without-their-knowing

## Square Building

Topics: Counting, addition, subtraction, geometry, data, area
Materials: Square tiles, graph paper
Common Core: 3.OA.D.9, 3.MD.6, 3.MD.7, MP1, MP3, MP6, MP7, MP8
How many squares does it take to build bigger squares?

## Why We Love Square Building

Square tiles create a fluid ladder from concrete counting to the abstraction of geometric arrays, multiplication, and division. This lesson lays out a dynamic pathway that allows kids to explore these ideas at the right depth for them.

## Launch

To launch the lessons, build a few squares out of square tiles, and count how many tiles you need to make each one, and records those numbers.

Then you ask the class: Is there a pattern in the number of square tiles it takes to build a larger square?

## Example Launch

Teacher: I've got all these square tiles. I wonder if I can build a square out of them. Of course, I could build a square using just one tile. But what if I wanted to build the next largest square? How many tiles would I need? Think about it, then share with the person next to you. [Students hold whispered discussion.] How many tiles would I need?
Student: 4.
Teacher: [demonstrating] I see. I'd just put them together like this. Now let's get tougher. How many tiles would it take to make the next biggest square? Try to imagine it, then share with the person next to you. [Students discuss.]
Teacher: Any thoughts?
Students: [Expect multiple answers here] 8, 12, 16, 9.
Teacher: This is exciting! It seems like there could be a lot of different answers here.
Who thought 8 ? Can you show us how 8 square might fit together. [Student volunteers, ends up with a 2 by 4 rectangle.] So eight tiles built a rectangle, but not a square. Can anyone tell me why this isn't a square? What needs to be true about squares?
Student: All the sides have to be the same length.
Teacher: Right. All the side have to be the same length. So this side is 2 edges long, and this one is 4 edges long. Not a square. But maybe we could move them around... [moves tiles to make a 3 by 3 square missing a corner.] Is this a square?
Student: It needs one more!
Teacher: So it wasn't 8 tiles we needed. It was 9 ! This looks like a pattern starting. I'll write it down. [Writes: Squares we can build: 1, 4, 9, ...]

Teacher: Your job today will be to find the next numbers in the sequence by building larger squares. Make sure your squares are actually squares! All the sides need to be the same length. Also, try to make your list complete. If we can, we always want to find the next largest square, and not jump ahead.


## The Work

Give students 15-20 minutes to build squares of different sizes and write down their list of numbers. They can also use graph paper and draw out the squares. After students have their lists, have them compare them with another classmate or two to see if they found the same numbers. Challenge students who have found patterns to make predictions for what number comes next, and then try to build or draw it to check that they're right.

## Questions and Prompts

- How do you know that is the next largest square?
- How do you know that's a square?
- What makes a square a square?
- How did you make that square? (Did you use the last square as a starting point?)
- How did you count the tiles? (One by one, or some other way?)
- How do you know that you didn't make a mistake in your counting?
- Do you all agree that this is the correct count?


## The Wrap

Bring the class together and have the students give you the numbers they found. The beginning should look like this: $1,4,9,16,25,36,49,64,81,100, \ldots$.

Discuss patterns, and have students share patterns they found.
Some patterns students may have found include:

- Looking at the pattern of odds and evens (odd, even, odd, even, etc.)
- Noticing how much each number in the pattern increases by.

For square numbers, this is $+1,+3,+5,+7$.

- Noticing that the numbers are $1 \times 1,2 \times 2,3 \times 3,4 \times 4$, etc.

Leave students with the question: how can all these patterns be true at once?

## Tips for the Classroom

1. It's worth mentioning to your students that these numbers, which represent the number of little squares it takes to make a bigger square, are called square numbers. Nifty name, and sensible too.
2. While a good target for students is to build and find the first 10 square numbers, it is okay if not all students get up to the highest numbers. It's also okay if students went higher without building every single square. In fact, it's great if they start finding patterns that help them predict what the larger square numbers will be. If students can use the patterns on the chart to predict which numbers they might expect to come next, then they can try to build those missing squares as well.
3. Another interesting pattern in the square numbers is in the last digits, where there is a repeating symmetrical pattern ( $1,4,9,6,5,6,9,4,1,0,1,4,9,6$, etc.). Some students might notice this, and there's definitely something to explore here. But in this context, it's better to leave this as an unanswered question for students to ponder on their own.
4. Use ten-frames to help students build to larger numbers.

## Mini-Lesson: What is Multiplication?

Write down $3 \times 4$ on the board. Ask the students what it means, and how to draw a picture of $3 \times 4$. Let students share their ideas. The main points to highlight in the conversation:

- The multiplication sign "x" can be read "groups of." The expression $3 \times 4$ can be understood as "three groups of four."
- There are several ways to draw "three groups of four." One is to actually draw three groups, and put four objects in each one.

- Another useful drawing is an array, with three rows and four columns.


4
8
12

- Once you've established the answer ( $3 \times 4=12$ ), demonstrate how to write in the answer on each of the multiplication tables students will be filling in today.
- Plan to do one or two more examples (i.e., $5 \times 5,1 \times 4$ ) with the class before letting them go to work on filling out the various 5 by 5 multiplication tables.
- Also, point out how the Dot Array Multiplication Table is there as a reference if they want it.

When students have finished with the 5 by 5 multiplication tables, encourage them to look for patterns in the numbers. A brief wrap up on their observations and discoveries is a good way to end at this station.

The 10 by 10 multiplication table is also available, if students whip through the 5 by 5 table. The 10 by 10 table should be theirs for the future, to fill out accurately, to use as a reference, and also to help themselves to memorize their math facts for the future.
$\qquad$

Part 1.
Fill in the table.


Part 2.
What is something you notice about this table?

## Dot Array Multiplication Table

| $\times$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bullet$ | －$\bullet$ | －官 |  | －ヤ・•• |
| 2 | $\bullet$ | $\bullet \bullet$ | $\bullet \bullet \bullet$ | $\begin{aligned} & \bullet \bullet \bullet \bullet \\ & \bullet \bullet \bullet \bullet \end{aligned}$ | $\begin{aligned} & \bullet \bullet \bullet \bullet \\ & \bullet \bullet \bullet \bullet \bullet \end{aligned}$ |
| 3 | $\stackrel{\bullet}{\bullet}$ | $\stackrel{\bullet \bullet}{\bullet \bullet}$ | $\begin{aligned} & \bullet \bullet \bullet \\ & \bullet \bullet \theta \\ & \bullet \bullet \bullet \end{aligned}$ | $\begin{aligned} & \bullet \bullet \bullet \theta \\ & \bullet \bullet \bullet \theta \\ & \bullet \bullet \bullet \bullet \end{aligned}$ | $\begin{aligned} & \bullet \bullet \bullet \bullet 日 \\ & \bullet \bullet \bullet \bullet \theta \\ & \bullet \bullet \bullet \bullet \end{aligned}$ |
| 4 | $\stackrel{\bullet}{\bullet}$ | $\begin{aligned} & \bullet 0 \\ & \bullet \bullet \\ & \bullet \bullet \end{aligned}$ | $\bullet \bullet 日$ $\bullet \bullet 日$ $\bullet \bullet 日$ | $\begin{array}{r} \bullet \bullet \bullet \theta \\ \bullet \bullet \theta \\ \bullet \bullet \bullet 日 \\ \bullet \bullet \bullet \theta \end{array}$ |  －०००० $\bullet \bullet \bullet \bullet$ |
| 5 | $\stackrel{-}{\bullet}$ | $\bullet \bullet$ $\bullet \bullet$ $\bullet 0$ $\bullet \bullet$ | $\begin{aligned} & \bullet \bullet \theta \\ & \bullet \bullet 日 \\ & \bullet 日 \theta \\ & \bullet \bullet 日 \\ & \bullet \bullet \theta \end{aligned}$ |  |  |

Fill in the table.


## Blockout

Topics: Multiplication, area, strategy, addition.
Materials: Crayons or colored pencils, Blockout game sheet
Common Core: 3.OA.A.1, 3.OA.C.7, 3.MD.C.6, 3.MD.C. 7
Roll the dice and shade in a rectangle. How can you claim the most space on the board?

## Why We Love Blockout

This is one of those rare games that reinforces both the skill of multiplication and the visual model that makes sense of it. Blockout can be played competitively or collaboratively, and is a wonderful game to introduce or reinforce the concepts behind multiplication.

## The Launch

Take a volunteer and demonstrate the first several turns of a game of Blockout. Players choose colors, then take turns rolling the dice, and shading in a rectangle given by the dice rolls. If you roll a 2 and a 5 , you can shade in a 2 by 5 (or 5 by 2 ) rectangle. No one can shade in a square that has already been colored. If there is no room to fit the rectangle you rolled on the board, you pass. If all players pass in a row, the game is over. Players get a point for each square they have colored in at the end of the game.

Students can play in groups of 2-4, though 2 is preferable. It is also possible to play individually or collaboratively. For a collaborative or solitaire game, players roll and try to cooperatively fill up as much of the board as possible. If every player must pass in a row, the game is over. The fewer the number of leftover squares, the better the game.

## Prompts and Questions

- How many points does that roll give you?
- Who's ahead?
- What roll are you hoping to get this turn?


## The Wrap

Discuss how students counted up their rolls. With a roll of 5 and 4, how would they have counted up the number of squares in their rectangle? (I.e., counting by 5 s? counting by 4? Other strategies?) Discuss other possible rolls, and how they're counted. How many points to you get for rolling 6 and 6 ?

## Tips for the Classroom

1. For the first time playing, students can play as above. For subsequent games, show students how to track their points as they go. For example, they can write $2 \times 5=10$ inside the 2 by 5 rectangle, and know that they have 10 points for that turn. This connects the game to multiplication without feeling to academic right away.
2. Once students are comfortable writing equations in the rectangles, you can abstract one step further and introduce the scoring sheet.
3. Blockout can also be played as a solo or collaborative game by trying to fill up as much of the board as possible. Play ends when a roll is made that cannot be placed.

## Blockout

For 2 players.
Rules. Players take turns rolling two dice, and drawing a rectangle on the game board with side lengths given by the two numbers they rolled. For example, if you rolled a 3 and a 6 , you would draw a 3 by 6 rectangle, placed horizontally or vertically on the board.

Your rectangle cannot intersect or be contained in any previously drawn rectangles. If you cannot add a rectangle to the board on your turn, pass the dice to the next player. If all players pass in a row, the game is over.

Players get a point for each square they've drawn a rectangle around. For example, a 3 by 4 rectangle is worth 12 points. Whoever boxes the most squares wins.

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Blockout Scoring Sheet

| Turn | Player 1 Equation | Player 1 Score | Player 2 Equation | Player 2 Score |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  |  |

## Example Game

Rules. Players take turns rolling two dice, and drawing a rectangle on the game board with side lengths given by the two numbers they rolled. For example, if you rolled a 3 and a 6 , you would draw a 3 by 6 rectangle, placed horizontally or vertically on the board.

Your rectangle cannot intersect or be contained in any previously drawn rectangles. If you cannot add a rectangle to the board on your turn, pass the dice to the next player. If all players pass in a row, the game is over. So Player 1 doesn't get too great an advantage, their first rectangle must be drawn in the corner. After that, rectangles may be drawn in any open spot.

Players get a point for each square they've drawn a rectangle around. For example, a 3 by 4 rectangle is worth 12 points. Whoever boxes the most squares wins.


## Day 3

## Goals

1. Connect units and unitizing to multiplication.
2. Explore multiplication intuitively using Upscale Pattern Blocks.

## Opener <br> Unit Chat

Note: the lesson plan for unit chats is below. All unit chat images are in Appendix 4. You can go through unit chats in order, or skip around if you'd like to try different types earlier.

You can also find more unit chats at mathforlove.com/lesson/unit-chats.

## Activity

Pattern Block Multiplication 1

## Game

Pig

## Choice Time

## Blockout

Pig
Challenge Problems: Times Table Counting Challenges
Note: Challenge Problems are a good regular option for Choice Time.
Find more Challenge Problems in Appendix 3.

## Closer

Pose this question to students:
If a blue rhombus is worth 1 , what does 1 hexagon, 2 hexagons, and 3 hexagons equal?
Let students discuss in pairs or small groups, then discuss as a class to see how students approached the problem.

## Unit Chats

Topics: Mental math, numerical fluency; argument \& critique
Materials: White board or projector
Common Core: Variable, but generally good for NBT, NF, OA, MP1, MP3, MP6.
Counting with respect to different units.

## Why We Love Unit Chats

Unit Chats are a kind of Number Talk that emphasizes not just how many, but also the unit involved. These are a fantastically productive, fun, differentiated, and delightful warm up for math. Perfect as a 5-10 minute opening or closing exercise.

## The Launch

Post a Unit Chat image. It should have different kinds of objects to count in it, and be arranged in arrays or other structures as appropriate for the student level. Students get some time to look at what is in the picture, and how many of which object they see. After they've had 20-60 seconds to look, ask students what they see. You'll receive different answers about what they saw, and how many. You can ask students to explain different ways of counting what they saw, and also different things that they see to count in the picture.

## Example Unit Chat

Teacher: Take a look at this picture. Think about how many you see. [Waits for 30 seconds.] Quietly turn to the person next to you and tell them how many you see. [Students quietly discuss.] Who would like to share what they saw?
Student: I see avocados.
Teacher: How many avocados do you see?
Student: I see fifteen.
Teacher: Fifteen avocados. I don't see that at all.


Photo credit: Christopher Danielson

Student: Look, there are five on the top, then another five, and then five on the bottom. So that's 15 .
Teacher: Ah! You're talking about the avocado halves. In that case, I agree. That's 5, 10, 15 avocado halves. What else do you see?
Student: They're in a checkerboard pattern.
Teacher: That's true. The pitted avocado halves and the unpitted avocado halves form a checkerboard. Does that mean there are the same number of each?
Student: Yes! / No!
Student: There are 8 with pits.
Teacher: Let's count. 1, 2, 3, 4, 5, 6, 7, 8. That's right. Did you count one by one?

Student: No, I saw the 3 on the top, plus 2, plus 3 .
Teacher: Ah, and $3+2+3=8$.
Student: There are only 7 without pits.
Teacher: It's strange that it would be different if they were in a checkerboard pattern. I still don't see why there are more with pits than without.
Student: Because the first and last have pits. If there were one more row, it would be the same.
Teacher: I think I see. You're saying it goes "pit, no pit, pit, no pit,..." But it ends on "pit" and starts on "pit," so there's an extra.
Student: One "no pit" got thrown away.
Teacher: What do you mean?
Student: There were eight avocados that got cut in half, but one no-pit half isn't there.
Teacher: How do you know?
Student: Because if you put all the halves together, it would make wholes, and there would be eight wholes. But the last no-pit half is missing.
Teacher: So how many whole avocados are there?
Student: Seven and a half.
Teacher: I see. So we could see this as 15 half avocados, or we could see it as 7 and a half whole avocados. Very neat!

## Prompts and Questions

- How did you see that?
- How did you count that?
- Does anyone else think they can explain what Therese is saying?
- Turn to the person next to you and see if you can see what Dwayne is describing.


## Tips for the Classroom

1. Use images that are accessible to everyone. The best images have some easy things to count and some harder things to count.
2. You can emphasize how students counted, or shift the conversation to what they counted, depending on what will be the most engaging and enlightening. It can be okay if Unit Chats turn into something that resembles a Number Talk.
3. Remember: doing more short Unit Chats is better than doing just a few long ones. Aim for 5-10 minutes. You can use multiple images if they go super short, but often one image is plenty.

## Pattern Block Multiplication 1

Topics: Multiplication, multi-step problems
Materials: Upscale Pattern Blocks, scratch paper and pencil
Common Core: 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8, 3.MD.7.d, MP1, MP3, MP7
Note: Alternatively, it's possible to use generic pattern blocks instead of Upscale Pattern Blocks. If doing so, be sure to remove the orange squares and tan rhombuses before starting this activity, or tell students not to use them.

If you know what one block equals, can you figure out the value of all the shapes?

## Why We Love Pattern Block Multiplication 1

This lesson involves fundamental ideas like changing the unit and multiplication in a hands-on context that prepares students for subtle concepts like division and fractions. Highly accessible, and easily differentiable.

## Launch

The game in this activity is to change the value of the triangle and see what the other blocks-and larger collections of blocks-are worth. Start by posing a simple series of questions:

- If the triangle equals $1 . .$.

What does the rhombus equal? (2)
What does the trapezoid equal? (3)
What does the hexagon equal? (6)
Let students prove these values are correct by covering the shapes with triangles, or making equivalent arguments ( 3 triangles in a trapezoid and two trapezoids make a hexagon, so $2 \times 3=6$ triangles in a hexagon).

Once these values are established, move on to some harder questions:

- If the triangle equals $1 . .$.

What is the value of 4 trapezoids? (12)
What is the value of 4 hexagons? (24)
Let students share their thinking on these questions as well. You can write out the arguments on the board or on scratch paper to demonstrate the kind of recording you'll expect from students.

Now we move in to the main part of the activity. Let students build a shape of their choosing, giving them a minute to build. When a minute is up, ask them to determine the value of their shape (given that the triangle is equal to 1 ), and the value of their
neighbor's shape. When they have written up their answer with a clear explanation, they can build a bigger, more complicated shape and solve that too. Repeat as time permits.

## Prompts and Questions

- How did you find that answer?
- What's the value of just your hexagons?
- Show me what you've written down so far.


## The Wrap

Find a design that's easy enough to be accessible to everyone, and pose it as a final problem. Let students attempt it on their own, writing down their work as clearly as they can. Then share some different student attempts to solve the problem.

For example, say your final problem was to find the value of 2 hexagons and 6 trapezoids. Students may have many different methods:

## Method 1

Hexagon $=6$, so the value of the hexagons is $2 \times 6=12$.
Trapezoid $=3$, so the value of the trapezoids is $6 \times 3=18$.
Total value is $12+18=30$.

## Method 2

Put together the 6 trapezoids to make 3 more hexagons, for a total of 5 .
That gives us a total value of 5 hexagons $=5 \times 6=30$.

## Method 3

Count each piece and add.
Hex + hex + trap + trap + trap + trap + trap + trap $=6+6+3+3+3+3+3+3=30$.

## Tips for the Classroom

1. An excellent uplevel for this activity is to ask a pair of students to find the sum of and difference between the shapes they built.
2. Don't try to keep all the students together and working on the same problem. Rather, let students work at the appropriate level of difficulty. Just make sure that everyone has attempted (or can do) the problem you discuss at the end.
3. Encourage students to write down their work with simple pictures and equations. A helpful way to encourage recording is to count all the hexagons, record that number, and use a multiplication equation to determine how many triangles that is, then repeat for other shapes, and find the sum. (Other methods work as well, of course.)
4. Some students may not be comfortable with multiplication. They can use addition to solve their problems.

## Pig

Topics: Probability, strategy, addition, estimation
Materials: One 6-sided die, pencil and paper
Common Core: 2.OA.B.2, 2.NBT.B.5, 2.NBT.B.6, MP1, MP7
Roll the dice and collect points. You can go as long as you want, but roll the wrong number and you lose all your points from that turn!

## Why We Love Pig

Pig is easy to learn and gives students lots of addition practice. Pig is also mathematically rich. Students get to articulate and defend strategies, and get practice with addition in a complex task.

## The Launch

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students give you "thumbs up/down" if they think you should keep rolling. If students aren't comfortable adding up all the numbers they roll by hand, have them take tiles or other counters to one spot when it is their turn, and place them in another spot (with ten frames or a hundred chart) when they "bank" them.

## How to Play

Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a $2,3,4,5$, or 6 , the player adds that many points to their score for the turn. A player may choose to end their turn at any time and "bank" their points. If a player rolls a 1, they lose all their unbanked points and their turn is over.

Beginner Game: The first player to score 50 or more points wins.
Advanced Game: The first player to score 100 or more points wins.

## Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?
- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?


## The Wrap

The question of strategy is a fascinating one for Pig. What strategies are students using? Does strategy even matter? Let students share their ideas for strategies, and discuss which ones they think are better or worse, and why.

## Tips for the Classroom

1. Demonstrate the game a couple times with the whole group. Solicit advice about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. For students who are less confident with addition, use ten frames and counters or a hundreds chart to keep track of the score. For example, students get to pick counters up as they roll. If they stop before they roll a one, then they transfer those counters to their ten frames. Fill up 5 ten frames to win.
3. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!
4. As kids play each other, circulate through the room and ask them about their strategies. It's okay for students simply to play, but there's an opportunity to probe deeper into the workings of chance and the strategy of the game too.
5. Remind students they can teach Pig to someone at home and play there.

Name

> Pig

Rolls
Rolls

Bank
$\qquad$

## Times Table Counting Challenge 1

How many dots are on this 3 by 3 dot array multiplication table?
Find the answer without counting one by one.


Answer: $\qquad$
Defend your answer:

## Times Table Counting Challenge 2

How many dots are on this 4 by 4 dot array multiplication table?
Find the answer without counting one by one.


Answer: $\qquad$
Defend your answer:

## Day 4

## Goals

1. Extend understanding of multiplication in different contexts, including openers, games, and story problems.

## Opener

Penny Nickel Dime

## Activity

The Ant and the Grasshopper
Note: this is a long-form story problem. Read the entire story to students as if it were story time. Then you can ask the students some questions they might have about the story. One that will hopefully come up is: how many seeds did the ant collect? This can transition into having students solve the problems on the accompanying sheet. Make sure students have access to the original story text as well, since that's what they'll need to turn toward to solve the problems on the sheet.

## Game

Big Blockout

## Choice Time

Big Blockout
Pig
Penny Nickel Dime
Challenge Problems - see Appendix 3

## Closer

Ask students how they could get the most points in Big Blockout if they rolled a 4, 5, 6 . Have them defend their response to each other, in pairs of small groups, and then to the class.

## Penny, Nickel, Dime

Topics: Addition to 100, multiplication by 5 and 10, money, estimation
Materials: One 6-sided dice, pencil and paper, pennies, nickel, and dimes (optional) Common Core: 3.OA.3, 3.OA.5, MP1, MP6, MP7

Roll the die 7 times. For every roll, you get to take that many pennies, nickels, or dimes. Whoever gets closest to $\$ 1$ without going over wins.

## Why We Love Penny Nickel Dime

This version of Don't Break the Bank is a hit with students, and also helps give a concrete meaning to place value by linking it to both money, addition, estimation, and multiplying by fives and tens. It takes very little time, so it can be used as a warmup, station activity, or in those five minutes before class ends. While kids may break the bank their first few games, they'll inevitably start estimating and choosing good strategies for themselves.

## How to Play

Whole Class/Group Game: The teacher (or a student) rolls the die. Whatever number it lands on, each player can choose to take that many pennies, that many nickels, or that many dimes. More practiced players can just record the numbers in a T-chart like the one below. Repeat for six times total, with each player choosing whether each number goes in the dimes or pennies column as you go.

The winner is the person who comes as closest to $\$ 1$ without going over.
Partner Game: Same as whole class game, except students take turns rolling the die, and everyone ends up entering different numbers into their grid.

## Tips for the Classroom

1. Have students draw a chart (see following page) to track their choices.
2. When playing in stations or with a small group, you can demonstrate how to count your total after the game. Students can also check each other's work in pairs.
3. Let students bust (go over \$1) as they refine their strategies. They'll catch on as they find they end up too high or too low.
4. To keep the game novel, ask students what they'd do differently if there were only 5 rolls per game, or six. Or eight! Try those variations and see what happens.
5. For students who need more help, you can play with physical coins, or have them draw the coins. Other students may only need to count how many of each coin they take on a turn.
6. Downlevel the game by removing nickels as an option. Uplevel the game by adding quarters.

## Penny, Nickel, Dime




## The Ant and the Grasshopper

Winter was coming, and the Ant started to prepare. Every day for a week, it saved 3 tiny seeds.

At the end of the week, the Ant passed the Grasshopper. "Silly ant!" said the Grasshopper. "You work all day, when you could be having fun!" And the Grasshopper laughed and pointed at the ant.

In the second week, the Ant worked even harder. It saved 4 tiny seeds every day. But when the Ant passed the Grasshopper at the end of the week, the Grasshopper laughed and pointed, saying, "Silly Ant!"

In the third week, the Ant worked even harder. It save 6 tiny seeds every day. But when the Ant passed the Grasshopper at the end of the week, the Grasshopper laughed and pointed, saying, "Silly Ant!"

Then winter came. The Grasshopper had nothing to eat. It went to the Ant's hill, and asked if the Ant had any extra seeds for it to eat. And do you know what the Ant said?
"Silly Grasshopper!"

## The Ant and the Grasshopper: Questions

Answer the questions below. Defend your answers using the story. Use scratch paper and draw pictures as necessary.

1. How many seeds did the Ant save in the first week?
2. How many seeds did the Ant save in the second week?
3. How many seeds did the Ant save in the third week?
4. How many seeds did the Ant save in all?
5. Winter lasts for 90 days, and the Ant eats one seed every day. Does the ant have enough food for the winter?
6. Do you think the Ant should give the Grasshopper any seeds? If so, how many?
7. Bonus: If the ant saved 6 seeds every day for 5 weeks in a row, how many seeds would that be in all?

## Big Blockout

Topics: Multiplication, commutativity and associativity of multiplication
Materials: Three dice per game, board, colored pencils
Common Core: 3.OA.7, 3.OA.9, MP1
Roll three dice; add two and multiply by the third. How do you get the highest score?

## Why We Love Big Blockout

Big Blockout is a quick and fun game for multiplication practice that poses a fascinating question at the same time. This Blockout adaptation connects the game to the array model of multiplication.

## The Launch

Big Blockout can be played with 2-4, but fewer players is usually better. Players take turns rolling three dice on their turn. On your turn, draw an array on the board. One side of the array is the sum of two dice of your choice; the third die gives the other side. In other words, you add two of your rolls together, and multiply by the third. That is your score for the turn.

Example. You roll 3, 5, 6 on your turn. You could add $6+3$ to get 9 , and multiply by 5 to score 45 points on the turn. But wait! If you add $5+3$ to get 8 , and multiply by 6 you can get 48 points! So scoring 48 points is actually the better option. This means drawing an 8 by 6 array (if there is space for it) would be your best move. (You could have gotten 33 points as well--do you see how?)

## Prompts and Questions

- What's the best way to get the most points after you roll? Is there some rule for which numbers you should add and which you should multiply?
- Do some scores come up more often than others?


## The Wrap

The fundamental choice in Big Blockout is which two numbers to add and which number to multiply by. Let's try a few more examples-see if you can figure out the best move. Since we know from the last game that multiplication describes a rectangle, we can look build a rectangle for each of these problems to help us.

You roll 1, 4, 5 . What's your best move? There are three options.
$(1+4) \times 5=25$
$(1+5) \times 4=24$
$(4+5) \times 1=9$

## 25 is the best move.

You can pose as many of these followup questions as you have time for. After each one, give the students a minute to solve one or more of the problem below and discuss amongst themselves.

You roll 2, 4, 5. What's your best move?
You roll 3, 4, 5. What's your best move?
You roll 4, 4, 5. What's your best move?
You roll 5, 4, 5. What's your best move?
You roll 6, 4, 5. What's your best move?

## Variations

Big Blockout Place Value: Instead of adding two dice, make a 2-digit number out of them. So if you rolled 3,4 , and 2 , you could get $34 \times 2=68$ points. Or better, you could get $32 \times 4=128$ points. (Play without the board - just tally scores every turn.)

Big Blockout Pro: Roll four dice instead of three. Add three dice of your choice together and multiply by the fourth.

## Tips for the Classroom

8. As students get more accustomed to the game, increase the difficulty by adding in 8 , 10 , and 12 sided dice.
9. For students who are less confident, let them use their multiplication tables to help them with the game.
10. You can also play Big Blockout without the Board, and just keep track of the score. Play to 200, for example.

## Big Blockout Board

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