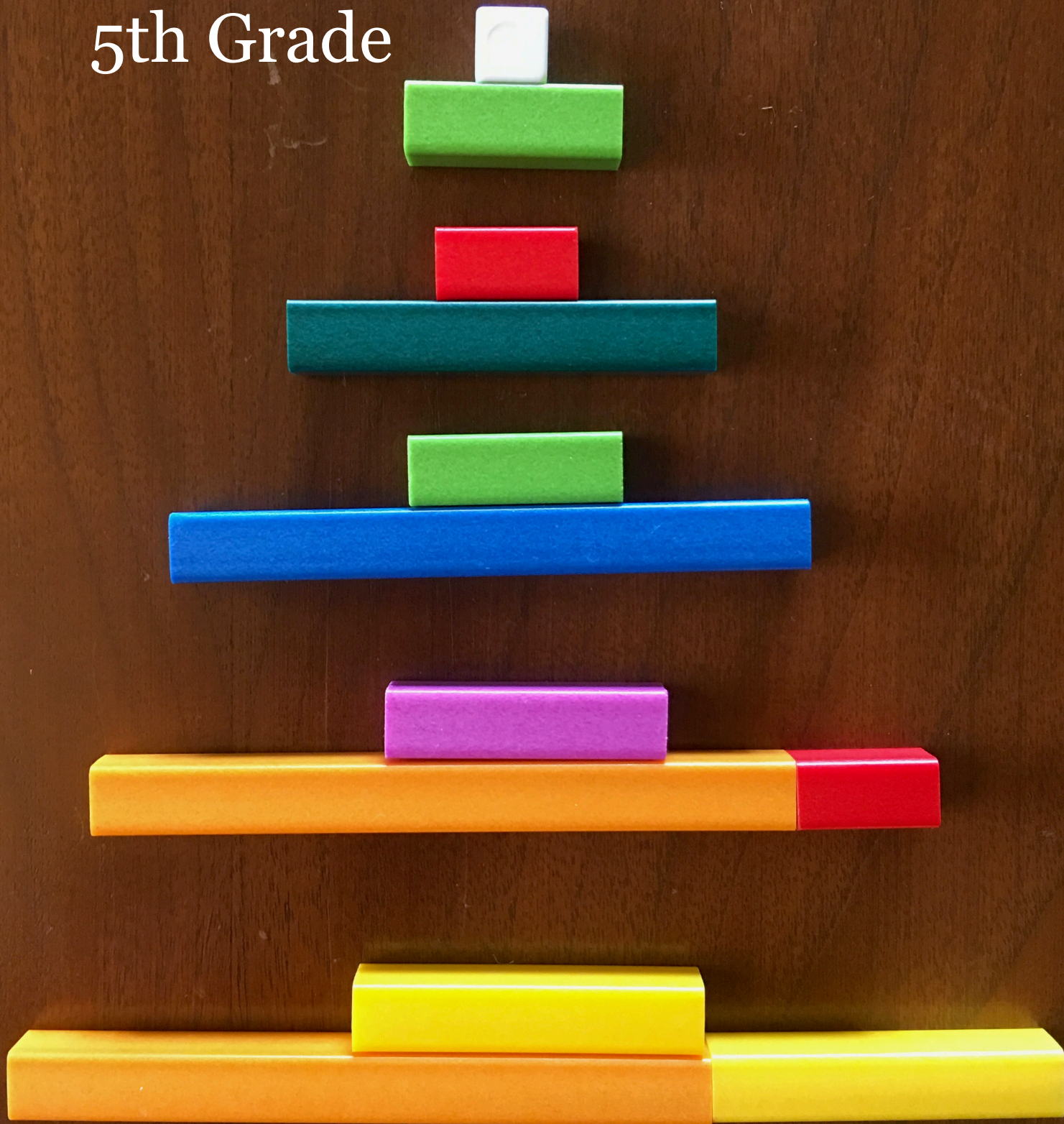


# Math for Love

5th Grade



by Dan Finkel & Katherine Cook





# Introduction

Welcome to the Math for Love Curriculum! This draft was adapted from a summer program we designed in 2017. The materials are ideal for use in a summer program, or as a supplemental curriculum to provide remediation or enrichment to students.

## Goals of the Curriculum

The goals of the program are two-fold:

- Improve student conceptual understanding of mathematics, while exercising skills and fluency
- Give everyone an opportunity to have fun and enjoy math

This curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorial while making connections to abstract reasoning.

## Program Values

The goals of this curriculum are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

## Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement.** Create an classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation.** Help students encounter problems, games, and activities of the right level of difficulty to create engagement.
3. **Thinking.** Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand and make meaning in a situation they don't yet fully understand.

4. **Positive Environment.** Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of the summer if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Ask students questions rather than telling them answers**

Rather than telling them whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates.

★ **Model how to play games, and teach how to win and lose**

Students can sometimes get really attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**

Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Number Talk or opening game (see the Warm Up in the daily plan). Use the Math Games and Movement Breaks from Appendix 1 for transitions between stations. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Have a *growth mindset* classroom**

Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. These students have what is known as 'fixed mindset' about math. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you learn to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Embrace mistakes**

One important way to encourage growth mindset is to embrace mistakes. They are a natural part of learning, and even more than that, they actually help us learn more and help us remember what we've already learned. Model for your students how to make mistakes, and how to use mistakes productively.

★ **Give your students *time* to think and explore**

Remember that many of the students in the program are here because they weren't given enough time to establish solid conceptual models. We are going to protect their time to develop those models in the summer. Make sure you don't push them too fast to drop the blocks or pictures. If you need to take more time on some lessons and don't make it through everything that's fine; this curriculum is built to give you more than you might need. Also note that a central place in the curriculum where the students practice fluency is in the games, and the goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**

We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students break down problems if necessary. However, don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students in this program will be more familiar with the "stuck" part, so try to start them with successes, and then slowly move them toward greater problem-solving stamina.

★ **Value play**

It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

## Other Notes and Best Practices

If you use this curriculum as a standalone for a summer program or other intervention, here are some ideas to help get the most out of it.

### ★ **Math Games and Movement Breaks**

Check out the math-based movement breaks in Appendix 1. These are great to mix in as breaks between activities.

### ★ **Folder for Worksheets**

Give each student a folder where they can keep their worksheets. If they finish another activity early, they can turn back to their unfinished worksheets and finish them.

### ★ **Choice Time**

Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should choose an activity that is right for them, and then stick with it for at least half of Choice Time.

### ★ **Number Talk Images and other warm ups**

For the Number Talks that require images, see Appendix 2. You can project these images to your class, or, where possible, create physical versions of them with magnetic ten frames or other blocks. Physical versions are sometimes preferable, since students can manipulate the blocks directly.

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# Day 1

## Goals

1. Establish class norms and community.
2. Explore strategy and probability while strengthening multiplication skills.
3. Learn and play a series of math games.

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## Part 1

Introduction, name games (i.e., Mingle), class agreements.

## Warm Up

Don't Break the Bank

## Pre-Assessment

## Exploration

Pattern Block Multiplication

Included below is a multi-part series of lessons.

Go as far into this series as comfortable.

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## Part 2

## Warm Up

Fraction Talk

## Game

Odd Pig Out

## Exploration

Cuisenaire Rod Multiplication

## Choice Time

Odd Pig Out; Don't Break the Bank; Challenge Problems

## Wrap Up

# Mingle

## Opening Game or Station Break

Mingle is a quick name game you can play on the first day of class. You can also return to its more mathematical versions later in the course as a station break.

### How to play

The teacher calls out a number (i.e., 3), and the students get themselves into groups of that size (or as near as possible to that size as possible) as quickly as they can. It might be impossible for everyone to get in a group every time, but each new number gives everyone another chance.

Once they are in groups, students can each learn each other's names. Then the teacher calls out a new number.

In the basic game, just call out single numbers. Once students get the idea, you can call out addition or subtraction problems (i.e., "get into groups of  $7-4$ ").

### Tips for the classroom

1. Call the adults in or out of the game depending on the number of students you have and what numbers you call, in order to give everyone at least one other person to have in their group.
2. Keep the game moving quickly to keep the energy up.
3. Don't forget to call out a group of 1 and a group of however many students are in the entire class at some point in the game.
4. For future games, once everyone knows each other's names, you can lead an optional skip-count with the class by counting the students in the class by group size (i.e., 3, 6, 9, ...).

# Don't Break the Bank!

**Topics:** Triple-digit Addition, Estimation, Probability

**Materials:** One 6-sided dice, pencil and paper

**Common Core:** 2.NBT.1, 2.NBT.3, 2.NBT.4, 2.NBT.5, 2.NBT.6, 2.NBT.7, 3.NBT.2, 4.NBT.B.4,

How close can you get to 999 without going over?

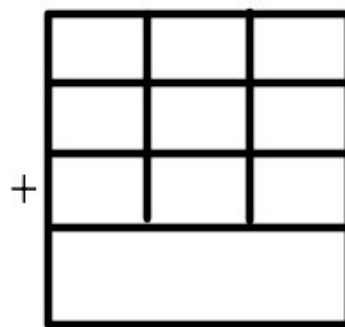
## Why We Love Don't Break the Bank

Don't Break the Bank is a Place Value powerhouse. It takes very little time, so it can be used as a warmup or in those five minutes before class ends. It's fun, and kids *love* it, even though it involves addition practice. And, while kids will usually break the bank (that is, go over 999) their first few games, they'll inevitably start estimating and choosing good strategies for themselves. Should the digits in the hundreds column add up to 9 or 8? How common is it to carry? The deeper thinking is almost inevitable.

## The Launch

Everyone makes a diagram like this on their paper:

Whole Class Game: The teacher (or a student) rolls the die. Whatever number it lands on, everyone enters it in one of the nine spots on the board. After nine turns, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum **without going over 999**. (See next page for example game.)



Small Group Game: Same as whole class game, except that you take turns rolling the die, and everyone ends up entering different numbers into their grid.

## Prompts and Questions

- What's a good strategy for this game?
- Where would you put this 5?
- Have you already "broken the bank?" How can you tell?

## Tips for the Classroom

1. When you are playing a game with the full class, let students take turns rolling.
2. You can narrate your own thoughts when placing digits in the grid. Remember to be clear that you're placing ones, tens, and hundreds.
3. Students may not entirely understand the game the first time through, but they should get the hang by the second game.
4. Extend the game to decimals by adding decimal points up and down one column.

# Example Game.

Turn 1: I roll a 4, and place it in my grid. So does the rest of the class.

	4		
+			

Turn 2: I roll a 2, and place it in the middle.

	4		
		2	
+			

Turns 3 - 8 pass in the same way. Perhaps I have a grid like this:

At this point, I see that I'll be in trouble if anything except a 1 is rolled, since I'll have broken the bank by going over 999.

	4		1
	2	2	1
+	3	6	6

Turn 9: A 5 is rolled, and I broke the bank! When I enter the 5 and add up my numbers, I'm over 999, and I'm out this game.

Now it's time to play again!

	4	5	1
	2	2	1
+	3	6	6
1	0	3	8

# Pattern Block Multiplication 1

Topics: Multiplication, multi-step problems

Materials: Pattern blocks, scratch paper and pencil

Common Core: 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8, 3.MD.7.d, 4.OA.1, 4.OA.2, MP1, MP3, MP4, MP6, MP7

If you know what one block equals, can you figure out the value of all the shapes?

## Why We Love Pattern Block Multiplication

This lesson involves fundamental math ideas like changing the unit and multiplication in a hands-on context that prepares students for subtle concepts like division and fractions. This is a highly accessible and easily differentiable lesson.

## Launch

The game in this activity is to change the value of the triangle and see what the other blocks—and larger collections of blocks—are worth. Start by posing a simple series of questions:

- If the triangle equals 1...  
What does the rhombus equal? (2)  
What does the trapezoid equal? (3)  
What does the hexagon equal? (6)

Let students prove these values are correct by covering the shapes with triangles, or making equivalent arguments (3 triangles in a trapezoid and two trapezoids make a hexagon, so  $2 \times 3 = 6$  triangles in a hexagon).

Once these values are established, move on to some harder questions:

- If the triangle equals 1...  
What is the value of 4 trapezoids? (12)  
What is the value of 4 hexagons? (24)

Let students share their thinking on these questions as well. You can write out the arguments on the board or on scratch paper to demonstrate the kind of recording you'll expect from students.

## The Work

Now we move to the main part of the activity. Let students build a shape of their choosing, giving them a minute to build. When a minute is up, ask them to determine the value of their shape (given that the triangle is equal to 1), and the value of their neighbor's shape. When they have written up their answer with a clear explanation, they can build a bigger, more complicated shape and solve that too. Repeat as time permits.

## Prompts and Questions

- How did you find that answer?
- What's the value of just your hexagons?
- Show me what you've written down so far.

## Wrap

Find a design that's easy enough to be accessible to everyone, and pose it as a final problem. Let students attempt it on their own, writing down their work as clearly as they can. Then share some different student attempts to solve the problem.

For example, say your final problem was to find the value of 2 hexagons and 6 trapezoids. Students may have many different methods:

### Method 1

Hexagon = 6, so the value of the hexagons is  $2 \times 6 = 12$ .

Trapezoid = 3, so the value of the trapezoids is  $6 \times 3 = 18$ .

Total value is  $12 + 18 = 30$ .

### Method 2

Put together the 6 trapezoids to make 3 more hexagons, for a total of 5.

That gives us a total value of 5 hexagons =  $5 \times 6 = 30$ .

### Method 3

Count each piece and add.

Hex + hex + trap + trap + trap + trap + trap + trap =  $6 + 6 + 3 + 3 + 3 + 3 + 3 + 3 = 30$ .

## Tips for the Classroom

1. An excellent uplevel for this activity is to ask a pair of students to find the sum of and difference between the shapes they built.
2. Don't try to keep all the students together and working on the same problem. Rather, let students work at the appropriate level of difficulty. Just make sure that everyone has attempted (or can do) the problem you discuss at the end.
3. Encourage students to write down their work with simple pictures and equations. A helpful way to encourage recording is to count all the hexagons, record that number, and use a multiplication equation to determine how many triangles that is, then repeat for other shapes, and find the sum. (Other methods work as well, of course.)
4. Some students may not be comfortable with multiplication. They can use addition to solve their problems.

# Pattern Block Multiplication 2

**Topics:** Multiplication, Multi-Step Problems

**Materials:** Pattern Blocks, scratch paper and pencil

**Common Core:** 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8, 3.MD.7.d, 4.OA.1, 4.OA.2, MP1, MP3, MP4, MP6, MP7

If you know one block, can you figure out all the shapes?

## Why We Love Pattern Block Multiplication

This lesson involves fundamental ideas like changing the unit and multiplication in a hands-on context that prepares students for subtle concepts like division and fractions. Highly accessible, and easily differentiable.

## Launch

The idea of this lesson is the same as with Pattern Block Multiplication Part 1—we now increase the difficulty and involve more steps in the problems. Note that tan rhombuses and orange squares are not used in this lesson.

Make three piles of pattern blocks: one with 5 hexagons, one with 11 trapezoids, and the last with 4 hexagons, 4 trapezoids, 3 blue rhombuses, and 2 triangles. Start with a review problem: if the triangle is worth 1, what is each pile worth? Students can solve the problems on their own or in pairs. Have students write down their answer for each pile on scratch paper. Briefly discuss how students solved (or might have solved) the problem.

From there, pose questions at the appropriate difficulty level for students:

What is each pile worth if...

1. The rhombus equals 4?
2. The trapezoid equals 9?
3. The triangle equals 5?
4. The hexagon equals 12?
5. The triangle equals 7?
6. The rhombus equals 8?

## Wrap

Pick the hardest problem everyone in the group attempted (or solved), and discuss different approaches to solving. For example, if the trapezoid equals 9, and you wanted to find what 5 hexagons equals, you might:

1. Think of the 5 hexagons as 10 trapezoids, and call it  $10 \times 9 = 90$ .

2. Say that each hexagon equals 2 trapezoids, so each hexagon equals  $2 \times 9 = 18$ . That means 5 hexagons =  $5 \times 18 = (5 \times 10) + (5 \times 8) = 50 + 40 = 90$
3. Think that triangles must equal 3 if trapezoids equal 9. Then imagine each hexagon as 6 triangles. This means there are 30 triangles in all, each equal to 3, which gives us  $30 \times 3 = 90$ .

## Tips for the Classroom

1. To adjust the difficulty, you can also make the piles larger, smaller, or more or less mixed with different blocks.
2. Don't try to keep all the students together and working on the same problem. Rather, let students work at the appropriate level of difficulty. Just make sure that everyone has attempted (or can do) the problem you discuss at the end.
3. Encourage students to write down their work with simple pictures and equations.
4. If some students are ready, call the hexagon equal to 3, or the trapezoid equal to 1 to get fractional answers.
5. A good hint for struggling students is to figure out all the individual blocks (triangle, rhombus, trapezoid, hexagon) before you tackle the larger shapes.

# Pattern Block Multiplication 3

Topics: Multiplication, Multi-Step Problems

Materials: Pattern Blocks, scratch paper and pencil

Common Core: 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8, 3.MD.7.d, 4.OA.1, 4.OA.2, MP1, MP3, MP4, MP6, MP7

If you know the value of one block, can you figure out big shapes?

## Why We Love Pattern Block Multiplication

This lesson involves fundamental ideas like changing the unit and multiplication in a hands-on context that prepares students for subtle concepts like division and fractions. Highly accessible, and easily differentiable.

## Launch

The idea of this lesson is the same as with Pattern Block Multiplication Parts 1 & 2, except we further increase the difficulty to involve larger numbers, and connect to the area model of multiplication more explicitly. Note that tan rhombuses and orange squares are not used in this lesson.

Pull out a stack of 24 hexagons, and note that if the hexagon is equal to 1, the stack is equal to 24. But what is the stack equal to if...

1. The trapezoid equals 1?
2. The rhombus equals 1?
3. The triangle equals 1?

To make the problem more visibly meaningful to students, group the 24 hexagons in two stacks of 10 and one stack of 4.

Have students solve alone or in pairs, writing their work down on scratch paper. Help as necessary. Then repeat, but this time, have 3 - 4 students each contribute a handful of hexagons: let them each grab a handful, and count how many they picked. For example, students might contribute 8, 11, and 12 hexagons to a new pile. Ask students to write down estimates of the value of the pile if the trapezoid equals 1, the rhombus equals 1, and the triangle equals 1; then, when their estimates are written, to find the answers. Encourage students to use the area model on paper to do their work.

Repeat as time permits. Increase the challenge by increasing the number of hexagons, and by changing the value of the trapezoid, rhombus, and triangle on subsequent rounds.

What if... the trapezoid equals 5? The rhombus equals 5? The triangle equals 5?

What if... the trapezoid equals 8? The rhombus equals 8? The triangle equals 8?

And so on. Adjust these numbers to match the difficulty the students can handle.

## Prompts and Questions

- Can you write down the problem you're trying to solve as a multiplication problem?
- Try using the area model to solve this problem.
- Can you figure out what this stack of 10 hexagons is worth?



= 30 if (rhombus = 1),  
since 1 hex = 3, so 10 hex =  $10 \times 3$ .

This connects nicely to the area model.

(Rhombus = 1) means (Hexagon = 3),

So 24 hexagons will be  $24 \times 3$ .

If we break them up in stacks of 10,  
it looks like an area model.

	10 hex	10 hex	4 hex
3	30	30	12

## Wrap

Pick the hardest problem everyone in the group attempted (or solved), and discuss different approaches to solving. For example, if you decided the trapezoid equaled 5, and you wanted to find what 21 hexagons equals, you might:

1. Calculate that the 21 hexagons are equal to  $21 \times 2 = 42$  trapezoids using the area model, and that those triangles are worth  $42 \times 5 = 210$ , using a new area model picture.
2. Say that each hexagon equals  $2 \times 5 = 10$ , and calculate  $21 \times 10 = 210$ , using an area model if necessary.

And so on.

## Tips for the Classroom

1. To adjust the difficulty, you can also make the piles larger, smaller, or more or less mixed with different blocks. You can also try a stack of, say, 14 hexagons and 14 trapezoids.
2. Let students work at the appropriate level of difficulty for themselves. Just make sure that everyone has attempted (or can do) the problem you discuss at the end.
3. Encourage students to write down their work with simple pictures and equations. Especially encourage using the area model.

# Fraction Talks

**Topics:** Mental math, numerical fluency; argument & critique

**Materials:** White board or projector

**Common Core:** Variable, and especially MP3 and NF

This mental math routine creates powerful positive habits for students to understand and think productively about fractions.

## Why We Love Fraction Talks

Fraction Talks are a powerful way to extend the Number Talk format to the domain of fractions. They get all students involved, help them strengthen fluency, intuition, and mental math strategies, improve students' ability to explain and critique solutions, and allow teachers a valuable window into their students' thinking, as well as giving a framework for students to develop a more visual, conceptual framework

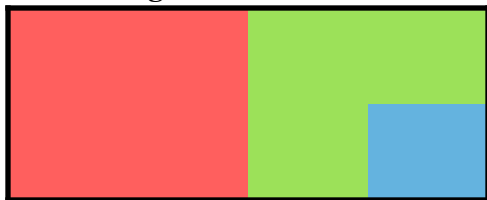
## The Launch

Refer to the write up of Number Talks for more on the mechanics of Number Talks, as a basis for Fraction Talks: <http://mathforlove.com/lesson/number-talks/>

Display a picture that represents a fractional relationship or relationships. Students consider the fraction being represented, then, after a brief “think time,” argue why they think the fraction is what they believe it to be. The teacher can facilitate the student discussion, underline powerful ideas, and encourage students to share multiple ways to solve problems.

### Example Fraction Talk

Teacher: I'm going to put up an picture, and you can tell me what fraction of the space is occupied by each color. Take some time to think about it, and about how you can defend your answer. [Teacher shows the image below. Students think about the image.]



Teacher: If you think you can tell me the fraction of the rectangle that is occupied by one—and just one—of those colors, raise your hand.

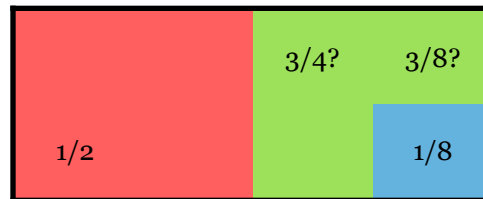
Student 1: Red is half. [Teacher adds this to the image.]

Student 2: Blue is one eighth. [Teacher adds this to the image.]

Student 3: Green is three quarters. [Teacher adds this to the image.]

Student 4: I have another answer: green is three eighths. [Teacher adds this to the image.]

Teacher: It sounds like we'll really need especially strong defenses for that green area to convince each other. Let's start with the other sections. Who can explain why they think the red section is half?



Student 5: Well, there are two halves. Like if the other side was all green, it's the same as the red, and that means there are two halves, since two halves make a whole.

Teacher: So you're imaging two equal pieces making the whole, meaning the red part is half.

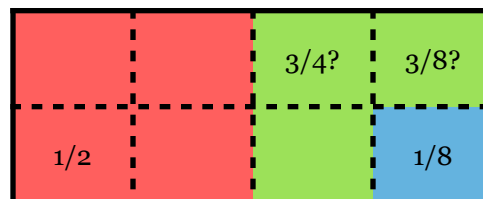
Student 5: Right!

Teacher: Great. Can anyone defend why the blue section is  $1/8$  of the whole?

Student 6: I imagined cutting the whole thing into eighths.

Teacher: I'm not sure I understand what you mean. Can you draw it for us?

Student 6: Sure. [Draws lines in on the drawing.] See? That makes eight pieces, and so blue is  $1/8$ .



Teacher: Very nice! By imaging those lines, we can see eight equal pieces—and you made sure the pieces were equal with how you chose to draw in the lines—and one of eight equal pieces is one eighth of the whole. Did anyone do this another way?

Student 2: I saw that it was one quarter of one half, but I like that way.

Teacher: I do too. I can really see clearly when we have eight equal pieces like that. Does anyone have a defense of either of the conjectures for the green area?

Student 3: I thought it was  $3/4$  since there are three green pieces, so it's  $3/4$  of a whole.

Student 7: I disagree, because if you had all four pieces, you wouldn't have a whole, you would have a half. So it is  $3/4$  of a half.

Student 3: Oh, I guess you're right. So I guess it's  $3/8$ .

Teacher: That's not clear to me. I guess I see that it is  $3/4$  of a half, but why is that  $3/8$ ?

Student 3: I guess I don't see why either.

Teacher: Yeah. And you're modeling exactly how to tackle these, because you're staking a claim, and then you're willing to change your answer when someone shows that it's wrong. But I still want to know if this is really  $3/8$ , and why.

Student 8: Here's how I saw it. We said blue was  $1/8$ . And so each of those boxes is one eighth. So that's one eighth, two eighths, three eighths.

Teacher: Ah, I think I see. You're saying the green is made of up three of these boxes.

How big is one box? One eighth. So that's [pointing at each green rectangle] one eighth, two eighths, three eighths. [Writes this on the image.] So that idea of dividing the entire

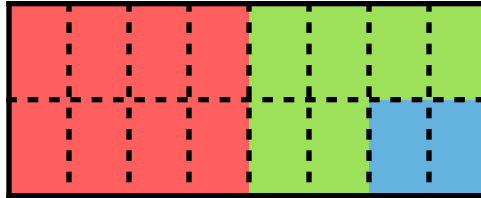
shape into equal pieces made this much easier to solve. So here's my question: if we follow the same logic, shouldn't the red section be four eighths? Because we could count, 1, 2, 3, 4 eighths. So why is one half, and not four eighths? Think about that, and then turn and talk to a neighbor. [Students discuss.] Yes?

Student 9: One half and four eighths are the same thing!

Teacher: Aha! [Writes  $1/2 = 4/8$ .] So our logic was right in both cases, because cutting it up into more pieces doesn't change what it is, even if it changes what we call it. Can anyone else give me another name for  $1/2$ ?

Student 10: Eight sixteenths?

Teacher: What would the picture look like for that? Can you draw it? [Student draws.]



Student 10: That makes 16 equal pieces, and 8 of them are red, so that's  $8/16$ .

Teacher: Great! I can imagine we could have made a lot of different versions of the same thing.

Student 11:  $16/32$ !

Teacher: Okay - let's move on to another talk...

## Prompts and Questions

- Who would like to defend this answer?
- I don't quite follow. Could you draw what you are seeing?
- How did you do that/know that?
- Does anyone else think they can explain what Shawn is saying?
- Turn to the person next to you and explain how you figure out the fraction.

## Tips for the Classroom

1. Start with questions that are accessible to everyone.
2. Students will be looking to see if you indicate what the right answer is. Don't favor right answers over wrong ones. Make sure that the explanations are what matters.
3. Give students constructive language to use in the discussion, like, "I respectfully disagree, because..." and "I agree with \_\_\_\_\_, because..."
4. Always keep the environment safe and positive.
5. Don't worry if you don't reach total consensus on every problem. Sometimes a student will need more time to process. You can move on when it feels like it is time.
6. Fraction Talks can sprawl if you're not careful. Doing short (5 - 10 minute) talks regularly is more powerful than long ones infrequently.

## Resources

Find more at [fractiontalks.com](http://fractiontalks.com).

# Odd Pig Out

**Topics:** probability, strategy, multiplication, addition

**Materials:** Two 6-sided dice, pencil and paper

**Common Core:** 3.OA.7, 3.NBT.2, MP1, MP5, MP6, MP7

Roll the dice and multiply. You can go as long as you want, but roll an odd number and you lose all your points from that turn!

## Why We Love Odd Pig Out

Odd Pig Out is a natural extension of Pig to multiplication. It is great practice for multiplication and addition in a fast-moving, fun game.

## The Launch

The teacher chooses a volunteer, explains the rules, and plays a demonstration game. Because students already know Pig, this game should be relatively intuitive to learn.

Players take turns rolling the dice as many times as they like. After each roll, they multiply the numbers they rolled together. If the product is even, they add that number to their current points for the turn. If the product is odd, players lose all their points from that turn and their turn is over. A player may choose to end their turn at any time and “bank” their points.

Play to 300.

## Prompts and Questions

- Is there an easier way to add up all those numbers?
- How many points to you have for this turn so far?
- Who’s ahead?
- Are you sure that’s the product of those two numbers? What does your multiplication table say?
- What strategy are you using?

## The Wrap

Ask students whether they’re more likely to roll odd products or even products. How many odd numbers are there on the multiplication table (up to 6 by 6)? How many even numbers? How are they distributed? Do students see any patterns?

## Tips for the classroom

1. Demonstrate the game a couple times with the whole class (or in a station). Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.

2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing.
3. Make sure students have a copy of the dot array multiplication table, or the multiplication tables that they have made, handy to help them if they need them.

# Cuisenaire Rod Multiplication

**Topics:** Multiplication, changing units

**Materials:** Cuisenaire rods, paper and pencil

**Common Core:** 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.6, 3.OA.7, 4.OA.2, MP1, MP2, MP3, MP7

You know the value of the white rod... how can you figure out the other pieces?

## Why We Love Cuisenaire Rod Multiplication

This lesson combines the fundamentals of multiplication with deeper problem solving in a context that's natural and hands-on.

## The Launch

This lesson is designed to alternate between the teacher posing problems by assembling groups of Cuisenaire rods physically and saying their value, and students solving the question on their own, and writing up their solutions. Give students time as needed—at least a few minute for the early problems, and more as they get harder.

*Problem 1. If the white Cuisenaire rod equals 1, what are the other rods worth?*

Note: You can pose problems with almost no words by placing the Cuisenaire rods on a white board, and writing the numbers underneath or beside them.

If student haven't thought through this kind of problem before, this is a good warmup problem. Students will likely build a staircase from the rods, and see that red = 2, light green = 3, and so on, up to orange = 10. Challenge them to determine what orange + blue + brown is (orange + blue + brown =  $10 + 9 + 8 = 27$ ).

Once students have found what all the rods are worth, you can ask them to prove how they know that the blue rod is 9. There are many ways to prove it using what you know about the smaller. For example, the blue rod is 4 reds (i.e., 4 twos) plus 1 white (one). That's 9. It's also a yellow plus a purple, which is  $5 + 4 = 9$ . It's also one white less than an orange rod, which gives  $10 - 1 = 9$ . And so on.

*Problem 2. If white equals 2, what are the other rods worth?*

In this case, every rod will be equal to a multiple of 2. Note that some students may mistakenly mistakenly believe that red = 3, light green = 4, etc. This can be proved wrong by noting that white + white = red, which would mean  $2 + 2 = 3$ . Clearly a mistake!

Once students have show their solutions to this problem, you may want to pose several questions at once, so students can work through to harder problems when they're ready.

*Problem 3. If white equals 5, what are the other rods?*

*Problem 4. If white equals 4, what are the other rods?*

*Problem 5. If white equals 6, what are the other rods?*

*Problem 6. If white equals 8, what are the other rods?*

*Problem 7. If white equals 12, what are the other rods?*

*Problem 8. If **red** equals 14, what are the other rods?*

If more problems are needed, let students make up their own.

## Prompts and Questions

- What if the red rod equaled 10? Is that too big or too small?
- How do you know that the brown rod has that value?

## Wrap Up

Take the last problem all students have attempted and spend a few minutes letting students share their answers with each other. You can have them share their methods with a partner, and then take one or two volunteers to share their method with everyone.

## Tips for the Classroom

1. Make sure students can build their own version of the problem and solve physically.
2. Adjust the difficulty of the problems as necessary.
3. Students can always guess and check. This is a good strategy to encourage, since it makes the connection between division and multiplication more explicit.

# Day 2

## Goals

1. Introduce models for understanding the meaning of fractions.
  2. Explore representations of fractions.
  3. Play games to practice multiplication and arithmetic.
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## Part 1

### Warm Up

Fraction Talk

### Exploration

Cuisenaire Rod Fractions & Challenges

### Game

Horseshoes

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## Part 2

### Warm Up

Don't Break the Bank

### Story Problems

1. The Ant and the Grasshopper
2. (optional) The Monster

### Game

Prime Climb

### Choice Time

Horseshoes; Prime Climb; Odd Pig Out; Challenge Problems

### Wrap Up