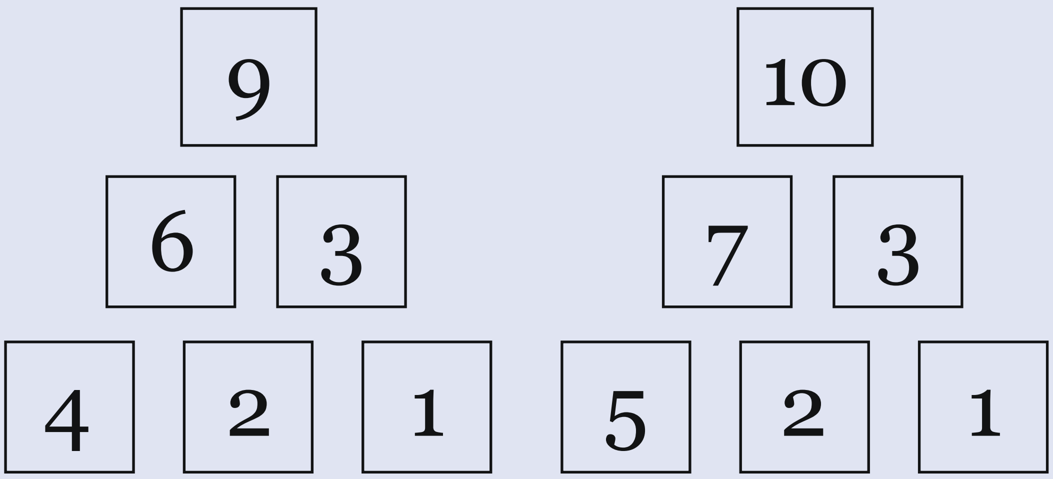
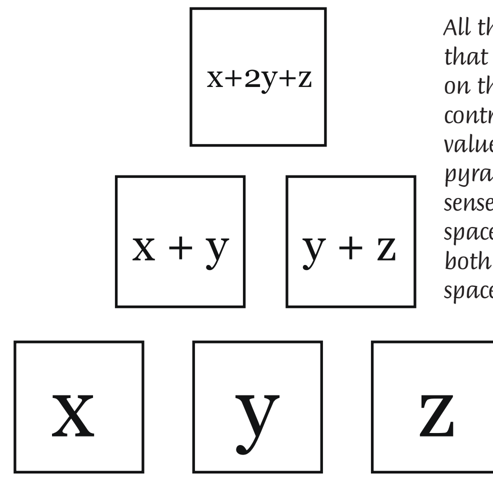
One way to approach this problem is just to mess around with it. And then, there's another way: looking at it mathematically...

PUZZLE 1 There are several answers. Here is one possibility for each.



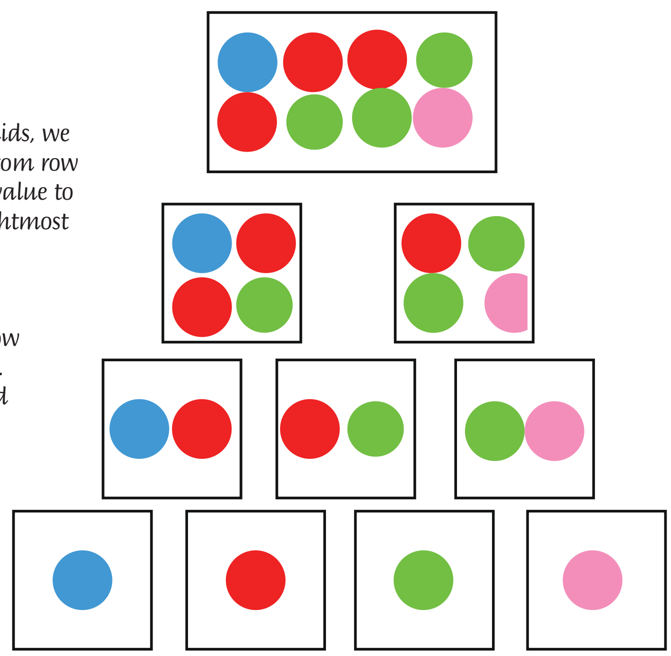
As you play with the numbers in the bottom row, however, you might notice that certain spaces have a larger impact than the others. We can see this explicitly with help from a little algebra. Put variables x, y, and z in the bottom row, and see what happens.



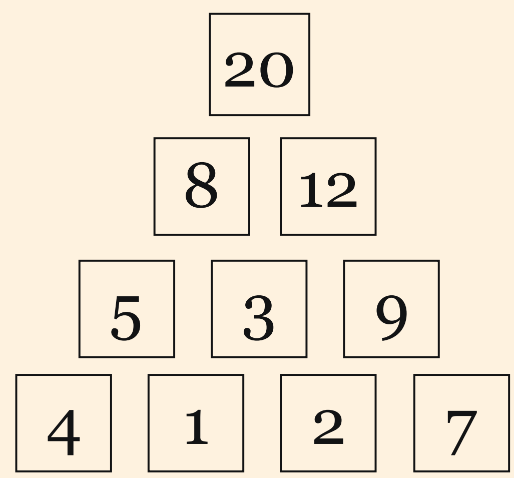
All this is telling us is that the middle space on the bottom contributes twice its value to the top of the pyramid. That makes sense, since the middle space passes its value to both the left and right spaces above.

Following this same logic in the larger pyramids, we can see that the middle two spaces in the bottom row of the 4-layer pyramid contribute triple their value to the spot at the top, while the leftmost and rightmost spot contribute their value just once.

Put 1 and 2 in the middle spots, and you know you'll be contributing 9 to the total at the top. That means the spots on the outside must add up to eleven in the second puzzle. And indeed, we can find a solution that works.

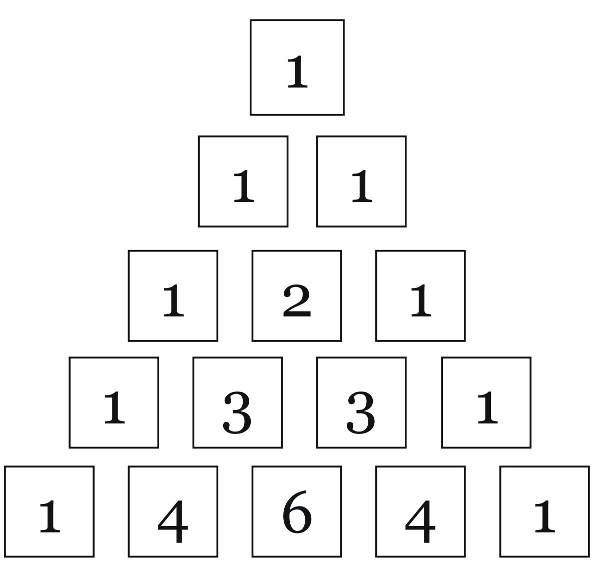


PUZZLE 2



There is at least one other solution to this puzzle. Can you find it?

If we want to find out how much each spot on the bottom row contributes to the value at the top of the pyramid, there's a mathematical object that can tell us, without us having to do the algebra or colour tracking like we did earlier. That object is **Pascal's Triangle**. Pascal's Triangle is a kind of Pyramid in reverse: each number is the sum of the two spots above it.

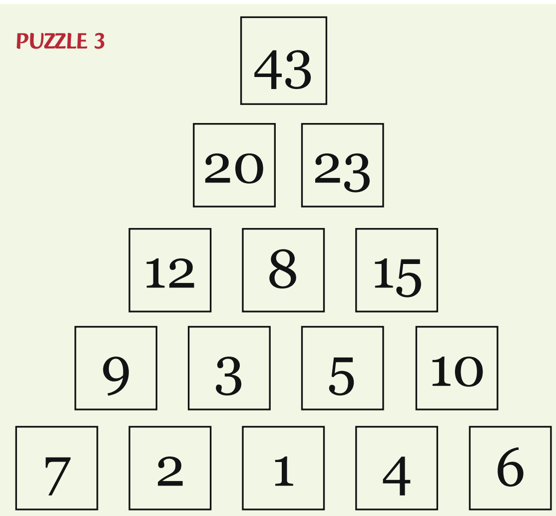


It turns out that these numbers keep track of exactly how much each spot contributes to the total at the top of our pyramid. In other words, the middle spot in the layer five pyramid contributes 6 times its value to the top spot. The next two spaces contribute 4 times their value to the top spot. The outside spaces contribute 1 times their value.

This means that we'll get the smallest outcome if we use the smallest possible numbers in the centre spots. If we didn't have to worry about repeating numbers in the higher layers, we might make the bottom row 5, 3, 1, 2, 4, which would give us 5 + (4 x 3) + (6 x 1) + (4 x 2) + 4 = 5 + 12 + 6 + 8 + 4 = 35 as the maximum. Avoiding repetitions will force us to be a bit larger.

Happy puzzling!

PUZZLE 3



It's possible to make 44 appear nicely at the top of a pyramid this size as well.

There's a natural **research question** here: continue finding the smallest possible top of the nice pyramids with 6 layers, 7 layers, and so on. Is there some pattern to these numbers?

You can write to Dan Finkel (dan@mathforlove.com) with your responses to the Research Question [subject: 14Play].