PUZZLE 1

Puzzle 1 is pretty hard, but some simplifications can make it easier. If you assume two players ended their game in a draw, that would mean all 15 edges (6 dots, therefore (6x5)/2 = 15) of the game were coloured in.

There are 2^15 = 32,768 ways to do that, though if we consider all the ways that they're equivalent to each other, by rotating the board, say, or changing all the blue edges to red and all the red edges to blue, there are really just 78 distinct ways to draw in all the edges between 6 dots.

You're welcome to try drawing them all in and seeing that there's no blue or red triangle in any of them (see image).

But as a mathematician, I want to avoid that tedious work and find a prettier way.

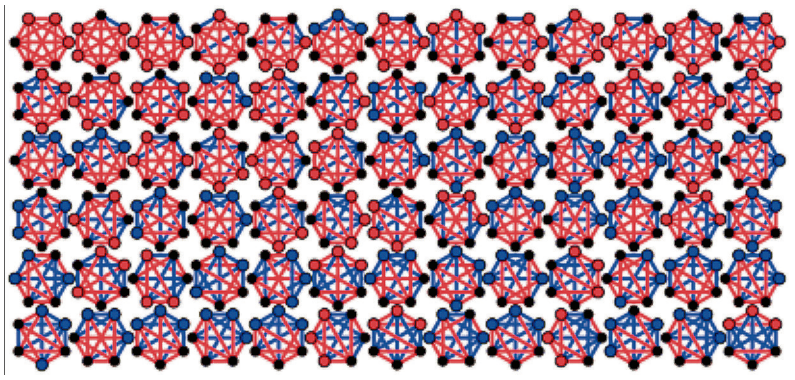
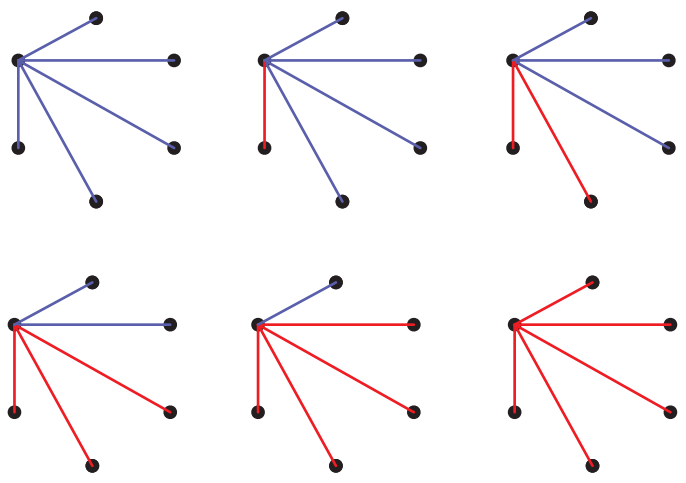


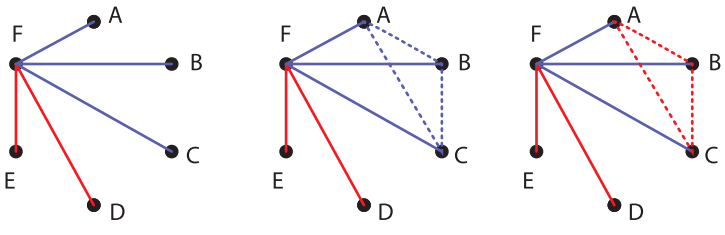
PHOTO: CLAUDIO ROCCHINI/WIKIMEDIA COMMONS

And here is a prettier way: imagine drawing in the five edges coming out of one of the points. There are five edges, so at least three of them must be the same colour.



While the top row has at least three blue edges, the bottom row has at least three red edges.

If, say, at least three edges are blue, that means none of the connecting edges between those three points could be blue, or else we would would have a blue triangle. And that means they are all red, which means we have a red triangle.



In this example, if AB or BC or CA is blue (centre), then triangle FAB or triangle FBC or triangle FAC will be a blue triangle. In order to avoid that, if we make AB, BC and CA red (right), then triangle ABC will be a red triangle.

That's it! Simple, but not easy.

PUZZLE 2

As for the game of Don't Make a Triangle on five dots that ends in a tie, a pentagram in two colours gives us the answer. There's no triangles in a single colour connecting the three of the five original dots.