A MATHEMATICIAN AT PLAY

Grids, polygons and acute angles...

Are you one of those who gets started with a game of "Dots and Boxes" with your neighbour the moment you get some free time at class? Or do you imagine countless possibilities whenever you are confronted with a pattern of dots? Either way, you'll love this one. For there's plenty to learn with **Daniel Finkel** *as he explores a grid of 16 dots...*

Of all the myths about mathematics, the one I find most blatantly wrong is the idea that some people are just born knowing the answers. In my experience, when you confront a genuine puzzle, you start out not knowing, no matter who you are. Moreover, "knowing" the answers can be a trap; learning mathematics is about looking at what you thought you understood and seeing that there's deeper mystery there than you realised.

For example, I've been thinking about grids a lot lately, like this 4 by 4 grid of dots (see figure 1).

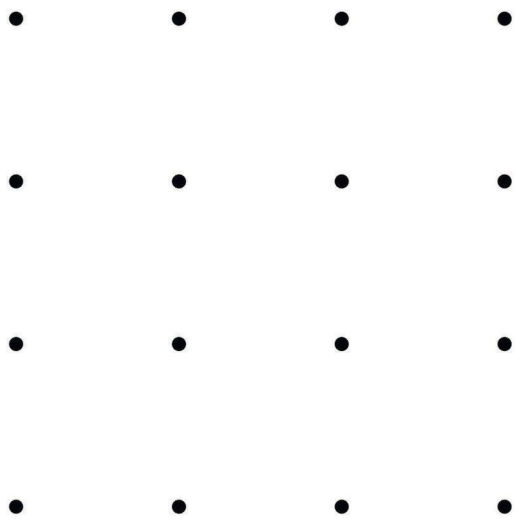
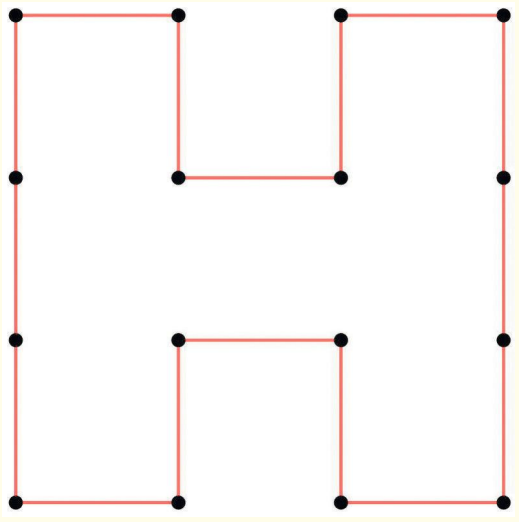


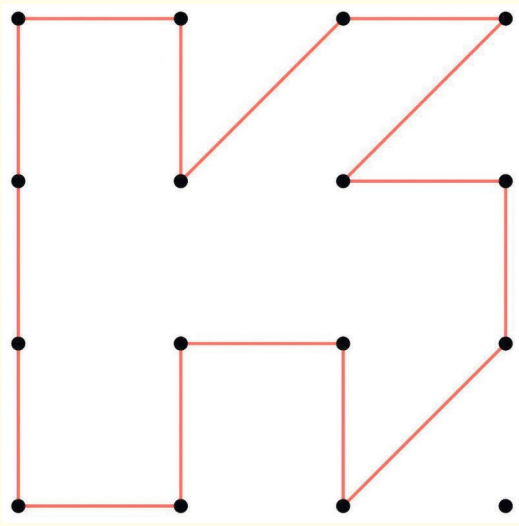
Figure 1 >>

In some ways, grids are one of the most familiar objects we know. We see them around us every day, in tile ceilings and floors, in the sides of buildings, and even in regularly planted gardens. And yet, it sometimes feels like we know nothing about them at all. There's so much mystery in such a simple arrangement!

Here's a series of puzzles about grids. Every puzzle here is, if you want it to be, just the beginning of a larger, more beautiful, exploration, so after each puzzle, I've included a direction for research.



^ Figure 2



1

I can connect the dots in the grid to form a polygon. (For the shape to be a polygon, there can be no overlapped lines, and each dot in the grid can be visited at most once.) I have formed polygons on the grid with 12 sides (**see Figure 2**) and with 13 sides (**see Figure 3**).

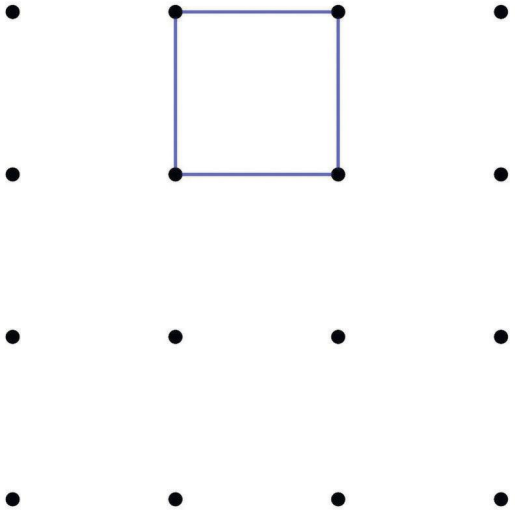
THE PUZZLE

Find a way to form a polygon with 16 sides.

RESEARCH QUESTION

For larger grids, is it always possible to make a polygon with as many sides as the grid has dots?

<< Figure 3



2

I can make a grid square by connecting four dots in the grid to make a square. At first, it seems like there will be nine grid squares. Then you realise there are more that you missed. And then you realise there are even more.

THE PUZZLE

Find all 20 grid squares that can be formed in the 4 by 4 dot grid.

RESEARCH QUESTION

Can you predict how many grid squares can be formed in a 5 by 5 grid? What about different sized grids?

3

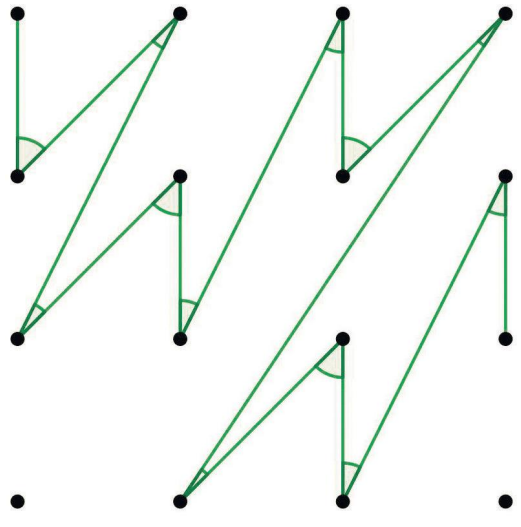
The grid is based on right angles. What if we push against this structure. For example, I've connected 14 of the points of the grid together with line segments so that the angle at every point is acute, i.e., less than 90°.

THE PUZZLE

Find a way to connect all 16 points with an acute angle at every point. (**Note:** a successful solution to this puzzle won't include any straight angles through points.)

RESEARCH QUESTION

Can you connect the points in larger grids with only acute angles? For what size grids is it possible?



You can write to Dan Finkel (dan@mathforlove.com) with your responses to the Research Questions [subject: 1Play]. Answers to the three puzzles can be found on page 47.

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling Prime Climb and Tiny Polka Dot.