## A MATHEMATICIAN AT PLAY

Partitions and pieces of eight

The problem of partitions is one that seems deceptively simple and yet eluded mathematical geniuses for the longest of times. Daniel Finkel gives you three such puzzles that will introduce you to the world of partitions...

Some problems in mathematics are easy to state and maddeningly difficult to solve. A classic example is the problem of partitions, which asks: given a number of objects, how many different ways can you divide those objects into groups?

For example with four stones, I could divide them into groups of (refer image)
4,
3 and 1,
2 and 2,
2 and 1 and 1,
or 1 and 1 and 1 and 1
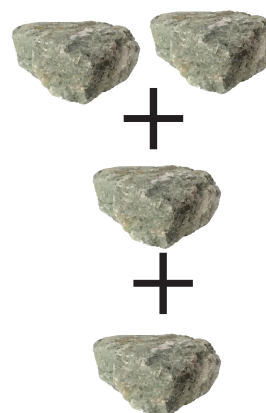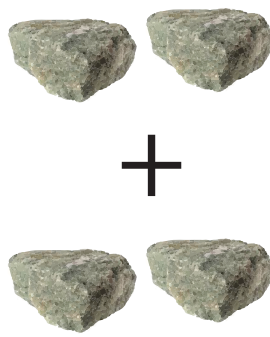
This problem of finding a method to count the ways of partitioning numbers seems like it should be simple – it's just addition! And yet, the solution eluded mathematicians for centuries. Euler worked on it in the 18th century, and Ramanujan and Hardy tackled it in the early 20th century. But the problem wasn't solved until 2011, when Ken Ono and Jan Bruinier put forth a solution using high-powered modern mathematical techniques, well and truly beyond the grouping of stones. To me, there's something beautiful about that counterpoint: simple question, elusive solution.

For today's puzzle, I'd like to share some simpler partition puzzles. A general hint: these can usually be solved by simplifying the problem and organising your results to look for patterns, and there will be many beautiful patterns to find. Also, in all the puzzles today, order matters. That means we'll count 5 + 3 as distinct from 3 + 5.

PUZZLE 1
There are 3 ways to write 3 as the sum of 1s and 2s:
3 = 1 + 2
3 = 2 + 1
3 = 1 + 1 + 1
How many ways are there to write 8 as a sum of 1s and 2s?

PUZZLE 2
There are 7 ways to write 8 as a sum of two positive integers:
8 = 1 + 7, 2 + 6, 3 + 5, 4 + 4, 5 + 3, 6 + 2, 7 + 1
How many ways can you write 8 as a sum of three positive integers?

PUZZLE 3
There are 4 ways to write 3 as a sum of any number of positive integers:
3,
1+2,
2 + 1,
1 + 1 + 1
How many ways can you write 8 as a sum of any number of positive integers?

RESEARCH QUESTION If you substitute any other number for 8 in the puzzles above, can you find a general method that will allow you to solve the puzzles?

You can write to Dan Finkel (dan@mathforlove.com) with your responses to the Research Question [subject: 10Play].

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling Prime Climb and Tiny Polka Dot.