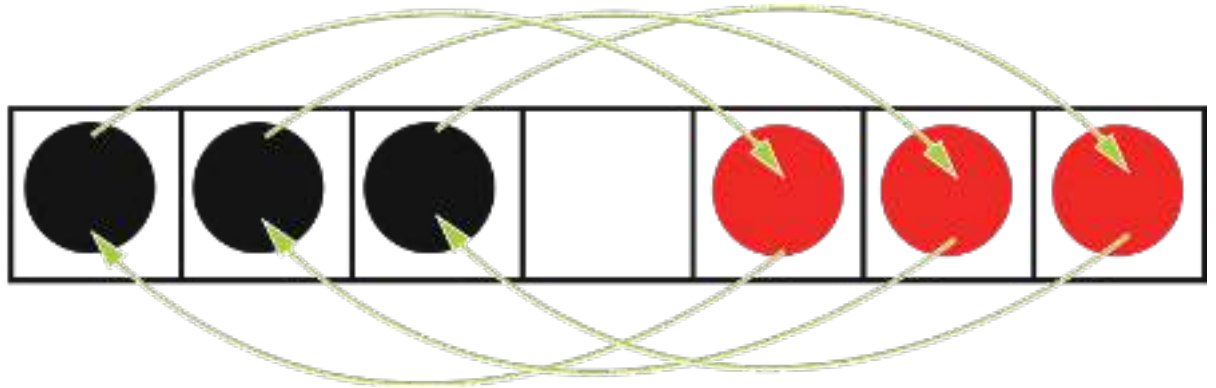


10 NUMBER GAMES

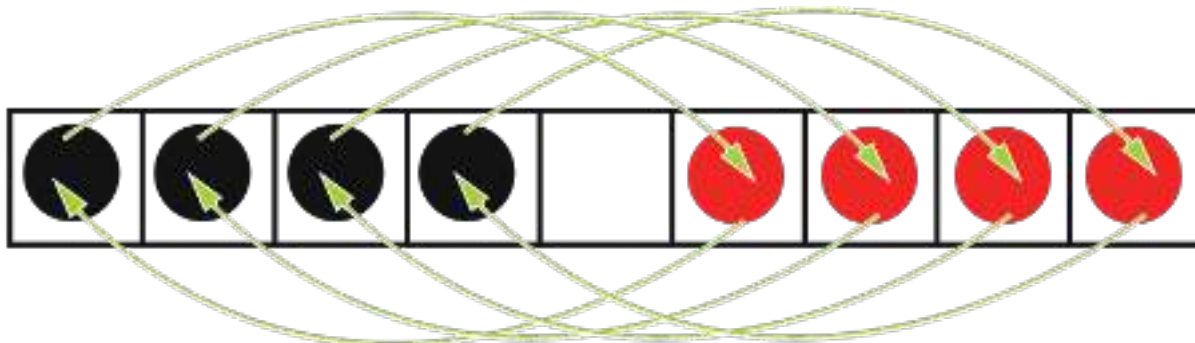
For puzzle 1, it takes 15 moves to switch the three red with the three black checkers. It can be somewhat tricky to actually perform the moves that make the switch happen, and easy to run into dead ends. Therefore, I want to start with an argument that proves it will take 15 moves if it can be done.

Consider any one of these checkers, and imagine how it would fair on a board by itself. Individually each one needs to travel exactly four spots to get home. With six pieces, that would require $6 \times 4 = 24$ moves.



But we're forgetting about the jumps! Is there some way to predict the number of jumps to expect? The answer is yes: every time a red checker and a black checker meet, there will be exactly one jump: either red jumps black or black jumps red. So how many jumps should we expect? Each red checker must meet each black checker to get to its end spot, so that is 3×3 meetings. Each jump moves a checker two spaces instead of one, which means they each save us one move. So the total should be $24 - 9 = 15$ moves.

Of course, this argument doesn't tell you how to solve the puzzle, or even that you can solve the puzzle. Still, it's quite powerful, since it allows us a framework to generalise, and essentially solve all the problems.



For example, for puzzle 2, with 4 black and 4 red checkers, each checker needs to move 5 spaces, and there will be 4×4 jumps. That gives $(8 \times 5) - (4 \times 4) = 24$ moves. We can generalise this calculation to any number of checkers.

Let C = the number of red checkers. Then each checker needs to travel $C + 1$ spaces, and there are $2C$ checkers (C red + C black). Any pair of checkers will meet once and create one jump, then that's $C \times C$ jumps.

$$\begin{aligned} \text{This means the total number of moves is:} \\ (\text{the number of checkers}) \times (\text{the space each checker needs to travel}) - (\text{the number of jumps}) \\ = 2C \times (C+1) - (C \times C). \end{aligned}$$

If you do a little algebra, this simplifies to $C^2 + 2C$, or $C \times (C + 2)$. This, in fact, is the answer to the research question posed with the second puzzle.

That's all wonderful, but can the problems actually be done? The answer is yes, and there's a nice structure to the ordering of moves as well. Here are instructions to solve each puzzle. Note that after the first move, I don't need to specify which colour moves; I just need to say whether the move is a slide (S) or a jump (J). So assume that black moves first, and see what you notice about the solutions below.

Puzzle 1: SJSJJSJJSJJSJS

Puzzle 2: SJSJJSJJSJJSJJSJJSJS

Lo and behold, there's a tidy pattern here as well. It might be easier to see if I abbreviate how many jumps happen in a row with a number. So instead of JJJ I'll just write 3.

Then the answer to puzzle 1 is S1S2S3S2S1S

And the answer to puzzle 2 is S1S2S3S4S3S2S1S

The jumps increase from 1 to the number of checkers, and then decreases back to 1. Keep this pattern going and you've got a recipe to solve a checker puzzle of any length.

What about two dimensions then? This problem is surprisingly tractable if you observe one thing: we already know how to solve the 1-D version in the middle (circled in the image), and if that middle horizontal line was clear, we could solve all the vertical lines as well. After all, each column is the same 1-D puzzle again as well!

This gives us the key to complete the puzzle. Solve the middle row, and when a gap opens up in a column, solve that column. So solve the middle column first (8 moves), then slide a black forward in the middle row. Then solve the second column now that you have a gap, and so on. In the end, you'll just be solving 6 versions of the 1-D puzzle, which will take $6 \times 8 = 48$ moves.

Of course, you don't have to stop there! You can find a formula to solve larger versions of this square puzzle, or take these puzzles to cubes and beyond! But I'll sign off for now. Till next time, happy puzzling!

