A MATHEMATICIAN AT PLAY

Climb up the stairway of numbers

There is a long, beautiful history of thinking about numbers and shapes as two aspects of the same idea. And there are all sorts of these geometric numbers. We will be looking at one of those today, the ones we call "step numbers"...

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Square numbers are often described using arithmetic:
$1^2 = 1 \times 1 = 1$,
$2^2 = 2 \times 2 = 4$,
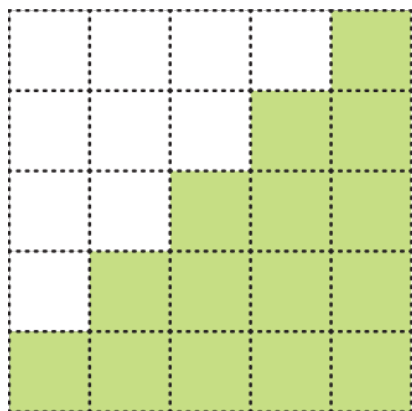$3^2 = 3 \times 3 = 9$,
$4^2 = 4 \times 4 = 16$, and so on.

Square numbers can geometrically be thought of as squares too. To visualise this, imagine a square number of dots —16, say – and notice that they form an array quite naturally. The name "square number" isn't just a coincidence!

There are all sorts of other geometric numbers: triangular numbers, cube numbers, pentagonal numbers, and so on, and often visualising the numbers as shapes gives us insight into how they work.

So today, I'd like to explore **Step Numbers**. A step number is a number that can be written as the sum of two or more consecutive positive integers. In other words, step numbers can be arranged to form a section of a staircase.

For example, 15 is a 2-step number since 7 + 8 = 15. In fact, 15 is also a 3-step number, since 4 + 5 + 6 = 15. And 15 is also a 5-step number, since 1 + 2 + 3 + 4 + 5 = 15.

Visually, we can see these quite literally as steps in a staircase.

15 as a 5-step number   15 as a 3-step number   15 as a 2-step number

PUZZLE 1

Part 1
Find a simple rule to determine if a number is a 2-step number

Part 2
Find a simple rule to determine if a number is a 3-step number.

Can you find rules for any number of steps?

PUZZLE 2

We saw that 15 was a 2-step, 3-step, and 5-step number. Find three other numbers that are also 2-step, 3-step, and 5-step numbers.

PUZZLE 3

Both 8 and 128 are not step numbers. How many other numbers less than 1000 are not step numbers?

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling Prime Climb and Tiny Polka Dot.