A MATHEMATICIAN AT PLAY

A tower of exponents and the power of divisibility

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Many students learn divisibility rules in school. These can be fun to play with, but they are often a little tame, and they won't help you much here. Today I'd like to play around with some stranger, more surprising divisibility puzzles.

Do numbers scare you? Are you petrified every time your math teacher turns to you for the answers? Try out these puzzles, mull over the answers and you'll soon realise that your fear for numbers is totally uncalled for...

PUZZLE 1
THE POWER OF 37

Let's play a quick game. Now follow these steps:
1) Pick any digit.
2) Write this digit three time in a row to get a 3-digit number.
3) Divide this three digit number by the same digit added to itself three times.
For example, if I picked 5, I would get 37 as my answer.

$$555 \div (5 + 5 + 5) = 37$$

Now here's the question: Can you get any number aside from 37 as your answer? Why or why not?

PUZZLE 2
THE 7, 11, 13 MYSTERY

We are going to be playing another game now. It's similar to the one in the first puzzle, and yet, it is different. Here are the steps:
1) Choose a three digit number.
2) Write it twice in a row to get a 6-digit number.
That's it. We are good to go now.

For our example, I choose 356, and write it twice to get 356,356.

I claim this number is divisible by 7. Indeed, $356{,}356 \div 7 = 50908$

I claim this quotient is divisible by 11. Indeed, $50908 \div 11 = 4628$

I claim this quotient is disable by 13. Indeed, $4628 \div 13 = 356$

And look! The final quotient is the original number!

Tackle this question:
Can you find a three digit number where this won't happen?
Or will it happen for every three-digit number. And if so, why?

Turn to the next page for answers to the three puzzles.

$$5^{10^{5^{10^{5^{10}}}}} + 10^{5^{10^{5^{10^{5}}}}}$$

PUZZLE 3
NUMBERS THAT BREAK CALCULATORS

One fun thing about exponents is that they get big fast, and this is especially true of towers of exponents.

To calculate a tower of exponents, start from the top and work your way down. For example,
$4^{3^{\wedge}2} = 4^9 = 262{,}144$.
Meanwhile, $5^{4^{\wedge}3}$ is forty-five digits long.
Now that you know how towers of exponents work, here's **the question:** To your left, you can see two towers of exponents being added to each other. Can you tell me if the sum of those two huge numbers divisible by 11?

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the bestselling Prime Climb and Tiny Polka Dot.