

Counterexamples

Math concepts: logic and deduction

Number of players: 4 - 30

Equipment: Nothing

Time: 5 - 10 minutes

How to Play

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher, though it can be a student) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form “All _____s are _____” or “No _____s are _____.” You can also play around with statements like “If it has _____, then it can _____.” For instance:

- All birds can fly. (Counterexample: penguins)
- No books have pictures in them.
- All books have pictures in them.
- If it produces light, then it is a light bulb.
- If it has stripes, then it is a zebra.
- (harder) No square has a perimeter equal to its area.
(Counterexample: a 4 by 4 square.)

Example Game

Teacher: I claim all animals have four legs. Who can think of a counterexample?

Student 1: A frog!

Student 2: A spider.

Student 3: A fish.

Teacher: Why is a frog a counterexample?

Student 4: Because it has two legs.

Teacher: Right. I said every animal has four legs, but a frog is an animal with just two legs. So I must have been wrong. What about this one: everything with four legs is an animal.

Student 5: A spider.



Teacher: A spider is an animal with eight legs, so it proves that not every animal has four legs. But I claimed that if you have something with four legs, it must be an animal. To prove me wrong, you have to give me something with four legs that isn't an animal.

Student 5: Like a table?

Teacher: Who can tell me if a table is a counterexample to my claim?
And so on.

Why it's a great game

It's possible to vary the difficulty to play with kids as young as kindergarten. What's great, though, is that you can segway to pretty substantial math concepts, and address common misconceptions. What can be as simple as a kind of reverse "I Spy" for youngsters ("I claim that anything green is edible," "Not a tree!") can lead to subtle points like disproving claims like "doubling a number always gives you a larger number" (not true for negative number or 0) or sorting out why every square is a rectangle, but not every rectangle is a square. For older kids, you can even go to much trickier problems like: every point on the number line is a rational number. The language of counterexamples is crucial to distinguish true and false claims in mathematics; this game makes it natural, fun, and plants the skills to be used later.

Tips for the Classroom

1. Start simple.
2. Try using silly claims, e.g., "The only one who likes cookies is cookie monster."
3. Use Number Talk mechanics. Have students raise a thumb at their chest quietly when they have a counterexample, and raise more fingers if they can think of more.
4. Kids can think of their own false claims, but sometimes these aren't the right kind, and they often have to be vetted.
5. Once you get the language of counterexamples out there, look for places to use it in the rest of your math discussions.