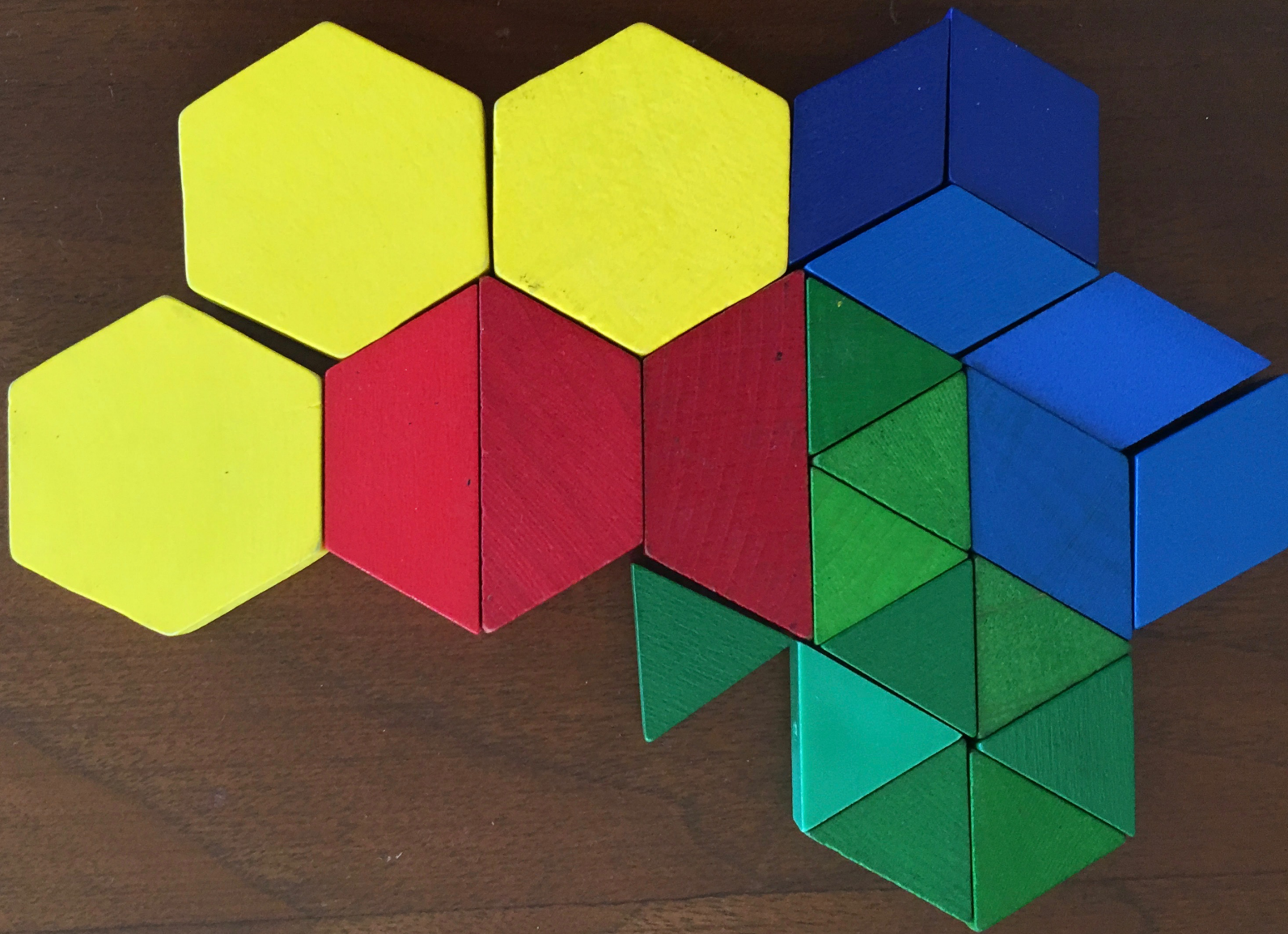


Sample

# Math for Love

## Fractions - Grade 3



By Dan Finkel



# Introduction

This work was part of an effort to create a compelling and comprehensive resource for fractions in grade 3. This guide includes 25 lessons, with activities, lessons, and games to help students learn everything they should know about fractions by the end of third grade.

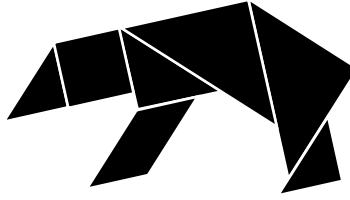
These lessons are best employed as the main work in a classroom setting over a five-week period of approximately hour-long lessons. While sample dialogs between teachers and students abound, they should serve as inspiration, not a script. Ideally, both students and teacher will feel like the lessons give them a sense of ownership over the content in these pages.

What you have in your hands (or on your computer screen) was commissioned and piloted by the Renton School District in Washington State, and is being used throughout that district as of this writing. These activities were written by Dan Finkel of [mathforlove.com](http://mathforlove.com), drawing on principles from Cognitively Guided Instruction (CGI), as well as the principles put forward in the author's TED Talk ( [ted.com/talks/dan\\_finkel\\_5\\_ways\\_to\\_approach\\_math\\_better](http://ted.com/talks/dan_finkel_5_ways_to_approach_math_better)) and his own work with students. Big thanks go to Ivan Flores of the Renton School District and educator-consultant Becca Lewis, who contributed ideas abstract and concrete, large and small to this project.

Every lesson is designed to launch by inviting the curiosity of students. Questions begin with equal sharing and manipulative-heavy explorations (pattern blocks are a central tool), and progress to develop subtler and more abstract tools, including working on number lines and reading and manipulating written fractions.

If you have feedback, I'd love to hear it. Email me at [dan@mathforlove.com](mailto:dan@mathforlove.com).

Enjoy!

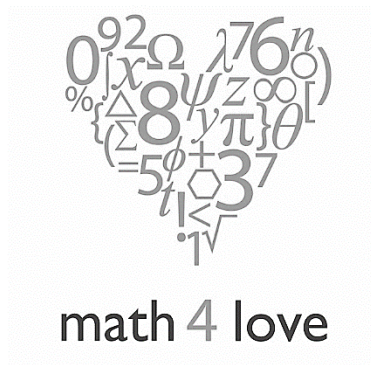


# Teachers Guide

## GRADE 3 – FRACTIONS SUPPLEMENT

By Daniel Finkel

Consultants: Ivan Flores and Becca Lewis



**Becca Lewis**  
Math Educator



# Equal Sharing Problems 1

**Topics:** Multiplication, division, fractions

**Materials:** Pencil & paper, pretend “brownies” made from paper (optional)

**Common Core:** 1.OA.1 | 1.G.3 | 2.G.2 | 2.G.3 | 3.OA.3 | 3.NF.1 | 3.G.2 | MP1 | MP2 | MP3 | MP4 | MP6 | MP7 | MP8

**Essential Question:** How can everyone get an equal share?

## Why we love Equal Sharing Problems

Equal sharing problems are an excellent way to introduce fractions meaningfully and appropriately. Changes in the numbers used can adjust the level of challenge in the problems.

## The Launch

For each of the following questions, choose two students to come up for a physical demonstration. You can use the same two students, or take different volunteers each time. Draw a picture of the brownies, or use pretend brownies made from paper.

Question 1. These two want to share 6 brownies so they both get an equal share. How can they do it?

Let students think, pair, and share to discuss their ideas with a neighbor. After a brief conversation, discuss ideas as a group. You might hear:

- “Give three to each of them.”
- “Give one to her, then one to him, and then one to her again, and one to him, and one to her, and one to him, and then you’re done.”

After students are convinced that they have at least one strategy they could use to divide the brownies, follow up with:

Question 2. These two want to share 7 brownies so they both get an equal share. How can they do it?

Again, let students think, pair and share their ideas with a neighbor, then discuss as a group. The conversation might go like this:

**Student 1:** Just share them one by one until the brownies are all gone.

**Teacher:** Okay. [Dealing out paper “brownies.”] You get one, then you get one, and you get one... now they’re all gone. Do they both have an equal share?

**Student 2:** No! She has 4 and he has 3.

**Student 3:** She should give one to him.

**Teacher:** Let's try that. Could you give one of your brownies to him? [She does.] Now do they have an equal share?

**Students:** Yes.

**Teacher:** That's good. Do you mind if I make sure? Let's count each side. She has 1, 2, 3. He has 1, 2, 3, 4. Wait a minute! What happened?

**Student 4:** Wait! He has to give her one now.

**Teacher:** Let's try that. [They do.] Do they have an equal share now?

**Student 5:** Now she has more! They can't have the same amount of brownies? You should give away 1 to the teacher.

**Teacher:** That's a fun idea. But sometimes we want to share everything between ourselves and not have any leftover, or give any away. Is there any way to share these 7 brownies so they both get an equal share?

**Student 5:** No. Any way they do it, somebody will always have more.

**Teacher:** Interesting theory! Do you agree or disagree? Talk to a neighbor, and tell them why. [Students discuss.]

**Student 6:** I think there is a way to share the brownies. They should cut the last one in half.

**Teacher:** What do you mean, the last one?

**Student 6:** Well, there's 7 to start. Then she takes 3 and he takes 3, so they have the same amount, but there's one left. So if they cut that one in half, they can each take half.

**Teacher:** What an interesting idea. Has anyone here ever cut a brownie or a cookie in half to share it before? [Hands are raised.] Let's try that. We'll share 3 with each of you, and then there's one left. And if we cut it in half [tears the paper], you get half, and you get half. How many brownies do you have now?

**Volunteer 1:** I have 4. No, wait! That's 3 and a half.

**Volunteer 2:** I have 3 and a half.

**Teacher:** So you have 3 brownies, and a half more. That's 3 and a half brownies. And you have 3 and a half brownies. Do they each get an equal share?

**Student 7:** That half is bigger.

**Teacher:** Good point. Maybe I didn't do a perfect job of cutting it. But if you imagine that I cut the brownie perfectly in half, would it be fair?

**Student 7:** That would be fair.

**Teacher:** I think so too. So there was a way of dividing these 7 brownies up so these two could get an equal share after all, without having any brownies leftover, or giving any away. And they each got three and a half. Did anyone try something totally different?

**Student 8:** I did. I split the first brownie in half, and then the second brownie in half, and then the third brownie in half, and I kept going.

**Teacher:** So how many halves did he get?

**Student 8:** 7 halves.

**Teacher:** And how many halves did she get?

**Student 8:** 7 halves.

**Teacher:** Very interesting. Everyone, find a partner for a quick turn and talk. One method of dividing these brownies led to each person getting 3 and a half. Another method led to them each getting 7 halves. What's going on here? Why is it 3 and a half one way and 7 halves the other way? Is someone making a mistake, or is there some connection here that I'm missing?  
[Conversation continues as appropriate.]

## The Work

After the students share their ideas on the equal sharing problems in the launch. Set them to work in pairs on the following problems. Encourage them to draw their solutions.

1. Two kids want to share 10 cheese slices equally. How can they do it? Draw your answer.
2. Two kids want to share 11 cheese slices equally. How can they do it? Draw your answer.
3. (Challenge) Four kids want to share 9 cheese slices equally. How can they do it? Draw your answer.

## Questions and Prompts

- Show me how you could start splitting these up.
- Can you draw a picture?
- Have you used up all the cheese slices? Do both kids have an equal amount?
- Why is there a leftover cheese slice that needs to be split in some cases, but not in others?
- What would happen if there were 12 slices or 13 slices? Can you predict how many cheese slices each person will get?

## The Wrap

Show the students the following brief video. Show: <https://youtu.be/-DXzv019RDQ?t=23s>

Let students discuss what they saw. Was the kid in the video right? Was the sharing fair? Why or why not? How could it have been fair?

Following the discussion of the fractions involved, ask students what they think confused the kid in the video. What makes these equal sharing problems tricky?

## Extension

For students who need extra challenge, try the following questions:

- 4 friends want to share 33 pieces of candy so everyone gets an equal share. How can they do it?
- 3 friends want to share 34 slices of cheese so everyone gets an equal share. How do they do it?
- Pick a problem you've done, and find a different way to do the sharing.

## Tips for the Classroom

1. Drawings are helpful. Students may also be helped by physically taking paper to stand in for brownies or cookies and sharing it physically, especially when they're trying to solve these kinds of problems for the first time.
2. Note that the key idea in equal sharing problems is that everyone should get an equal share, and also that there should be no leftovers; everything being divided should go to the people sharing it. Students may suggest giving a brownie to the teacher, giving it away to someone else, giving it to the dog, etc. These are fun suggestions, but make it clear that the solutions we're after should involve using up all the brownies/cheese slices/etc. without including more people, and with everyone getting an equal share.
3. If problems are too easy or too hard for students, change the numbers they use accordingly, or let them try changing the numbers. Be aware that using numbers of kids aside from 2 or 4 will be more challenging.



# Smooth and Lumpy & Equal Sharing by Twos

**Topics:** Multiplication, division, fractions

**Materials:** Pencil & paper, rectangle pieces of paper to act as “brownies” (optional)

**Common Core:** 1.OA.1 | 1.G.3 | 2.G.2 | 2.G.3 | 3.OA.3 | 3.NF.1 | 3.G.2 | MP1 | MP2 | MP3 | MP4 | MP6 | MP7 | MP8

**Essential Question:** What can be shared? What can't?

## Why we love Smooth and Lumpy

This terminology was presented by Paul Lockhart in his book *Arithmetic*. Bringing attention to the physical reality of what can and cannot be shared makes fractions that much more tangible and meaningful.

## The Launch

Lead a conversation about *smoothness* and *lumpiness*. It might go like this:

**Teacher:** Yesterday we started with these two questions.

**Question 1.** *Two friends want to share 6 brownies so they both get an equal share. How can they do it?*

**Question 2.** *Two friends want to share 7 brownies so they both get an equal share. How can they do it?*

What if we changed the word “brownie” out for another word? Would we always be able to give the two an equal share? What if the two friends were sharing 6 pieces of cheese. Would be they able to share them equally?

**Student 1:** Yes. They each get 3 pieces.

**Teacher:** What if they were sharing 7 pieces of cheese?

**Student 2:** They each get 3 and a half pieces of cheese.

**Teacher:** So we can share pieces of cheese just like we share brownies. What if we were sharing puppies? If they had 6 puppies, could they share them? Discuss with the person next to you. [Students discuss.]

**Student 3:** They could still get 3 puppies each.

**Teacher:** What if they were sharing 7 puppies?

**Student 4:** [laughing] They'd have 3 and a half puppies?

**Student 5:** ewww!

**Teacher:** I think we can agree we don't want to cut puppies in half. But there's something even deeper here, which is that when you cut it in half, it's not a puppy anymore. Some things can be divided, or shared, and some things can't. So let's call things that can be shared *smooth*, and things that can't be shared *lumpy*. Then puppies would be...

**Students:** lumpy.

**Teacher:** And brownies would be...

**Students:** Smooth.

**Teacher:** I'm going to say different things, and after each one, discuss with the person next to you whether you think it's lumpy or smooth. Ready? Let's try... milk.

**Students:** [after brief discussion] smooth.

**Teacher:** Right. It seems like we could divide that up however we wanted. Cars.

**Students:** [after brief discussion] lumpy.

[Note - if students disagree, solicit their thinking to explore their take on why something is or is not smooth or lumpy. They likely have interesting reasoning to share.]

**Teacher:** Balloons.

**Students:** [after brief discussion] lumpy.

**Teacher:** Airplanes.

**Students:** [after brief discussion] lumpy.

**Teacher:** Candy bars.

**Students:** [after brief discussion] smooth.

[Note: continue with as many more examples as you deem necessary. Ideas include apples, bananas, people, trees, houses, dolls, legos, bottles, cans, apple juice, t-shirts, pencils, money, water, gum, and so on.]

**Teacher:** What about numbers?

**Students:** Smooth/lumpy.

**Teacher:** Interesting. What if there was something I made up. I'm imagining it right now. It's like an imaginary shape, but it only exists in my head. Is it smooth or lumpy?

**Student 6:** Well, it depends what you want it to be.

**Teacher:** And guess what? Numbers are something we made up, and they exist in our heads. So they're smooth when we want them to be smooth, and lumpy when we want them to be lumpy. Sometimes we share things and leave remainders aside, because we don't want to cut them up. And sometimes we divide up the remainders into fractions. It just depends whether we've decided we're working with something lumpy or smooth.

Today, I've got another challenge for you. Let's pick something smooth, like cookies. I'm curious when we even need to cut up the cookies or not when we're sharing them. So here's your job: you're going to figure out how to actually share any number of cookies evenly between two people. If there is 1 cookie, how would you do it? Talk quietly to a partner.

[Students discuss]

**Student 7:** Each person gets half a cookie.

**Teacher:** [Drawing on the board] Right. Just cut the cookie in half, and each person gets half. What if we wanted to share 2 cookies?

**Student 8:** Each person gets a whole cookie.

**Student 9:** Or you could cut the first cookie in half and the second cookie in half.

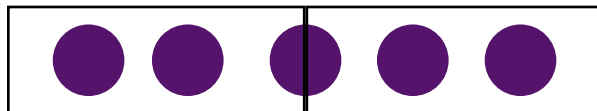
**Teacher:** So each person could get a whole cookie, or they could get two half cookies, depending on how we decide to do it.

**Teacher:** What about two people sharing 3 cookies? 4 cookies? 5 cookies? 6? Your job today is to take this list all the way up to 12, and beyond if you can. You'll have a chart to fill out. Let's get to work!

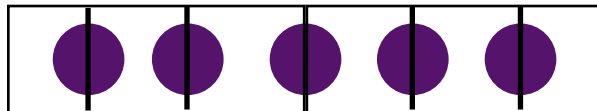
## Exploration

Hand out (or draw on the board) a chart like the one on the following page. Students can work in pairs to fill out how many cookies each person gets as the number of cookies grows larger. For students who find patterns and finish early, challenge them to include pictures for specific examples, or to make up a more challenging problem for themselves to tackle.

This is also a great opportunity to lead students to work together, especially when they have different looking answers. For example, a student might draw dividing up 5 cookies between two people using the following kind of drawing to show each share as 2 and a half.



However, students also might cut all five cookies in half and give each student half of each cookie.



That's 5 halves each.

These different sharing strategies are very valuable to compare together as students explore and during the wrap up discussion.

## Questions and Prompts

- What would a picture of dividing 5 cookies between 2 friends look like?
- How would you write out in words what each person gets?
- How do the words/numbers you wrote match with your picture?
- Do you see any patterns in this chart?
- Based on your patterns, what do you predict the chart would look like if you extended it up to 20 cookies? Can you write in your predictions?
- Are your predictions actually true? How do you know?

## Summarize/Discuss

Bring students together to fill in the chart as a group. Once you have a copy everyone can see, ask them what they notice about it. Are there patterns that could help them make predictions about what comes next on the chart? Is there anything else they notice about how sharing works?

This is a great time to take student conjectures to leave open for the future. For example, students may notice that you only need to cut cookies in half when sharing an odd number of cookies. Will this be true when you share with three or four people as well, or is it only true for sharing with two people? Also students might describe the pattern as 1 half, 2 halves, 3 halves, 4 halves, instead of using mixed numbers. There's a very valuable discussion to have if this comes up, about whether these are the same numbers or different numbers. In fact, an excellent question to close on is the following:

**Question.** Two friends are sharing 3 cookies. one friend suggests they each take 1 and a half. The other suggests they cut all three cookies in half, and each take 3 halves. Are these both fair? Why or why not?

## Extension

For students who finish their chart early and exhibit a clear understanding of the division process, challenge them to make a similar chart for sharing cookies among 4 friends instead of 2. What will their chart look like now?

## Tips for the Classroom

1. Remember to encourage drawings and written descriptions, and help connect their meanings. This is especially important when it comes to differentiating between a drawing of “3 halves” vs. “1 and a half.” The values are equal, but the drawings should differ.
2. If a student can’t tackle the whole graph, get them focused on drawing and solving a single situation (i.e., sharing 5 cookies between 2 friends).

# Equal Sharing Problems 2

**Topics:** Division, fractions

**Materials:** Pencil & paper

**Common Core:** 1.OA.1 | 1.G.3 | 2.G.2 | 2.G.3 | 3.OA.3 | 3.NF.1 | 3.G.2 | MP1 | MP2 | MP3 | MP4 | MP6 | MP7 | MP8



**Essential Question:** How can everyone get an equal share?

## Why we love Equal Sharing Problems 2

A continuation of lessons creating a foundational understanding how fractions are related to the problem of equal sharing. We generalize the process of equal sharing for more participants, and introduce language in a meaningful context.

## The Launch

Tell students you want to tell (or remind) them about some notation for naming and writing down fractions. Ask students to help you fill out the following chart, soliciting their thoughts on what should go in the second, third, and fourth columns of each row.

Sharing 1 sandwich between...	Each person gets...	Also known as	Possible Picture
2 friends	1 half	$\frac{1}{2}$	 or 
3 friends			
4 friends			
5 friends			
6 friends			

### Example Launch

**Teacher:** We've been working on how to share equally in a bunch of different situations, and we've gotten practice at drawing those situations. Let's look more at how to talk about it. Let's say some friends are sharing a sandwich equally. If there are two friends, how can they divide the sandwich?

**Student 1:** In half.

**Teacher:** Right. So if there are 2 friends, each of them gets 1 half. I can write it like this [writes "1 half"], and there's also a way to write it entirely with numbers, like this [writes  $\frac{1}{2}$ ]. All this means is we have 1 thing divided equally between 2 people. Can you think of some ways we might draw this? [Students share possible drawings.] Now, if we had 3 friends instead of 2, how would that be different? In other words, how would three friends evenly split a sandwich. Can we draw it?



**Student 2:** I can draw it. [Comes to the board, draws a square cut into 3 equal parts]. **Teacher:** Perfect. When it is cut into 3 equal parts, each part is called a “third.” And we can also write it “ $\frac{1}{3}$ ”, meaning 1 thing divided equally among 3 people. Which one of these is more? The half or the third?

**Student 2:** The third.

**Teacher:** Why?

**Student 2:** Because 3 is bigger than 2.

**Teacher:** That’s true. But why does that mean a third is larger than a half? What do the drawings of those things look like?

**Student 2:** Wait... the half is bigger.

**Teacher:** How do you know the half is bigger than the third?

**Student 2:** It just is bigger. Look at the picture.

**Teacher:** Okay. What do you see in the picture?

**Student:** If you cut the sandwich in 3 pieces, then each one is smaller.

**Teacher:** Yes! If we’re sharing the same thing with more people, then we each get a smaller piece. Let’s see if that keeps working. So these numbers below the line are really telling us how big the pieces are. If I’m sharing 1 sandwich with 2 people, they each get a half. If I’m sharing a sandwich with 3 people, they each get a third. More people seems to mean smaller pieces of sandwich. I wonder if that pattern will continue. What if there were 4 friends? Can anyone draw that?

**Student 3:** [raises hand, draws a square cut into four equal parts.]

**Teacher:** Does anyone know what we could call each part?

**Student 4:** A quarter.

**Teacher:** Right. And this is also called a fourth. There’s something nice about that, right? Four equal parts, and each one is a *fourth*. Let’s fill out the chart for that one too. What would the fraction look like?

**Student 5:** There’s a “1” over a “4.”

**Teacher:** [Writing “ $\frac{1}{4}$ ”] And that means *one* thing divided between *four* people. What is the size of this piece? Is it bigger or smaller than a third? [Students discuss and share.]

**Teacher:** On your own, write down what we might write if there were 5 friends sharing the sandwich.

[Draws 1 square split into 5 equal pieces.] What do you think each of these pieces will be called, and how do we write it?

The conversation can continue in this fashion until the chart is filled out up to sixths. Then ask students what we would call it if we split a sandwich into 8 pieces [1 eighth, or  $\frac{1}{8}$ ]. Or 100 pieces [1 *hundredth*, or  $\frac{1}{100}$ ]. If time remains, you can choose other numbers (25, 10, 66, etc.) and ask what we would call the fraction if there were that many people sharing the sandwich equally. In terms of vocabulary, the pattern to notice is that splitting one thing between some *number* of people gives 1 *numberth* or  $\frac{1}{\text{number}}$  as the fraction describing that situation.

Once students are comfortable with the language, pose the following two questions:

**Question 1.** *Four friends want to share 9 apples so they all get an equal share. How can they do it?*

Let students think, pair, and share to discuss their ideas with a neighbor. After a brief conversation, discuss ideas as a group. From last time, there may be ideas involving dividing into equal groups as well as breaking individual apples into halves or “halves or halves,” i.e. quarters, or fourths.

You should be able to help students construct this kind of argument: each friend takes 2 apples, which leaves 1 leftover. Then they cut that apple into four equal pieces, and each take 1 piece. That gives each friend 2 whole apples plus 1 fourth of an apple, or 2 and a quarter apples.

Students may also argue that you could cut every apple up into fourths, and everyone takes 1 fourth from each apple, which would give them each 9 fourths.

**Question 2. Three friends want to share 10 candy bars so they all get an equal share. How can they do it?**

Again, let students think, pair and share their ideas with a neighbor, then discuss as a group.

## Exploration

Have students work on the accompanying worksheets in pairs. All students should finish the “mild” version, and students who finish early can challenge themselves with the “medium” and “spicy” versions. Challenge students to draw pictures and explain what they notice about the questions and their approach to them.

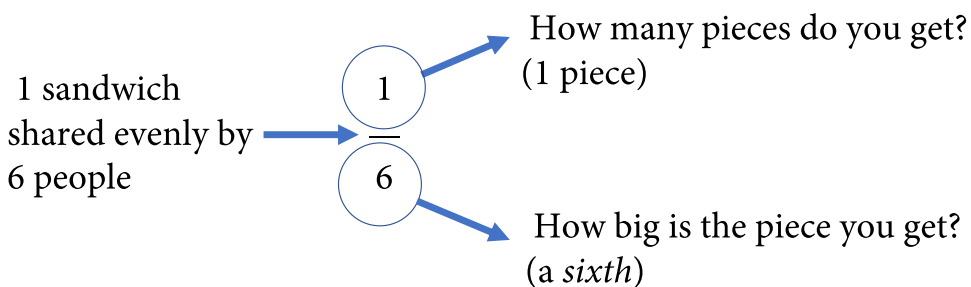
## Questions and Prompts

- Remember to draw a picture.
- Have you divided up all the pizzas? Do all kids have an equal amount?
- What’s different about the situation where there are two friends or three friends?
- How big are the pieces involved? If the number below the line is larger, does that make the pieces larger or smaller?
- [To the student who needs more challenge] Make up your own equal sharing problem that you think you can do, but which would be more difficult.

## Summarize/Discuss

Pose this story: someone comes up to you and asks whether you’d rather have 1 fourth ( $\frac{1}{4}$ ) of a cupcake, or 1 *hundredth* of a cupcake ( $\frac{1}{100}$ ). They say 1 hundredth is way better, because 100 is a bigger number. Do students agree? Have students discuss with a partner, and then the whole group. What does 1 fourth of a cupcake actually look like? (A drawing can help here). What does 1 hundredth of a cupcake look like (basically a crumb).

A nice way to conclude the summary of today's ideas is to construct an anchor chart together with students. Today, you can just begin it by choosing a unit fraction, i.e.,  $\frac{1}{6}$ , and indicating what it actually means. For example:



## Tips for the Classroom

1. As much as possible, connect the drawings [i.e., a square divided into three parts], verbal descriptions of the situation [3 friends sharing a sandwich], written language for the situation [each gets *1 third*] and fraction [ $\frac{1}{3}$ ].
2. Let students do as much of the explaining during the discussions as possible.
3. While we're waiting to introduce the vocabulary for *numerator* and *denominator*, you can use them if it helps clarify what's happening. But it's likely to be easier to just talk about the numbers today, and what they mean. Emphasize that the number below the line [the denominator] actually tells us what size the pieces end up being.
4. Make sure you describe the language to them so everyone understands. There are some exceptional cases in small numbers (especially halves and quarters, which don't follow the rest of the pattern in terms of vocabulary), so make sure you get far enough so students can see the patterns. It's nice that we say "fourths" as well as "quarters." We might say "twoths" as well as "halves" but we don't, maybe because it would sound too strange.

# Pattern Block Fractions 1

**Math Concepts:** Fractions

**Materials:** Pattern blocks, pencil & paper

**Common Core:** 3.NF.1 | 3.NF.3 | MP1 | MP2 | MP3 | MP6 | MP7 | MP8

**Essential Question:** What fraction is each pattern block shape?

## Why we love Pattern Block Fractions

This lesson series presents a hands-on approach to understanding fractions that is fun and illuminating to students.

## The Launch

All these activities are designed for triangles, blue rhombuses, trapezoids, and hexagons only.

**Remove the squares and tan rhombuses.**

### Part 1.

The teacher introduces everyone to the pattern blocks and declares that the triangle has a value of 1. What then, are the values for the other shapes? Students can quickly determine, by building, that the rhombus is then equal to 2 triangles, the trapezoid is equal to 3 triangles, and the hexagon is equal to 6 triangles. Place out stack of 3 hexagons and ask students what it equals in triangles (18).

### Part 2.

Now declare that we're going to try something different, and measure in terms of hexagons instead. In other words, for the rest of today, the hexagon will be the shape that has a value of 1. That means a collection of 3 hexagons will have a value of 3, as opposed to 18 (triangles). Explain that we could choose any shape to be 1 whole but that for us to agree on our answers then we also need to agree on what shape we call 1 whole.

As an opening question, the teacher whether the trapezoid is more than, less than or equal to 1 whole (hexagon). Students can think, then turn and talk. Then the whole group can discuss and reach the conclusion that the trapezoid is less than 1 whole and it has a value of 1 half.

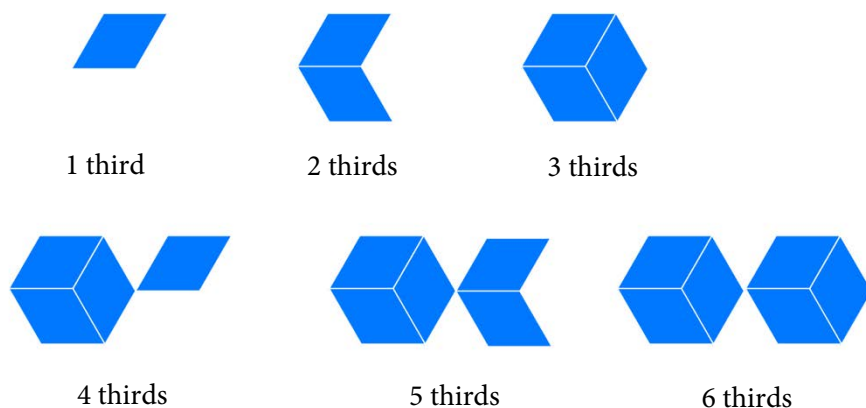
*How do you know?*

A mathematically sound argument: *"Since it takes two trapezoids to make the hexagon, and the hexagon is 1, the trapezoid must be half, because it takes two halves to make a whole."*

Follow up by asking students what the value of the blue rhombus is if the hexagon is 1.

A mathematically sound argument: *"Since it takes three rhombuses to make the hexagon, and the hexagon is 1, the rhombus must be 1 third, because it takes three thirds to make a whole."*

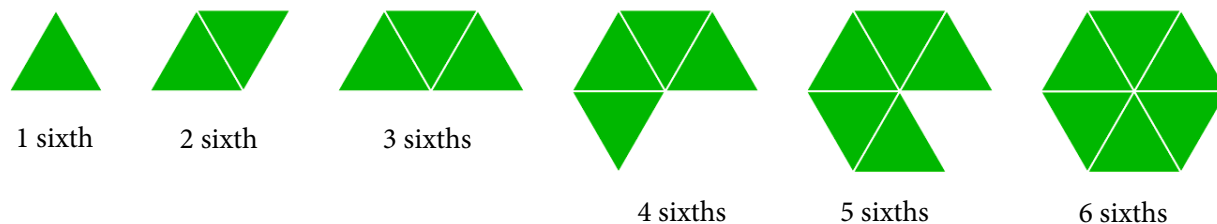
Emphasize the idea of noticing what unit we choose to use by counting out multiple rhombuses and counting with students how many thirds that is. For example, once students agree that 1 rhombus is 1 third, put out 2 rhombuses, and say that you could call this “2 thirds.” Then put out three rhombuses. Would students agree that you could call this “3 thirds?” What do students think you should call 4 rhombuses? [4 thirds.] Ask students to turn and talk to explain what we would call 7 rhombuses. [7 thirds.] The pattern student should notice is that once you know what you’re counting by, you just need to count them off.



Finally, ask what the triangle is equal to.

A mathematically sound argument: *“Since it takes six triangles to make the hexagon, and the hexagon is 1, the triangle must be 1 sixth, because it takes 6 sixths to make a whole.”*

Follow up by asking what 2 triangles should be called [2 sixths.] You can continue adding triangles until students feel like the pattern of counting triangles as sixths is obvious.



Follow up by asking students what 3 trapezoids should be called. Students will likely answer “3 halves,” though “1 and a half” is also right, if they see 2 trapezoids as being joined to make a hexagon. Actually forming the hexagon from two of the trapezoids makes the argument for “1 and a half” clear, and also plants a seed for a great problem-solving strategy. Once students have a handle on the fractions given by the blocks themselves, move on to the main work of the day.



## Exploration

**Challenge 1:** Give students a few seconds to quickly build a shape with pattern blocks. Then student should figure out what number/fraction goes with the shape they built and record their answer. *Remember that we're measuring with hexagons being equal to 1.* (That is, hexagon is the whole, or the unit.)

Once they've named their shape, have them switch shapes with a partner and have each find the number that goes with the other's shape. Do they agree with each other? What makes a challenge harder or easier?

The teacher can time the first round of building, but after that students can work at their own pace. The teacher can repeat this activity many times. The teacher can also specify a maximum number of blocks to use (i.e., ten blocks) to prevent the numbers from getting out of control, at least to begin.

This first challenge could take the rest of class if you can keep students engaged. One especially effective way to do this is to keep challenging them to make shapes that will be challenging to their partner, but which they can successfully assign a fraction to themselves.

Note that we don't want to worry about finding common denominators at this point. If a student builds a shape with 5 trapezoids and 4 rhombuses, it is totally reasonable to call this "5 halves plus 4 thirds". If they rearrange the shapes to form hexagons, they could also call it "3 wholes plus a half plus a third" or "2 plus 1 half plus 4 third." Finding different names for the shapes is excellent, but the most important part is that the names actually describe the pictures.

**Challenge 2:** If students need more scaffolding to the task, challenge them to build shapes that match specific numbers or equations, still with the hexagon equal to 1. For example, they can build (and record) shapes that are

- 1 and a half
- 8 thirds
- 3 and a third
- 5 sixths

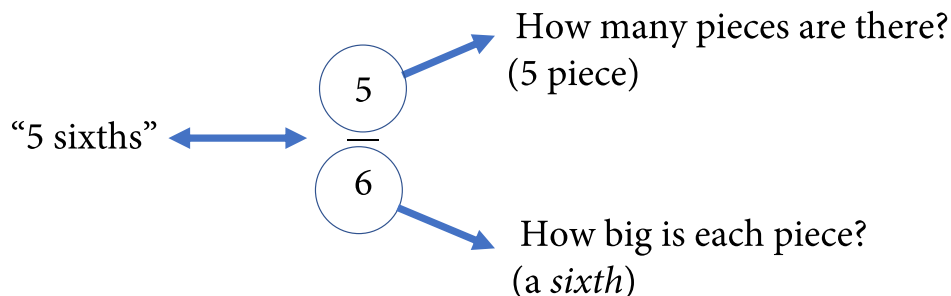
See optional accompanying worksheet.

If students are ready for a real challenge, there is also a "spicy" worksheet featuring more difficult problems. Please note that these are optional given that they do go beyond grade 3 standards.

- 3 and a half plus 1 and two thirds
- 6 minus 3 and five sixths
- 2 plus 3 halves.

## Summarize/Discuss

Ask students to share one new thing they noticed about fractions today. They can share in pairs, then take a few student comments. Conclude by extending the anchor chart from yesterday's lesson. In this case, we're less focused on equal sharing, and more on the idea of the size of the unit we're using (i.e., a sixth) and how many there are (i.e., 5 triangles). To co-construct this chart with students, you can take 5 triangles and ask what we should call it, and then help connect the written language (5 sixths) to the fraction ( $\frac{5}{6}$ ) to what those numbers mean (how many pieces there are, and how big each piece is). The language of *numerator* and *denominator* still isn't necessary at this



stage.

If time permits, ask students to share one thing they still wonder about fractions.

## Tips for the Classroom

1. Students may build extremely large, difficult to calculate structures. It's a good idea to give them a time limit if they do this, so they don't end up building for too long. You can also give them a limit on the number of blocks they can use (or have access to).
2. Connect the shapes to mathematical vocabulary as appropriate to the students. Don't worry about trying to get students to find common denominators yet.
3. Feel free to make up more fraction questions for students or encourage them to make up questions for each other.

# The Fraction Claim Game

**Math Concepts:** Fractions

**Materials:** Pattern blocks, pencil & paper, game sheets with spinners

**Common Core:** 3.NF.1 | 3.NF.3 | 4.NF.3 | 5.NF.2 | MP1 | MP2 | MP3 | MP6 | MP7 | MP8

**Essential Question:** What fractions combine to make a whole?

## Why we love The Fraction Claim Game

This simple game lets students playfully practice building ones, twos, and threes from fractional parts using pattern blocks.

## The Launch

This game is designed for triangles, blue rhombuses, and trapezoids only. Remove the squares and tan rhombuses. The hexagons are unnecessary, but don't need to be removed.

Choose a volunteer to demonstrate the game by playing against you, where all students can see. Each pair of players uses a single board.

### Rules

*Players take turns spinning the spinner. After each spin, they place a block or blocks equal to either one half of a hexagon, one third of a hexagon, or one sixth of a hexagon. Blocks may be placed in any unclaimed area inside any one of the outlines. For example, if you spun one third, you could place one rhombus or two triangles into a single outline, as long as it had space. You must place your block or blocks in a single shape. If there is no space to place your blocks, skip your turn.*

*Whoever completes a shape gets 1, 2, or 3 points for doing so, depending on its size. When all the shapes are complete, add up the scores to see who won.*

Play as many demonstration games are necessary until most students understand how the game is played. Then let them play on their own with partners.

## Exploration

Students can play in pairs, trading partners occasionally in whatever manner works in your classroom. Circulate to play with them and clarify rules when necessary. When students have played for 10 or 15 minutes, you can start asking them about their strategies for winning, and what they noticed about the fractions involved in the game.

Some specific prompts for this discussion might include one or more of the following:

- It's your first turn and you spun 1 half. Where should you place your block? Why?
- It's your first turn and you spun 1 third. Where should you place your block? Why?
- It's your first turn and you spun 1 sixth. Where should you place your block? Why?