Broken Calculators

I've got a collection of calculators in my basement--unfortunately, all of them are broken in particular ways. I've recently been curious if I can still use them to do calculations. Every calculator displays 0 at the beginning, except calculators 5 and 6.

Calculator 1
Only two buttons do anything on this calculator: button A adds 3 and button B adds 7. I can make the calculator display 20, for example, by hitting ABAB.
1. What's the largest positive integer you can't display on calculator 1?

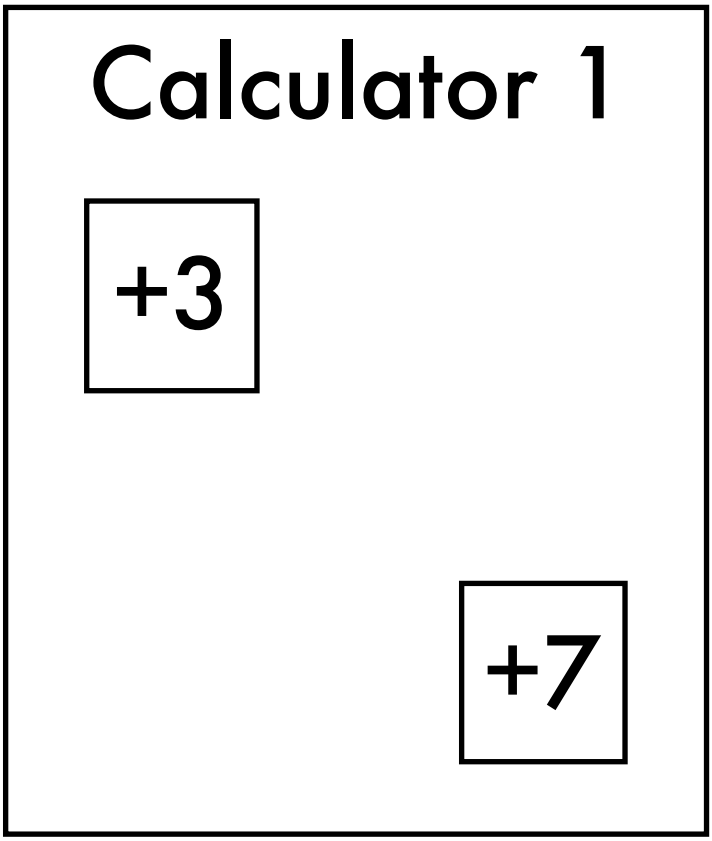
Calculator 2
On calculator 2, button A adds 1, and button B multiplies by 3.
2. What's the fewest number of button presses it takes to get to 102? What about 511?

Calculator 3
On calculator 3, button A adds 6, button B divides by 2, and button C divides by 3.
3. Prove that calculator 3 can display any positive integer.
4. What is the fewest number of button presses it takes to display 99 on calculator 3?

Calculator 4
On calculator 4, button A adds 5, button B adds 7, and button C takes the square root.
5. Prove that it's possible to display any positive integer greater than 1.

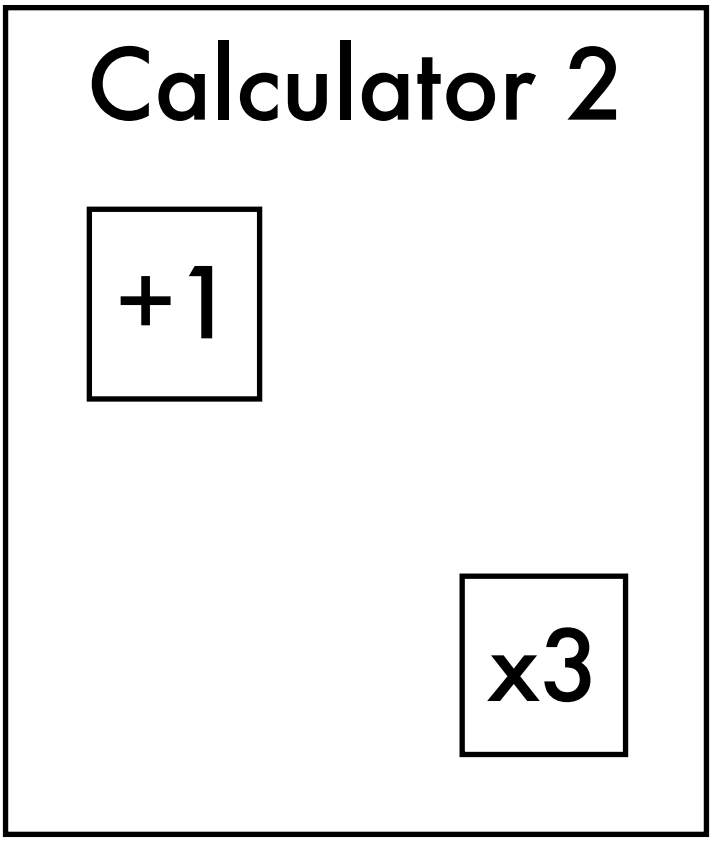
Calculator 5
On calculator 5, button A adds the last number to be displayed to the current number, and button B adds the two last numbers to be displayed to the current number. This calculator always starts at 1. For example, if you hit just button A, you'll display the Fibonacci numbers, 1, 1, 2, 3, 5, 8, 13, ...
If you hit just button B, you'll display the Tribonacci numbers 1, 1, 2, 4, 7, ...
6. Can calculator 5 display any positive integer? If yes, prove it; if not, what is the first number it can't display, and are there finitely or infinitely many numbers it can't display?

Calculator 6
7. Calculator 6 also starts at 1. Button A adds the last three numbers. Button B takes the positive difference of the last two numbers. Can the calculator display any positive integer? If yes, prove it; if not, what is the first number it can't display, and are there finitely or infinitely many numbers it can't display?



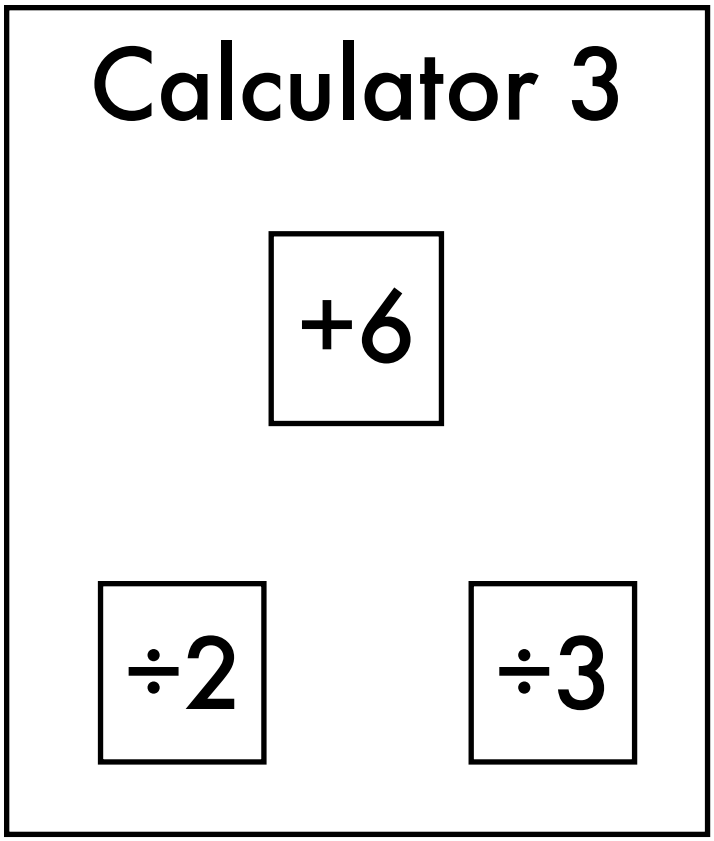
Challenges

1. Make 50 on Calculator 1.
2. Make 50 in the fewest number of button presses.
3. What is the largest number you can't make?



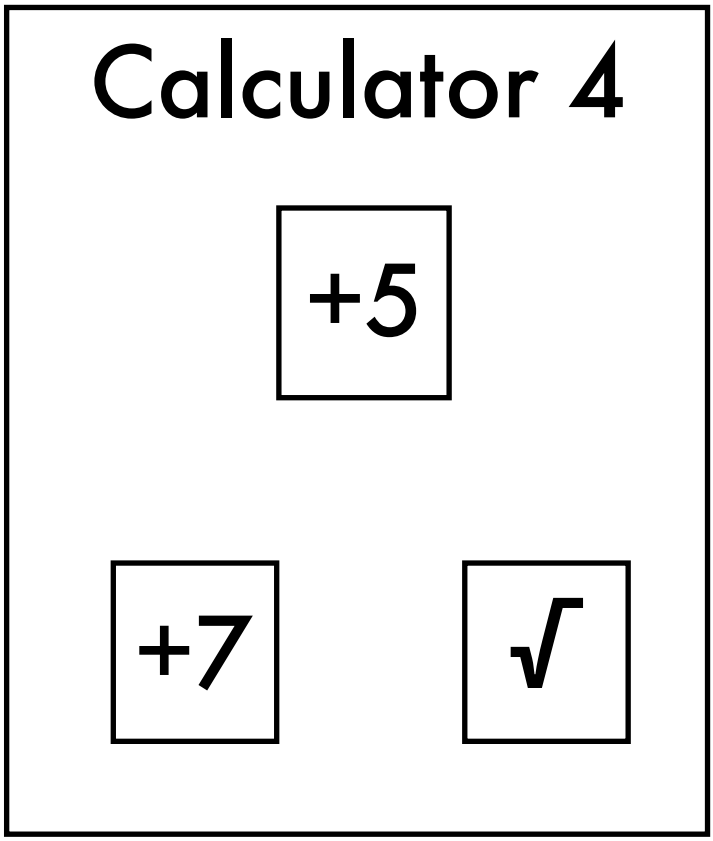
Challenges

1. Find the fewest button presses it takes to get to 50.
2. Find the fewest button presses it takes to make 102.
3. Find the fewest button presses it takes to make 511.



Challenges

1. Find the fewest button presses it takes to get to 50.
2. Find the fewest button presses it takes to make 101.
3. Can you make any number with Calculator 3?



Challenges

1. Find the fewest button presses it takes to get to 11.
2. Find the fewest button presses it takes to make 13.
3. Can you make any number with Calculator 4?

Broken Calculators
Teacher's Notes

Each calculator requires a different insight, but the key to helping students with this one is simply to encourage them to 1) play around with the given rules and 2) *organize* their results. The right organization can help immensely.

You can use the printouts of the calculators to make the puzzles more immediate for students.

For calculator 1, try writing numbers in three columns
1 2 3
4 5 6
7 8 9
etc.

Since adding 3 keeps you in a column, you just need to see what adding 7 does. As soon as you've visited all three columns, you've got a way to get to every number lower down in the chart. Another way to think about it: once you have three numbers in a row, you've got every number greater than them.

For calculator 2, try finding quick routes to smaller numbers. Or, imagine where you would want to be in the move before you reach 102.

Another general hint with some of these is to try to figure out if there's a way to add 1 to the previous number you could get. "You got to 3, can you get to 4?" That's helpful for Calculator 3.