A MATHEMATICIAN AT PLAY

Behaviour patterns and parity puzzles

There is nothing more exciting to a mathematician than when the same behaviour emerges in dissimilar situations. The three puzzles in today's article are notable for exactly this strange behaviour. As you ponder them, ask yourselves if it seems like they have anything in common. They look and feel very different, but they are united by an underlying theme that's simple and surprising! **Daniel Finkel** presents you three puzzles that are different and similar at the same time...

PUZZLE 1

If you put + or - signs in the blanks in the equation below, what are the possible answers? What answers are impossible?

__1__2__3__4__5 = ?

Note that if we use all + signs we get 1 + 2 + 3 + 4 + 5 = 15; if we use all - signs, we get -1 - 2 - 3 - 4 - 5 = -15. Every other answer must be between -15 and 15. How many of those numbers can you get?

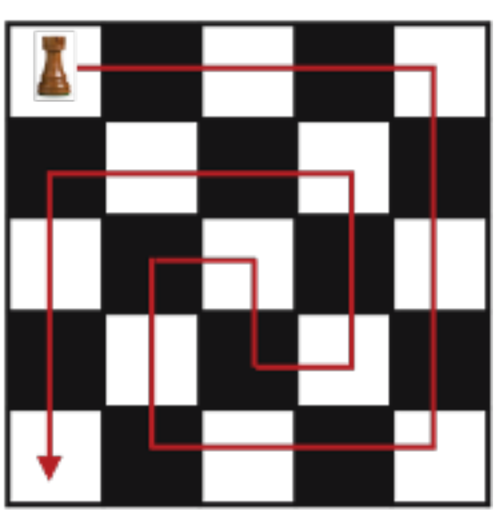
BONUS PUZZLE: using all four operations + - x ÷, and parentheses ( ), make an equation using 1, 2, 3, 4, 5 each exactly once that equals 99.

PUZZLE 2

Let's take a rook's tour of a 5 by 5 chessboard. The rook can move vertically or horizontally as many spaces as you wish. However, on this tour, it can never cross the same square twice.

It's not hard to see that a rook in the top left corner can easily make the tour, and pass through every square exactly once. Can it be done from every square?

Find the squares the rook can start in for a successful tour. Why do some squares seem to be impossible?



Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling *Prime Climb* and *Tiny Polka Dot*.

PUZZLE 3

This one is a classic. Twenty-five prisoners are to be executed at dawn. However, they are given a special opportunity to survive, if they can guess the colour of the hat they are wearing.

Each prisoner will receive a black or white hat to wear, and then be placed in a straight line. The prisoner at the back of the line faces forward, and sees the other 24 prisoners, along with their hats.

He then must say a single word that would correspond to his guess for his own hat colour: "white" or "black." Then the prisoner second from the back looks at the 23 prisoners in front of him and says "white" or "black."

This continues one by one until all the prisoners have guessed their own hat colour, after which everyone who guessed correctly is freed, and everyone who guessed wrong is put to death.

The prisoners talk the night before to discuss a plan. What is the best strategy they can employ, and how many of the prisoners can be sure to survive?

When I first heard this problem, I thought it must be a trick question. In fact, it has an entirely honest answer.

Here's an example of the type of strategy the prisoners might try: every other prisoner, starting from the one in the back, could say the colour of the hat in front of them. Then the prisoners in front would know their hat colour, and be free. That would save just under half of the prisoners for sure – the others have a 50/50 chance.

That's an example of a strategy that saves at least some of the prisoners for sure. In fact, it's possible to find a strategy that saves nearly everyone for sure. Can you figure it out?

One last quandary to ponder: what do all these puzzles have in common?