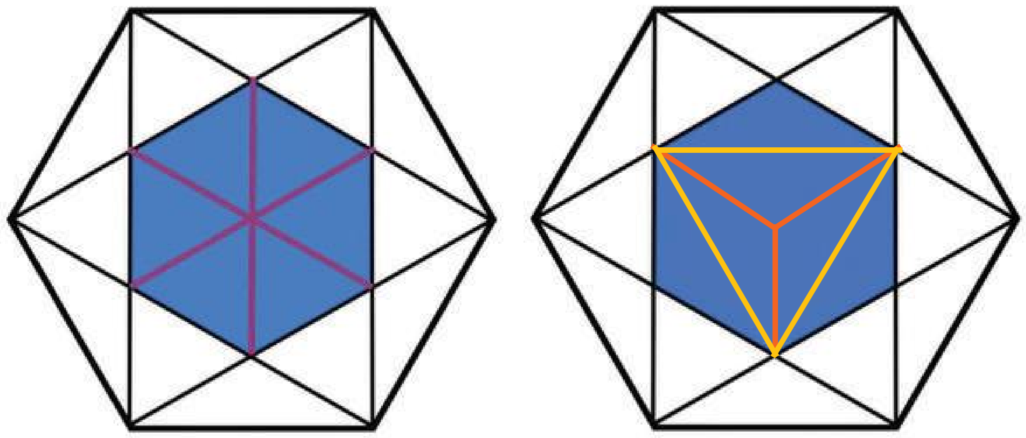
PUZZLE 1

Hexagon in hexagon: It's possible to cut the blue hexagon up into six equilateral triangles or six of the other, wider triangles as shown in these figures. Since these are precisely the shapes surrounding the blue hexagon in the larger hexagon, there is twice as much area uncoloured as there is blue. That means the blue part takes up 1/3 of the entire shape.



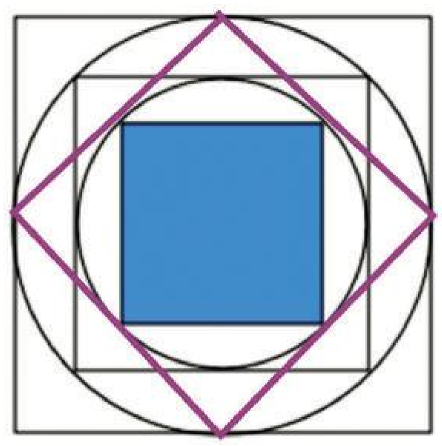
PUZZLE 3

This one was the simplest, wasn't it? So I'll just leave you with the answers...

The areas of the triangles are, from least to greatest: 9/8, 9/4, and 27/8.

Or, to see the pattern more clearly: 9/8, 18/8, 27/8. That's 1 1/8, 2 2/8 and 3 3/8.

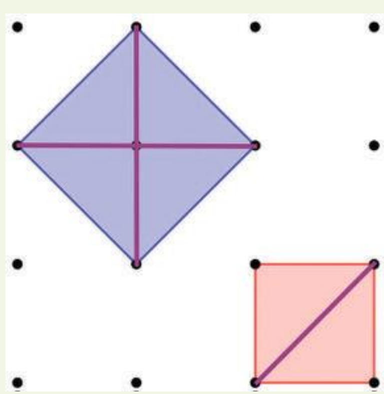
Square in a square: Rotate the middle square and its corners will touch the sides of the larger square, and its sides will touch the corners of the smaller square. You can fold in the corners of the larger square and it will completely cover the medium, rotated square, so the middle square is half the area of the large square. Similarly, the small blue square is half the area of the middle square. So 1/4 of the entire shape is blue.



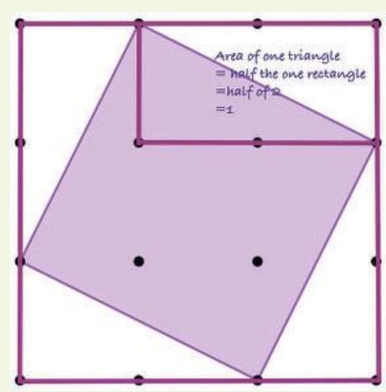
I like these puzzles because they give you another way to think about area than our typical, formula-based approach. Area can be cut into pieces and reassembled, or shifted and rotated. There is a long history of beautiful geometric ideas in this kind of kinaesthetic approach.

PUZZLE 2

Part 1 Divide the blue square into 4 triangles. Each triangle is half the size of the red square, so the total area of the blue square is 2.



Part 2 Draw a three by three square around the tilted square, and cut away the four white triangles. Each of the white triangles is half of a 2 by 1 rectangle, which mean each triangle has area 1 square unit. The larger square has area 9 square units, so the tilted square must have area 9 - 4 = 5 square units.



For me, finding these kinds of deep, beautiful patterns are what draws me to mathematics. Playing with tilted squares on grids can lead you to all kinds of amazing discoveries.

Part 3 Only 19 is impossible. The reader can check that the following squares have areas 17 (Figure 1), 18 (Figure 2), and 20 (Figure 3). There are several layers of theory here.

The first, which some may be familiar with, is that every tilted square turns out to have area equal to the sum of two square numbers, given by the tilts themselves [see Pythagorean theorem image in Figure 4]. In the example, the red tilted square has area 5, which is exactly the sum of the areas of the two blue, non-tilted squares.

This isn't a coincidence, and in fact, if you play around with pictures like the one in the answer to part 2, you'll see why it has to be true in every case.

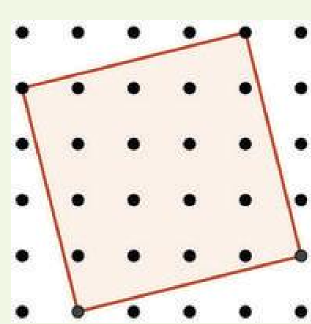


Figure 1

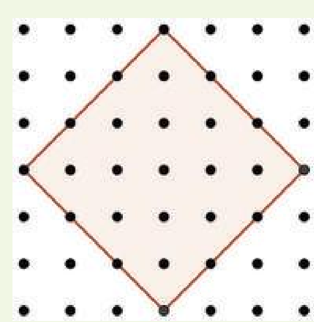


Figure 2

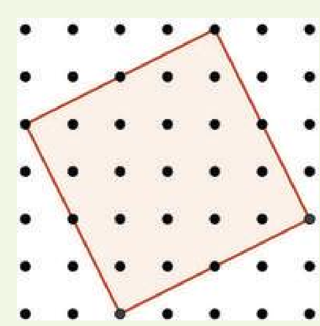


Figure 3

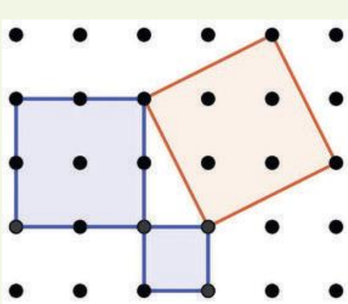


Figure 4

Since
$17 = 4^2 + 1^2$,
$18 = 3^2 + 3^2$, and
$20 = 4^2 + 2^2$,
we can find a tilted square for each of those areas. There is no way to express 19 as a sum of two perfect squares (as you can check), so no tilted square of area 19 exists.

The second, deeper layer fewer people are familiar with, but it is remarkable. It turns out that if you list prime numbers greater than 2, you can find a tilted square of that prime numbers area if, and only if, the prime number is one more than a multiple of four. So we can find tilted squares with areas of 5, 13, 17, 37, and 101 but not of areas of 3, 7, 11, 19, or 103. This pattern – and many general forms of it – was first proved by the mathematician Gauss in 1801, and he called it his "**golden theorem**".