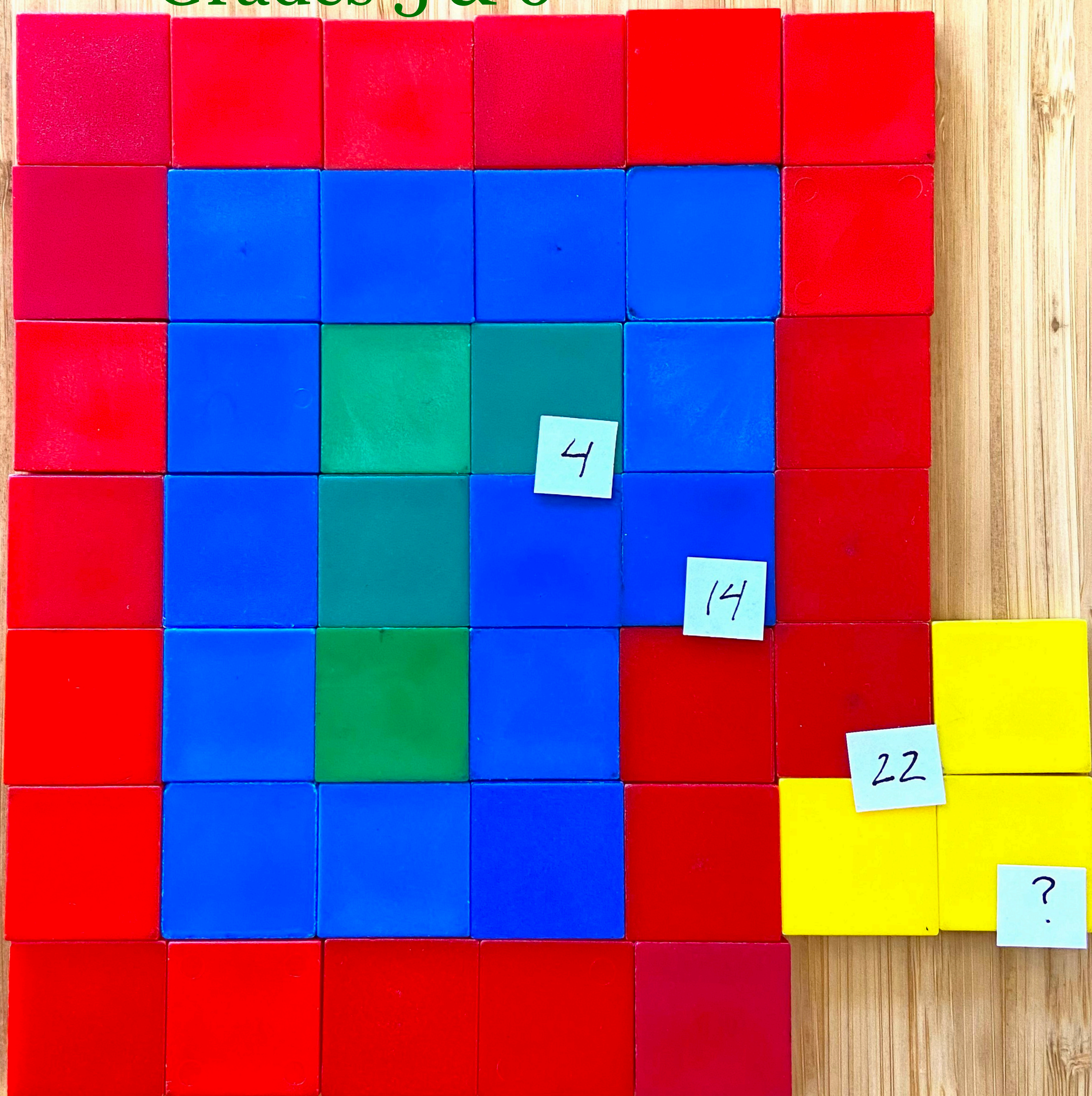
Math for Love

Grades 5 & 6



Dan Finkel & Katherine Cook

A word about using this book

This book was designed to support a summer math program lasting sixteen 75 - 90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. Games can be played for 5 - 30 minutes. Deeper tasks might be good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this curriculum that should be helpful: mathforlove.com/video/math-for-love-video-pd.

Enjoy!

A word about the copyright

We want this book to be used by teachers to help students explore math in a positive way. Feel free to make photocopies, share ideas with parents and colleagues, and use this as a resource draw on. In general, we support this kind of fair use of our materials. Please don't post elements from this book online without citing the source, share large chunks of the book electronically, or sell parts of the book anyone.

Table of Activities

Where to find each activity the FIRST time it's used in this curriculum

Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

Goals of the Math For Love Curriculum

We wrote this program to be both *play-based* and *rigorous*. The goals of the program are two-fold:

- Improve conceptual understanding of and fluency in mathematics
- Give everyone an opportunity to have fun and enjoy math

Many students haven't had enough time working with conceptual models of mathematics before being pushed into abstraction. To remedy this, the curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorially while making connections to abstract reasoning.

Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement**. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation**. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.

3. **Thinking**. Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. **Positive Environment**. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Be ready with questions**
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value *their* thinking and contributions, not just the answer.

★ **Model how to play games, and teach how to win and lose**
Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Unit Chat, Fraction Talk, or opening game (see the Opener in the daily plan). Use the Math Games and Station Breaks for transitions between Activities. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Have a *growth mindset* classroom**
Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you *learn* to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Encourage making and breaking conjectures**
Establish a habit of supporting students' conjectures, hypotheses and predictions, and students will learn more and commit to the thinking process. Help them break

and improve conjectures (using **counterexamples** especially—see Day 2), and they'll begin behaving like true mathematicians. Making conjectures normalizes mistakes as part of the learning process, and gives students a practical way to learn from them. It also makes doing and thinking mathematics the central activity of your class.

★ **Give your students *time* to think and explore**
Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**
We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

★ **Value play**
It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less, you can end sooner. If you need something longer, you should find many of the activities extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- *Goals*
- *Opener*
- *Activity*
- *Game*
- *Choice Time*
- *Closer*

Goals

These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students.

Opener

The Opener is the first activity of math class. The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

In general, the Opener should last about 5 - 10 minutes.

Activity, Game, Choice Time
Following the opener, there is a suggestion for an activity, a game, and Choice Time. This is where the bulk of class time will be spent. There are two recommended ways to approach these three elements.

1. Have students rotate between three stations. This is especially recommended when you have additional adults (instructional aides, parent volunteers, tutors) in the room aside from the teacher.
2. Take the whole class through the activities one by one. This is recommended when the teacher is the only adult in the classroom.

Either way you run your classroom, the elements are designed to give students the maximum opportunity to think & engage, practice skills, explore questions, and have fun.

Choice Time includes a suggestion of a small group of past games and activities for the students to try. This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

Closer
The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer should take 5 minutes or less.

Other Notes and Best Practices

- ★ **Math Breaks and Physical Games**
  Check out the math-based movement breaks in **Appendix 2**. These are excellent as transitions.

- ★ **Folder for Worksheets**
  Give each student a folder where they can keep their worksheets. If they finish an activity early, they can turn back to their unfinished worksheets and finish them. They can also work on them during Choice Time.

- ★ **Choice Time**
  Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should both choose the activity that is right for them, and then stick with it for at least half of the time.

- ★ **Challenge Problems**
  Challenge problems (see **Appendix 3**) are great options for Choice Time any day. Offering “spicy" variations of worksheets or unfinished activities as Choice Time activities can be another nice option.

- ★ **Games to send home**
  See **Appendix 1** for games to send home. These will help parents/guardians and students play math games at home. You can always send other favorite games home, or encourage students to share games they've learned with people at home. Note that there is no homework for this program otherwise.

The Day 16 Transition

The first 15 days of this curriculum come from our grade 5 book. These lessons focus especially on activities and games designed to firm up upper elementary level skills. There’s a particular focus on multiplication, division, fractions, and decimal fractions. In our experience, even the content that targets earlier grade standards is enormously useful for students in both grades 5 and 6.

Day 16 represents a transition into deeper problem-solving tasks. (These show up occasionally in the first 15 days too; the Math Magic Trick and The Calendar Problem are good examples.) The activities grow richer, and are designed to take an hour or more, or even extend into multiple days. It won’t be unusual to have some questions answered and other mysteries remaining at the end of a class. The arithmetic may be simpler, but the opportunities for mathematical thinking are deeper.

Because the activities are longer and richer, we dispense with teaching new games starting at Day 16. Choice Time is still a good option, but may be dropped if the activity still has students' attention and engagement. We stop recommending specifics for Choice Time here - any of the activities and games from the first 15 days are excellent options, and you can curate them as you wish. We include Closers, but they may follow the Wrap section of the lesson plan closely.

We also feature Counterexamples more commonly as an Opener. This primes students to get into the hypothesizing/predicting/conjecturing/defending/critiquing mindset.

This transition to building students' staying power, persistence, and problem-solving skills is one of the best ways to help them succeed in mathematics over the long term. Students may not be able to focus on one activity for so long right away, but the more you invest in their ability to do so, the more it will pay off.

If you're interested in exploring more about rich tasks, check out the links below for a video series on the topic:

Four-part series on rich learning: mathforlove.com/video_category/rich-learning
Three-part series on specific tasks: mathforlove.com/video_category/rich-tasks

Teaching a shorter program

If you are teaching a shorter program and want to emphasize skills and games, and especially if you're working with 5th graders, you could use the following schedule: Days 1 - 15, Day 27.

If you are teaching a shorter program and want to emphasize a problem-based approach, or are teaching grade 6 only, you could use the following schedule:
Day 1, Day 2, Day 5, Day 10, Day 15 - 27.

In the latter case, make sure you teach games from days 1 - 15 as options for Choice Time.

Day 1

Goals

1. Establish classroom values and community.
2. Learn and play arithmetic games
3. Take preassessment
4. Explore pattern blocks and Cuisenaire rods to make creative designs satisfying numerical conditions.

Opener

Don't Break the Bank!

Activity

1) Preassessment
2) Forty Faces

Make sure to let students know that this preassessment is not a "test," and not something they're expected to know any or all of the answers to. It's just a way for you, the teacher, to see what ideas they are familiar with, so you can make sure you keep them challenged and interested. They definitely shouldn't worry if they can't get all, or even most, of the answers. (We've included some very challenging questions!) So just tell them to do their best and not to sweat it.

Note: we encourage doing Forty Faces with Cuisenaire rods, and making 60, 80, and 100 faces, if the students are ready.

Game/Puzzle

Pig

Choice Time

Don't Break the Bank!
Pig
Hundred Faces (with Cuisenaire rods)

Note: Choice Time will be a regular feature of class, but if students are deeply engrossed in activities, it's okay to shorten or skip Choice Time. You may need to balance the value of practicing automaticity through math games in Choice Time versus exploring deeper mathematical content in the activities.

Today, Choice Time is partly about practicing making and sticking with the choice, since the only options are the three from today.

Closer

Ask students to make a list (with a partner, a trio, or on their own) of traits that you need to do math. When they're done, discuss their lists and their ideas.

In the discussion, ask them to consider the activities of the day. There was a lot of math in Forty Faces, but also a lot of choice and creativity. There was a lot of math in Don't Break the Bank and Pig, and they're also both games, and hopefully are fun to play! How does this fit in with their conception of what math is, and what you need in order to do it well?

Close by letting them know that this program is designed to have them playing and exploring a lot, and also thinking deeply. The most important thing they'll need to know is that getting frustrated sometimes is part of the process, and if they can keep engaged and playing and thinking, they'll learn what they need to learn, and get better at what they're doing.

Don’t Break the Bank!

Topics: Triple-digit Addition, Estimation, Probability
Materials: One 6-sided dice, pencil and paper
Common Core: 4.NBT.4, 5.NBT.1, MP6

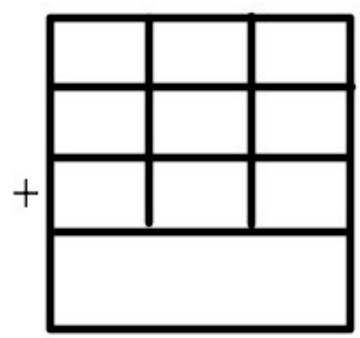
How close can you get to 999 without going over?

Why We Love Don’t Break the Bank

Don’t Break the Bank is a Place Value powerhouse. It takes very little time, so it can be used as a warmup or in those five minutes before class ends. It’s fun, and kids *love* it, even though it involves addition practice. And, while kids will usually break the bank (that is, go over 999) their first few games, they’ll inevitably start estimating and choosing good strategies for themselves. Should the digits in the hundreds column add up to 9 or 8? How common is it to carry? The deeper thinking is almost inevitable.

The Launch

Everyone makes a diagram like this on their paper:



Whole Class Game: The teacher (or a student) rolls the die. Whatever number it lands on, everyone enters it in one of the nine spots on the board. After nine turns, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum **without going over 999**. (See next page for example game.)

Small Group Game: Same as whole class game, except that you take turns rolling the die, and everyone ends up entering different numbers into their grid.

Prompts and Questions

- What’s a good strategy for this game?
- Where would you put this 5?
- Have you already “broken the bank?” How can you tell?

Tips for the Classroom

1. When you are playing a game with the full class, let students take turns rolling.
2. You can narrate your own thoughts when placing digits in the grid. Remember to be clear that you are placing ones, tens, and hundreds.
3. Students may not entirely understand the game the first time through, but they should get the hang by the second game.
4. Extend the game to decimals by adding decimal points up and down one column.

Name________________________________

Example Game.

Turn 1: I roll a 4, and place it in my grid. So does the rest of the class.

	4		
+			

Turn 2: I roll a 2, and place it in the middle.

	4		
		2	
+			

Turns 3 - 8 pass in the same way. Perhaps I have a grid like this:

At this point, I see that I'll be in trouble if anything except a 1 is rolled, since I'll have broken the bank by going over 999.

	4		1
	2	2	1
+	3	6	6

Turn 9: A 5 is rolled, and I broke the bank! When I enter the 5 and add up my numbers, I'm over 999, and I'm out this game.

Now it's time to play again!

	4	5	1
	2	2	1
+	3	6	6
1	0	3	8

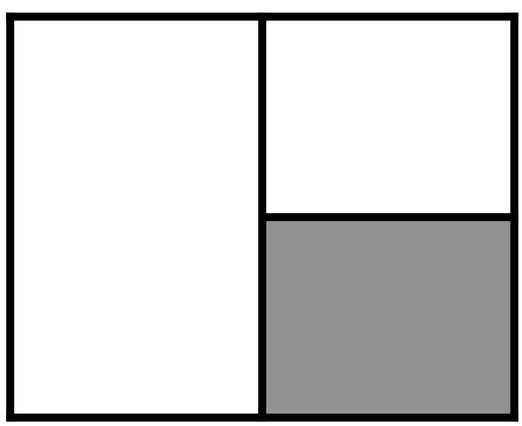
Name____________________________

Preassessment

1) I had 9 cartons of eggs. Each carton had 12 eggs.
   Then I cooked 15 eggs.
   How many eggs were left?

   Explain with equations, words and/or pictures.

2) What fraction of this image is shaded? ________________



Fill in the blanks.

3) $3\frac{1}{3} - 1\frac{1}{6} =$ _______________

4) $5\frac{1}{6} = 2\frac{1}{2} +$ ________________

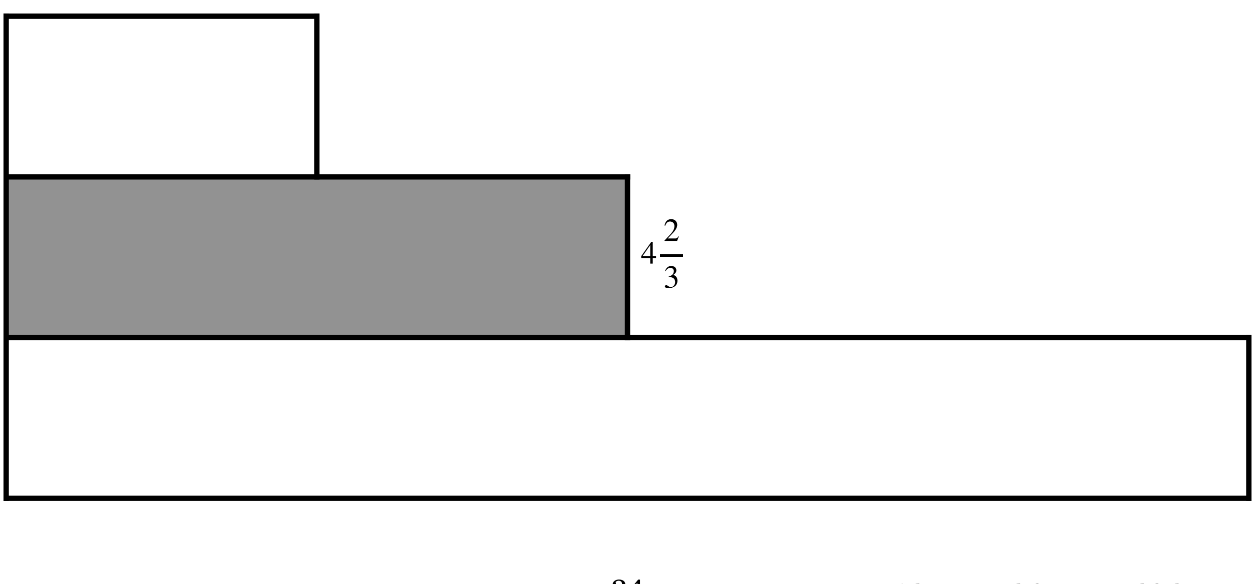
Place the numbers in order from least to greatest.

5) $\frac{4}{3}$, 0.8, 0.29, $1\frac{1}{8}$

____________ < ____________ < ____________ < ____________

6) The shaded area is $4\frac{2}{3}$. Label the other two regions.

One is half the shaded region. The other is double.



7) 1.2 × _________ = ___________ + 15

Fill in the blanks to make the equation true.

8) There are 41 adults and 35 children going to the festival.
Each van can hold 6 people.

How many vans do they need to get everyone to the festival?

Explain with equations, words and/or pictures.

9) Gum balls cost 4¢ each, and licorice costs 9¢ each.
I want to buy 23 gum balls and 12 pieces of licorice.
I have 5 quarters and 7 dimes.

Can I afford to buy the candy I want? Explain with equations, words and/or pictures

10) Juan mowed $\frac{1}{6}$ the lawn. Lynn mowed $\frac{1}{3}$ of the lawn. How much still needs to be mowed?

Label the lawn and defend your answer.

The Lawn

Name______________________________

Preassessment
Solutions and Rubric

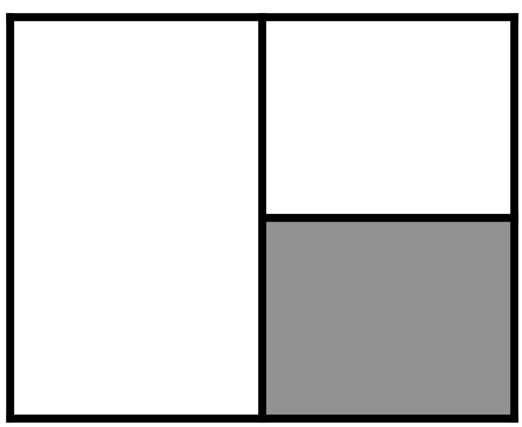
Preassessment

1) I had 9 cartons of eggs. Each carton had 12 eggs.
   Then I cooked 15 eggs.
   How many eggs were left?

   Explain with equations, words and/or pictures.

Answer: $9 \times 12 - 15 = 93$ eggs.
5 points for the correct answer.
5 points for clear equations, explanation, or picture

2) What fraction of this image is shaded? ________________



Answer: 1/4.
10 points for the correct answer.

Fill in the blanks.

3) $3\frac{1}{3} - 1\frac{1}{6} =$ ______________

Answer: $2\frac{1}{6}$ **or** $\frac{13}{6}$**. 10 points for a correct answer.**

4) $5\frac{1}{6} = 2\frac{1}{2} +$ ______________

Answer: $2\frac{2}{3}$ **or** $\frac{8}{3}$**. 10 points for a correct answer, even if not in lowest terms.**

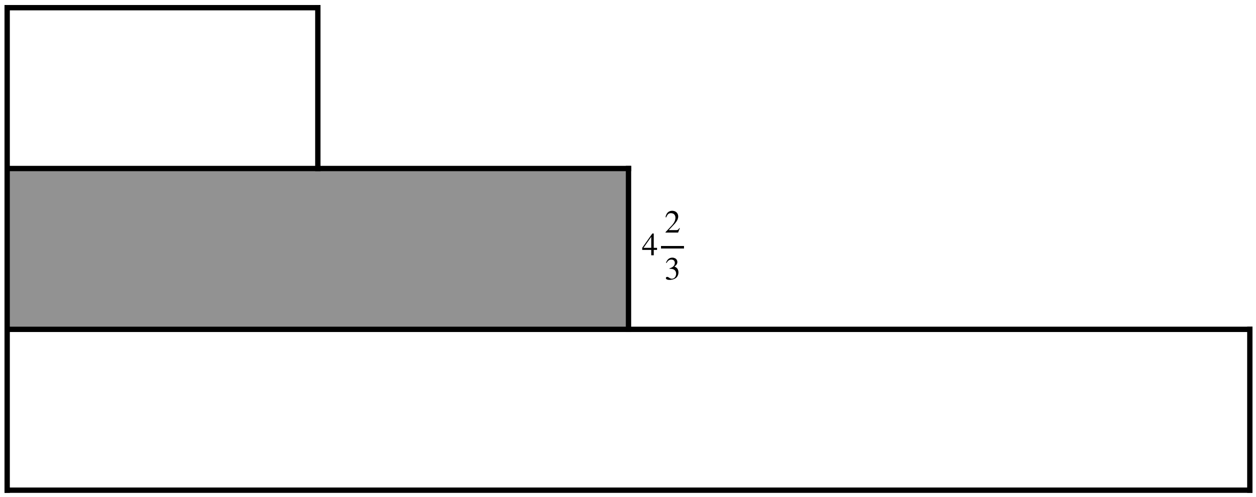
Place the numbers in order from least to greatest.

5) $\frac{4}{3}$, 0.8, 0.29, $1\frac{1}{8}$

___________ < ___________ < ___________ < ___________

Answer: 0.29 < 0.8 < $1\frac{1}{8}$ **<** $\frac{4}{3}$**. Ten points for all correct. Five points partial credit for just one out of place.**

6) The shaded area is $4\frac{2}{3}$. Label the other two regions.
One is half the shaded region. The other is double.



Answer: Top bar is $2\frac{1}{3}$. Bottom bar is $9\frac{1}{3}$.
Five points for each correct answer. (10 points total)
Other equivalent fractions also acceptable.

7) 1.2 × _________ = ___________ + 15

Fill in the blanks to make the equation true.

Ten points for any correct answer.
Example: 1.2 × 20 = 9 + 15

8) There are 41 adults and 35 children going to the festival.
Each van can hold 6 people.

How many vans do they need to get everyone to the festival?

Explain with equations, words and/or pictures.

5 points for clear, correct explanation and/or drawing
5 points for correct answer: 13 vans
Subtract 1 point if there are no units in final answer

9) Gum balls cost 4¢ each, and licorice costs 9¢ each.
I want to buy 23 gum balls and 12 pieces of licorice.
I have 5 quarters and 7 dimes.

Can I afford to buy the candy I want? Explain with equations, words and/or pictures

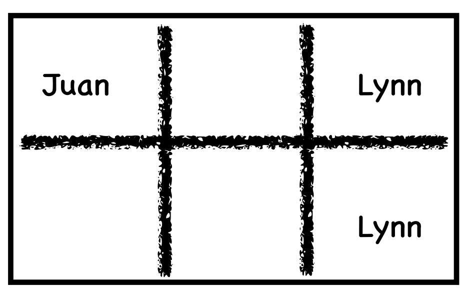
5 points for clear, correct explanation and/or drawing
5 points for correct answer: Total cost of candy is $2.00. Total money is $1.95. So I cannot buy all the candy I want.

Subtract 1 point if there are no units in final answer.

10) Juan mowed $\frac{1}{6}$ the lawn. Lynn mowed $\frac{1}{3}$ of the lawn. How much still needs to be mowed?

Label the lawn and defend your answer.

5 points for labeling and/or written argument
5 points for correct answer



The Lawn

Answer: 1/2 or 3/6. I divided the lawn into sixths. Juan mowed 1 sixth and Lynn mowed 2 sixths, so there were 3 sixths still left to mow.

Forty Faces

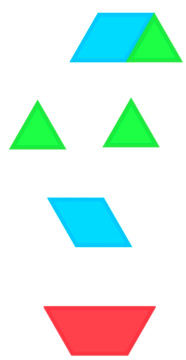
Topics: Addition, subtraction, skip counting, multiplication, logic
Materials: Pattern Blocks, Scratch paper and pencil, Cuisenaire rods (optional)
Common Core: 3.OA.3, 3.OA.8, 3.NBT.2, MP1, MP6, MP7

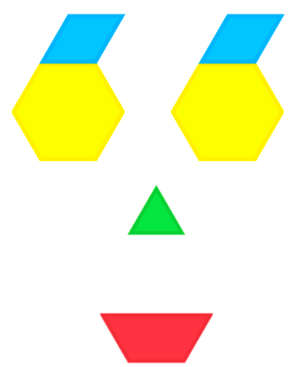
Why We Love Forty Faces

This delightful challenge provides an artistic exploration of ways to construct numbers by repeated addition or multiplication.

The Launch

Prepare the pattern blocks so they contain only green triangles, blue rhombuses, red trapezoids, and yellow hexagons. Ask students how many triangles it takes to build the blue rhombus (2), the red trapezoid (3), and the yellow hexagon (6). Then show them the faces below, either by building them or by projecting images of them.





Briefly discuss how these faces are made by putting together the equivalent of 10 or 20 triangles worth of area. For the second face, for example, there are 2 hexagons, 2 rhombuses, one trapezoid, and one triangle. In terms of triangle area, the total "value" would be 12 (in hexagons)+ 3 (in trapezoids) + 4 (in rhombuses) + 1 (in triangles)
= 12 + 3 + 4 + 1 = 20 triangles worth of area.

Once students understand how to count the "value" of the face, challenge them to create their own faces from pattern blocks that have value (i.e., area) 10, 20, 30, or 40.

Prompts and Questions

- How much more area do you need to add to get to 30?
- Show me how you found the area.
- Let's count how much the hexagons are worth.
- The trapezoids came to 18 area? Let's write that down.
- Do you think the two of you could make a face with an area of 75?

The Wrap

Share a pattern block face that has area forty. Tell students that there's someone else who doesn't count faces in terms of

Tips for the Classroom

1. Remove the orange squares and tan rhombuses from the pattern blocks before the lesson begins, or tell students not to use them (unless you'd like to go on some potentially interesting diversions).
2. Let students challenge themselves when they're ready. Can they make a "100 face"?
3. Encourage students to use pencil and paper to actually track the arithmetic. It gets difficult to find the answer without making a mistake once the faces get larger.
4. You can easily use Cuisenaire rods to make "forty faces" as well. Just use the white cube as the unit. Below is an example of a face with a value of 30.

Pig

Topics: Probability, strategy, addition, estimation (optional: fractions and probability)
Materials: One 6-sided die, pencil and paper
Common Core: 2.OA.B.2, 2.NBT.B.5, 2.NBT.B.6, MP1, MP7

Roll the dice and collect points. You can go as long as you want, but roll the wrong number and you lose all your points from that turn!

Why We Love Pig

Pig is easy to learn and gives students practice with addition (and multiplication, with Odd Pig Out). Pig is mathematically rich. Students must articulate and defend strategies relating to handling chance.

The Launch

Before you start, remind students of the importance of winning and losing gracefully. They'll do a lot of both with Pig, and it's important to take it lightly, since it's so easy to have setbacks.

Pig is a game that will sometimes punish a good decision and reward a bad one, which presents a real challenge for students: how can you tell if you made a good decision or a bad decision? Does strategy even matter in a game of luck like Pig? The teacher can bring these questions out over the course of the lesson, and let students grapple with them. In the long term, taking a scientific approach by running experiments and collecting data is one excellent way to handle the problem.

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students give you "thumbs up/down" if they think you should keep rolling.

How to Play

Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a 2, 3, 4, 5, or 6, the player adds that many points to their score for the turn. A player may choose to end their turn at any time and "bank" their points. If a player rolls a 1, they lose all their unbanked points and their turn is over.

Beginner Game: The first player to score 50 or more points wins.
Advanced Game: The first player to score 100 or more points wins.

Demonstrate enough turns so that students can see how rolling a 1 will lose them unbanked points, and that points in their bank will be safe even when a 1 is rolled.

The Work

Students can play Pig for fun anytime. Games can be quick and light.

The deeper work of Pig comes when we start to examine strategy. As students play, ask them to notice reflect on what strategy they're using as they as they play. Are they taking big risks, or is their play more conservative? After students have had enough time to play, discuss strategy for Pig as a class. What strategies did students use? Does strategy matter? How do you know? Pig is clearly a game of chance, but does that mean strategy makes no difference?

The stage is being set to actually run an experiment. How can we determine for sure whether strategy matters or not? We could pit two strategies against each other, and see which one wins; the more extreme the strategies, the more clearly we could see the difference. And what are the most extreme strategies? The most conservative we call *Better Safe than Sorry*: roll once and immediately bank your points. The most extreme we call *Let It Ride*: keep rolling until you get 50 points and win, or roll a 1 and lose all your points.

So if one person uses the *Better Safe than Sorry* strategy, and the other plays the *Let It Ride* strategy, who is more likely to win? Let students vote, and collect the number of votes from students as to which strategy they think will win. It might look like this:

Better Safe than Sorry	*Let It Ride*	No Difference
13	5	4

Now you can run an actual experiment. Have students play in pairs, each playing one of the opposing strategies. They should play to 50 points, and keep track of how many times each strategy wins. If students have 10-15 minutes to collect data, you'll likely have a good number of finished games. Collect all the data together on the board, and add up how many games were won for each strategy. In my experience, and quite surprisingly, *Let It Ride* tends to win about 75% of the time.

There's a big discussion here about probability, statistics, certainty, and uncertainty. Is the classroom data convincing to students? Has it settled the questions about whether strategy matters and which strategy is better? What would be a better strategy to pit against *Let It Ride* in a future game (for example, roll 3 times and then bank your points)? It's possible to run successive experiments, or to have students program a computer to run experiments for them.

And of course, you can always let students just play the game for fun when you have some extra time in class.

Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?

- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?

The Wrap

In addition to the experimental approach described above, we can wrap up playing the game by discussing the probabilities of outcomes, and how they can help us make predictions for good moves. Consider these questions:

- What is the probability that you roll a 1 on a given roll? (Answer: 1/6)
- What is the probability you won't roll a 1? (Answer: 5/6)
- If you don't roll a 1, what is your average point gain? (Answer: 4)

Considering these values, you can reframe the question for each roll in the following way: is it worth risk losing the points you haven't banked (1/6 chance) yet in order to have a of gaining about 4 points (5/6 chance). That means your chances of gaining an average of 4 points on a given roll is 5 times greater than your chance of losing all your points. When is this worth it?

If you have 10 points unbanked, should you risk them at 5:1 odds in order to gain 4 more? That actually seems like a good bet. In fact, anything up to 20 points ($20 = 5 \times 4$) seems like a good bet when you're considering the problem in this framework. This gives a mathematical rule of thumb for how you might want to proceed. Another experiment to try, then, is to play to 100, but have some people try the "Bank when you have 20 points or more" strategy, and others play some other strategy of their choice. Who will tend to win? (The difference may be subtle.) Still, this is another example of how mathematical analysis can give us some control over a situation, even when there's a great element of chance involved.

Tips for the classroom

1. Demonstrate the game a couple times with the whole group. Solicit advice about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!
3. As students play each other, circulate through the room and ask them about their strategies. It's ok for students simply to play, but there's an opportunity to probe deeper into the workings of chance and the strategy of the game too.

Name________________________________

Pig

Rolls	Rolls
Bank	Bank

Day 2

Goals

1. Connect units and unitizing to multiplication and division.
2. Learn and play Odd Pig out, a multiplication game.

Opener

Unit Chat - See Appendix 4 for images

Activity

Cuisenaire Rod Multiplication and Division

Game/Puzzle

Odd Pig Out

Choice Time

Don't Break the Bank!
Odd Pig Out
Hundred Faces (with Cuisenaire rods)
Challenge Problems - see Appendix 3

Closer

Play Counterexamples with students.

Depending on time available, you could do just the non mathematical examples (e.g., everything that flies is a bird) or basic mathematical examples (every shape with right angles is a square). The main move in the game is to refine the conjectures as students give counterexamples.

When the game is done, ask students if they see a connection between this game and the work we did today with Cuisenaire rods. In fact, it may have been similar. We made a conjecture (I think the yellow rod must be worth 24) and then adjusted if it didn't fit with other facts we knew (but if that were true, the orange rod must be worth 48. But I know the orange rod is 50, so that can't be right.)

In fact, tell students that the work of making conjectures and breaking them with counterexamples is some of the key work that mathematicians do. And it's a big part of what this class will be about too!

Unit Chats

Topics: Mental math, numerical fluency; argument & critique
Materials: White board or projector
Common Core: Variable, but generally good for NBT, NF, OA, MP1, MP3, MP6.

Counting with respect to different units.

Why We Love Unit Chats

Unit Chats are a kind of Number Talk that emphasizes not just how many, but also the unit involved. These are a fantastically productive, fun, differentiated, and delightful warm up for math. Perfect as a 5 - 10 minute opening or closing exercise.

The Launch

Post a Unit Chat image. It should have different kinds of objects to count in it, and be arranged in arrays or other structures as appropriate for the student level. Students get some time to look at what is in the picture, and how many of which object they see. After they've had 20 - 60 seconds to look, ask students what they see. You'll receive different answers about what they saw, and how many. You can ask students to explain different ways of counting what they saw, and also different things that they see to count in the picture.

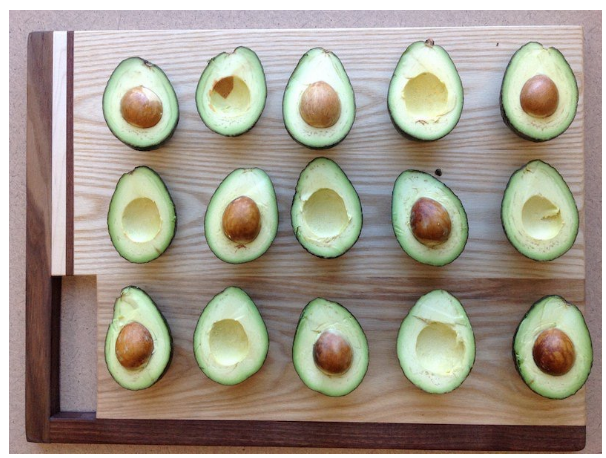


Photo credit: Christopher Danielson

Example Unit Chat
Teacher: Take a look at this picture. Think about how many you see. [Waits for 30 seconds.] Quietly turn to the person next to you and tell them how many you see. [Students quietly discuss.] Who would like to share what they saw?
Student: I see avocados.
Teacher: How many avocados do you see?
Student: I see fifteen.
Teacher: Fifteen avocados. I don't see that at all.
Student: Look, there are five on the top, then another five, and then five on the bottom. So that's 15.
Teacher: Ah! You're talking about the *avocado halves*. In that case, I agree. That's 5, 10, 15 avocado halves. What else do you see?
Student: They're in a checkerboard pattern.
Teacher: That's true. The pitted avocado halves and the unpitted avocado halves form a checkerboard. Does that mean there are the same number of each?
Student: Yes! / No!
Student: There are 8 with pits.
Teacher: Let's count. 1, 2, 3, 4, 5, 6, 7, 8. That's right. Did you count one by one?

Student: No, I saw the 3 on the top, plus 2, plus 3.
Teacher: Ah, and 3 + 2 + 3 = 8.
Student: There are only 7 without pits.
Teacher: It's strange that it would be different if they were in a checkerboard pattern. I still don't see why there are more with pits than without.
Student: Because the first and last have pits. If there were one more row, it would be the same.
Teacher: I think I see. You're saying it goes "pit, no pit, pit, no pit,..." But it ends on "pit" and starts on "pit," so there's an extra.
Student: One "no pit" got thrown away.
Teacher: What do you mean?
Student: There were eight avocados that got cut in half, but one no-pit half isn't there.
Teacher: How do you know?
Student: Because if you put all the halves together, it would make wholes, and there would be eight wholes. But the last no-pit half is missing.
Teacher: So how many whole avocados are there?
Student: Seven and a half.
Teacher: I see. So we could see this as 15 half avocados, or we could see it as 7 and a half whole avocados. Very neat!

Prompts and Questions

- How did you see that?
- How did you count that?
- Does anyone else think they can explain what Therese is saying?
- Turn to the person next to you and see if you can see what Dwayne is describing.

Tips for the Classroom

1. Use images that are accessible to everyone. The best images have some easy things to count and some harder things to count.
2. You can emphasize how students counted, or shift the conversation to what they counted, depending on what will be the most engaging and enlightening. It can be okay if Unit Chats turn into something that resembles a Number Talk.
3. Remember: doing more short Unit Chats is better than doing just a few long ones. Aim for 5 - 10 minutes. You can use multiple images if they go super short, but often one image is plenty.

Cuisenaire Rod Multiplication and Division

Topics: Multiplication, Division, changing units
Materials: Cuisenaire rods, paper and pencil
Common Core: 4.OA.2, MP1, MP2, MP3, MP7

You know the value of the one shape… how can you figure out the other pieces?

Why We Love Cuisenaire Rod Multiplication and Division

This lesson combines the fundamentals of division with deeper problem solving in a context that's natural and hands-on.

The Launch

This lesson is designed to alternate between the teacher posing problems by assembling groups of Cuisenaire rods physically and saying their value, and students solving the question on their own, and writing up their solutions. Give students time as needed —at least a minute or two for the early problems, and more as they get harder.

Problem 1. If you've used these before, you may be used to the white Cuisenaire rod being worth 1, the red being worth 2, since it's as long as two whites, and so on. But for now, let's try something different and say the red Cuisenaire rod is worth 10.
What are the other Cuisenaire rod worth?

Note: You can pose problems with almost no words by placing/drawing/projecting the Cuisenaire rods on a white board, and writing the numbers underneath or beside them.

If student haven't thought through this kind of problem before, you can solve some examples. For example, the purple rod equals 2 red rods, or 2 tens. That means purple equals 20. Similarly dark green would equal 30. Since light green is half of the dark green, it must also be half of 30; so light green is 15. And so on. Students should get the idea quickly enough.

Once students have found what all the rods are worth, you can ask them to prove how they know that the blue rod is 45, say. There are many ways to prove it using what you know about the smaller. For example, the blue rod is 4 reds (i.e., 4 tens) plus 1 white (1 five). That's 45. It's also a yellow plus a purple, which is 25 + 20 = 45. It's also one white less than an orange rod, which gives 50 - 5 = 45. And so on.

Problem 2. If purple equals 32, what are the other rods?

In this case, every rod will be equal to a multiple of 8.

Once students have shown their solutions to this problem, you may want to pose several questions at once, so students can work through to harder problems when they're ready.

Problem 3. If dark green equals 48, what are the other rods?

Problem 4. If brown equals 72, what are the other rods?

Problem 5. If black equals 56, what are the other rods?

Problem 6. If orange plus yellow equals 60, what are the other rods?

If more problems are needed, let students make up their own, and challenge each other to solve them.

Prompts and Questions

- You know the brown rod equals 72. What if the white rod equaled 6? Would that work, or is that too big or too small?
- How do you know that the yellow rod has that value?

Wrap Up

Take the last problem all students have attempted and spend a few minutes letting students share their answers with each other. You can have them share their methods with a partner, and then take one or two volunteers to share their method with everyone.

Tips for the Classroom

1. Make sure students can build their own version of the problem and solve physically.
2. Adjust the difficulty of the problems as necessary.
3. Students can always guess and check. This is a good strategy to encourage at first, since it makes the connection between division and multiplication more explicit.

Odd Pig Out

Topics: probability, strategy, multiplication, addition
Materials: Two 6-sided dice, pencil and paper
Common Core: 3.OA.7, 3.NBT.2, MP1, MP5, MP6, MP7

Roll the dice and multiply. You can go as long as you want, but roll an odd number and you lose all your points from that turn!

Why We Love Odd Pig Out

Odd Pig Out is a natural extension of Pig to multiplication. It is great practice for multiplication and addition in a fast-moving, fun game.

The Launch

The teacher chooses a volunteer, explains the rules, and plays a demonstration game. Because students already know Pig, this game should be relatively intuitive to learn.

Players take turns rolling the dice as many times as they like. After each roll, they multiply the numbers they rolled together. If the product is even, they add that number to their current points for the turn. If the product is odd, players lose all their points from that turn and their turn is over. A player may choose to end their turn at any time and "bank" their points.

Play to 300.

Prompts and Questions

- Is there an easier way to add up all those numbers?
- How many points to you have for this turn so far?
- Who's ahead?
- Are you sure that's the product of those two numbers? What does your multiplication table say?
- What strategy are you using?

The Wrap

Ask students whether they're more likely to roll odd products or even products. How many odd numbers are there on the multiplication table (up to 6 by 6)? How many even numbers? How are they distributed? Do students see any patterns?

(Optional) If you'd like to dig into the probabilities, the same mathematical approach you took with Pig can give a good rule of thumb for a strategy for Odd Pig Out. Students will need to do the harder mathematical work of figuring out:

- What is the probability that you roll an odd product on a given roll? (Answer: 1/4)
- What is the probability that you roll an even product? (Answer: 3/4)

- If you don't roll an odd product what is your average point gain?
  (Answer: the sum of the even numbers on a multiplication table of the appropriate size, divided by the number of even products. This is, in fact, a fascinating mathematical counting problem. For a 6 by 6 multiplication table, the sum of the even numbers is $21^2 - 9^2 = 441 - 81 = 360$.
  There are 36 - 9 =27 even products, so the average is $360/27 = 13^1/_3$.)

If we use the same argument as with Pig, you should be willing to risk up to 40 points if you have 3 to 1 odds of winning $13^1/_3$ so that's a good estimate of how risky you should be willing to be in Odd Pig Out.

Tips for the classroom

1. Demonstrate the game a couple times with the whole class (or in a station). Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing.
3. Note that the deeper mathematics discussed in the Wrap are optional, or can be returned to when students are ready. Just playing the game is great for multiplication fact practice.

Odd Pig Out

Roll two dice and write down their product. You may choose to continue rolling as long as the products are even. End your turn to bank your points.
If you roll an odd product, end your turn and lose all unbanked points.

Products	Products
Bank	Bank

Counterexamples

Topics: logic, deduction, mathematical argument, communication
Materials: None
Common Core: Variable, but especially MP1 and MP3.

Prove the teacher wrong. Rigorously.

Why We Love Counterexamples

Every kid loves to prove the teacher wrong. With Counterexamples, they get to do this in a productive way, and learn appropriate mathematical skepticism and communication skills at the same time.

It is possible to play Counterexamples with kids as young as kindergarteners as a kind of reverse "I Spy" ("I claim are no squares in this classroom. Who can find a counterexample?"). What's great, though, is that you can transition to substantial math concepts, and address common misconceptions. Counterexamples is a perfect way to disprove claims like "doubling a number always makes it larger" (not true for negative number or 0) or sorting out why every square is a rectangle, but not every rectangle is a square. For older kids, you can even go into much deeper topics, like: "every point on the number line is a rational number."

The language of counterexamples is crucial to distinguish true and false claims in mathematics; this game makes it natural, fun, and plants the skills to be used later. Counterexamples is also a great way to practice constructing viable arguments and critiquing the reasoning of others.

How it works

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher, though it can be a student) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All ______s are _______" or "No ______s are _______." You can also play around with statements like "If it has ______, then it can _______." For instance:

It's often best to start with non-mathematical examples.

- All birds can fly. (Counterexample: penguins)
- If something produces light, then it is a light bulb.
- If something has stripes, then it is a zebra.

Once students have the hang of it, make the examples more mathematical.

- Doubling any number makes it bigger. (Counterexample: -1 doubled is -2, which is smaller. 0 doubled is 0, which is the same size.)
- Multiplying two numbers gives a product that's larger than either of the starting numbers.
- Multiplying two numbers never gives the same answer as adding them. (Counterexample: $2 + 2 = 2 \times 2$. Or $3 + 1.5 = 3 \times 1.5$.)
- Fractions are always between 0 and 1.
- If shape A has a larger area than shape B, it has a larger perimeter also .
- If a shape has all its sides the same, then it's a square. (Counterexample: a rhombus. Squares need four equal sides AND four equal angles.)

Example

Teacher: I claim all animals have four legs. Who can think of a counterexample?
Student 1: A chicken!
Teacher: Why is a chicken a counterexample?
Student 2: Because it has two legs.
Teacher: Right. I said every animal has four legs, but a chicken is an animal with just two legs. So I must have been wrong. Let me try to refine my conjecture then. I should have said that animal must have 2 or 4 legs. That feels right.
Student 3: What about a fish?
Teacher: Aha. A fish is an animal with no legs. Thank you for showing me the error of my ways. What I should have said is that animals have *at most* four legs.
Student: 4: What about insects?
And so on.

Tips for the Classroom

1. It's good to make up false conjectures that are right for your students. But start simple.
2. Kids can think of their own false claims, but sometimes these aren't the right kind, and they often have to be vetted.
3. Once you introduce the language of counterexamples, look for places to use it in the rest of your math discussions.
4. You can also use Counterexamples to motivate a normal math question. Instead of saying "draw a triangle with the same area as this square," you can say, "I claim there is no triangle with the same area as this square." If students know to look for counterexamples, this will set them to work trying to disprove the claim right away.

Day 3

Goals

1. Play arithmetic games.
2. Solve story problems in creative ways.

Opener

Target Number

Activity

Story Problems - Spending Spree

Note: students don't necessarily need to answer all the questions on this worksheet. You can do the first question with them to play up the story of a Spending Spree, and also to see how there may be different approaches and solutions. The "Spicy" version of the worksheet is for students who want an extra challenge, or for Choice Time on a future day.

Game/Puzzle

Bowling

Choice Time

Don't Break the Bank!
Odd Pig Out
Bowling
Challenge Problems - see Appendix 3

Closer

Ask students if they'd rather have half of $700, or 700 half-dollars. Have them defend their reasoning using math or other observations (i.e., 700 half-dollars are really heavy!). They can talk to a partner or partners first, and then discuss with the whole group.

Target Number

Math concepts: Arithmetic, equivalencies
Equipment: pencil & paper
Common Core: Variable, but especially OA

You know the answer. What's the question?

Why We Love Target Number

This is a quick check-in that adjusts to the abilities of each student, allows for creativity and arithmetic practice together, and is a lot of fun. Target Number is a perfect warm-up.

The Activity

The teacher writes a "target" number on the board. The students try to write down as many different equations as they can that have that the target number as the answer. Then students share their favorite answers. For younger students, drawing different pictures or arrangements of ways to see/understand that number is an ok alternative.

Example

The teacher writes 7 on the board, and lets kids write on their own paper for about a minute, then asks students to share what questions they found. Students raise their hands to volunteer solutions while the teacher writes them on the board. These equations may go from simple equations like $6 + 1 = 7$ to the more complex $(4 \times 3) - 5 = 7$. The great thing is, anyone can start, but the sky is the limit!

Questions

Don't pursue these questions the first day you play Target Number. When your students are ready to go deeper with this activity, these questions will lead to interesting patterns to explore.

- If we only add 2 numbers, how many answers can we find?
- What if we add 3 numbers, or 4 numbers?
- What about any number of numbers?
- What if we only subtract, or only multiply, or only divide?
- What's the longest number sequence you can find that hits the target number?
- Can you hit the target number if you only use a single number, such as the number 4, in your equation?

Tips for the Classroom

1. Resist the temptation to praise answers with many steps as “smart.” This activity gives everyone a chance to contribute and be valued. You can describe those answers as “long,” or as having many parts.

2. If answers are wrong or unclear, you can take the opportunity to do the arithmetic with the class. On the other hand, if a student uses terms (like square root) that the class isn’t ready for yet, you can write down their answer but move on to other solutions.

3. One opportunity this lesson gives you is the chance to emphasize equivalency. If one student knows that $6+1 = 7$, and someone else knows that $(3 \times 4) - 5 = 7$, then that means that $6 + 1 = (3 \times 4) - 5$. It’s nice to underline the point that there are many ways to equal 7, and that these ways are all equal to each other.

4. To further emphasize equivalency, write $7 = 6 + 1$, rather than $6 + 1 = 7$.

5. **THIS TIP IS ESPECIALLY USEFUL**. Let’s say someone says that $7 = 5 + 3$. Rather than just saying “wrong,” say that $5 + 3$ gets us close to 7, but we need to do something else to get all the way there, then challenge students to find what still needs to be done. If someone can explain that $5 + 3$ is 8, and so you need to take 1 away, you have the number sentence $7 = 5 + 3 - 1$. This is both more sophisticated and accepts the original students wrong answer as a path toward a better, accurate answer, rather than a dead end.

Name________________________

Spending Spree

Karin won a spending spree at bookstore!
The only catch is, they has to spend every dollar in each round. If any is left over, she doesn't get to keep anything she bought.

Hardback books cost $15 each.
Paperback books cost $8 each.
Comic books cost $3 each.

What should she buy in each round so that every dollar gets spent?

Round 1. $50

Round 2. $100

Round 3. $200

Round 4. $500

Bonus Round: $654

Name________________________

Spending Spree

Spicy!

Karin won a spending spree at bookstore!
The only catch is, they has to spend every dollar in each round. If any is left over, she doesn't get to keep anything she bought.

Hardback books cost $14.50 each.
Paperback books cost $8.25 each.
Comic books cost $3.75 each.

What should she buy in each round so that every dollar gets spent?

Round 1. $48

Round 2. $70

Round 3. $155

Round 4. $225

Bonus Round: $660

Bowling

Topics: Equality, substitution, addition, skip counting and multiplication
Materials: Pattern blocks, paper and pencil, worksheet with table
Common Core: 3.OA.7, MP1

Can you knock over all the pins?

Why We Love Bowling

This quick, simple, fun game is great for collaborative or competitive play. It's also a great chance to practice all the operations in a very simple context with small numbers.

Launch

The teacher can do a first game with the entire class, demonstrating as they go. Subsequent games can be played with students deciding whether they want to play on their own or with a partner or small group. Either way, the teacher can roll the numbers at the beginning of play so everyone is using the same roll. (For future games in Choice Time, students will roll on their own.)

Roll three dice. Everyone gets to use the three numbers rolled at most once each to "knock over" the pins labeled 1 - 10. Every time you can make an equation that has a number from 1 to 10 as its solution, that pin gets "knocked over." The goal is to knock over as many as you can!

Prompts and Questions

- How did you find that answer?
- Did you have a way to get 7?
- You got 8… is there a way to change something to get 9 too?

The Wrap

No major wrap required, though it can be nice to see if there was a number most students couldn't get, and see if anyone got it.

Tips for the Classroom

1. If students are less confident with multiplication, division, or parentheses, it's all right for them to start with what's comfortable. But highlight and demonstrate the power of those more advanced feeling arithmetic moves.
2. Pair students up after they've been working alone for a minute or two to see if they can get each other unstuck.

Bowling

Roll 3 dice. You knock down a bowling pin if you can make the number on it using the numbers you rolled, with addition, subtraction, multiplication, and division.

EXAMPLE

I rolled 1, 3, 4. I can knock down almost all the numbers from 1 to 10:

$1 = 4 - 3$

$2 = 4 - 3 + 1$

$3 = 4 - 1$

$4 = 4 \times 1$

$5 = 4 + 1$

$6 = 4 + 3 - 1$

$7 = 4 + 3$

$8 = 4 + 3 + 1$

$9 = 3 \times (4 - 1)$

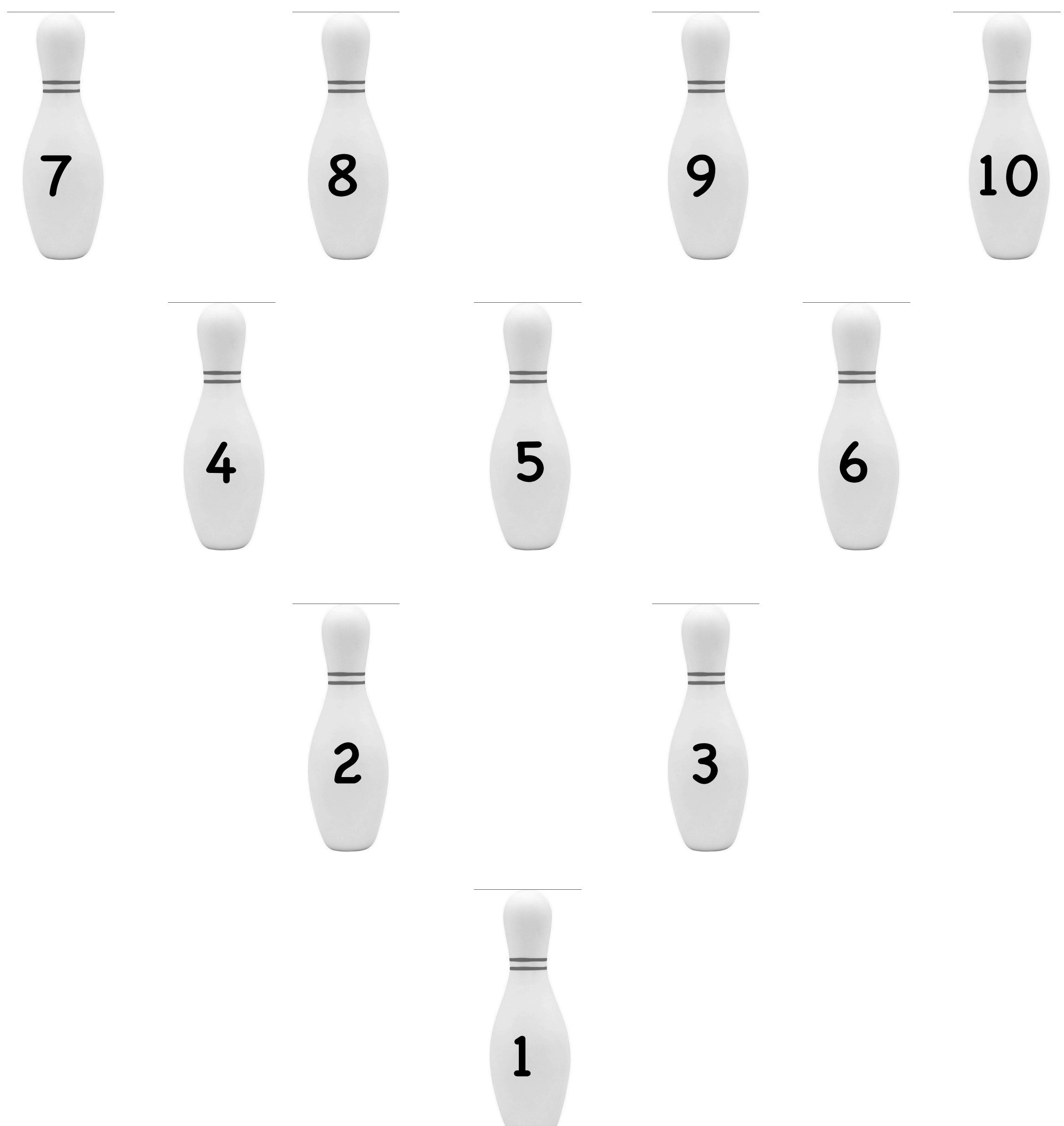
10 = ????

That's 9 pins down. Pretty good!

Now you try. Roll three dice, and see how many of the pins you can knock over.

Bowling

Roll 3 dice. Use the numbers you roll to make equations to knock down as many pins as you can.



Day 4

Goals

1. Learn and use the area model to solve 2-digit by 2-digit multiplication problems.
2. Play logic and arithmetic games.

Opener

Pico Fermi Bagels

Activities

1. Mini-lesson on the Area Model
2. Pose to students: what is the largest product you can make with the digits 1, 2, 3, 4?

Note: the mini-lesson gives a visual model for understanding and solving 2-digit by 2-digit multiplication problems. The question following it gives a fascinating challenge to solve, which should provide plenty of practice with the area model. (The answer is 41 × 32 = 1312, though it's not obvious that this is largest without doing much more work.) This question extends naturally to: what is the largest product you can make with the digits 2, 4, 6, 7 (or any collection of four digits).

Game/Puzzle

Big Blockout

Choice Time

Don't Break the Bank!
Odd Pig Out
Big Blockout
Pico Fermi Bagels
Challenge Problems - see Appendix 3

Closer

Ask students what the largest product they can make with the digits 2, 3, 4, 5 is. Does having solved the same question for 1, 2, 3, 4 help?

Pico Fermi Bagels

Math concepts: Logic and deduction, place value
Equipment: Paper or whiteboard to record guesses
Common Core: 1.NBT.B.2, 2.NBT.A.1, 2.NBT.A.3, MP1, MP3

Can you use the clues to get the number with the fewest possible guesses?

Why We Love Pico Fermi Bagels

Once you get used to the funny words, this game is a wonderful exercise in logic, and a nice way to get kids playing with the ideas of digits and place value. Pico Fermi Bagels is a perfect warmup.

How to Play

The teacher secretly chooses a number with no repeated digits. Students attempt to guess the number. After each guess, the teacher gives feedback:

- If the guess has no numbers correct, the teacher responds: "Bagel."
- For each digit the guess has correct, but in the wrong place, the teacher says: "Pico."
- For each digit the guess has correct and in the correct place, teacher says: "Fermi."

Example Game
Let's say you wrote down the secret number 487.
Guess 1: 139. Response: "Bagel" — no digit is correct.
Guess 2: 820 Response: "Pico" — the 8 is right, but in the wrong place.
Guess 3: 468 Response: "Pico Fermi" — the 8 is right, but in the wrong place, the 4 is in the correct place.
Guess 4: 568 Response: "Pico" — the 8 is right, but in the wrong place.
Guess 5: 482 Response: "Fermi Fermi" — the 4 and 8 are in the correct place.
Guess 6: 487 Response: "Fermi Fermi Fermi" — all digits are in the correct place.
The guessers got it in six guesses! Can they do it in even fewer next time?

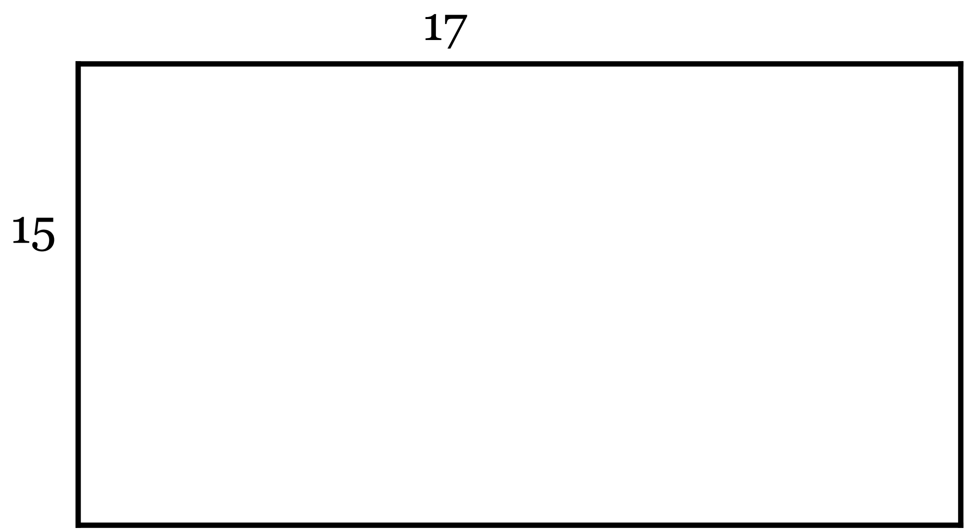
Tips for the Classroom

1. Note that students DON'T get a Pico, Fermi, or Bagel for each digit. The clue applies to the entire 2- or 3-digit number.
2. Start with 2-digit numbers. Go to three-digit numbers only when the 2-digit numbers have become straightforward.
3. Write the guesses and the responses somewhere that everyone can see them.
4. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses.
5. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

References: Play online at http://communicrossings.com/html/js/pfb.htm

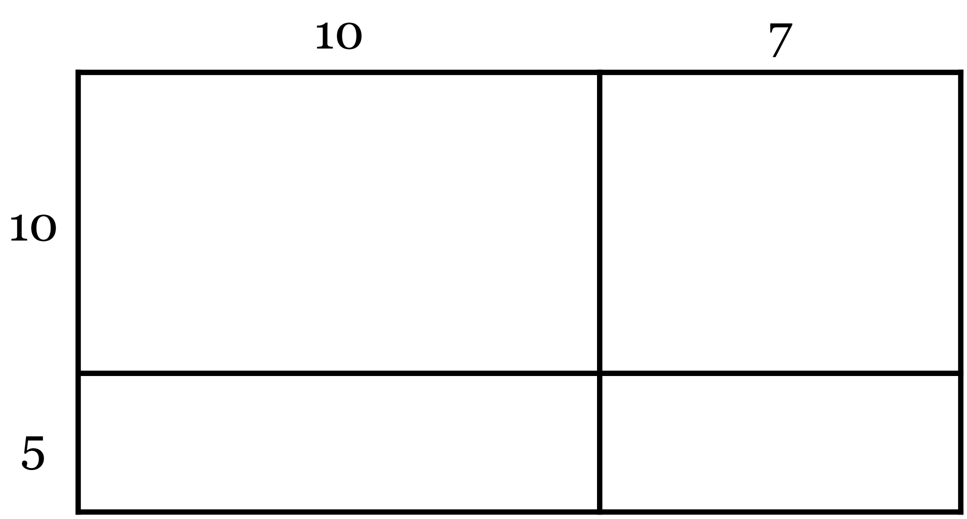
Mini-lesson: Area Model for 2-digit multiplication

Example Mini-lesson
Teacher: Here's an equation to solve: 17 × 15 = _____. Now this seems tricky, but I know how to draw a picture of it.



But this doesn't help quite enough. Really, we need to cut it up into pieces that are easy to work with. What's an easy piece to work with? (Ideally, students respond by suggesting cutting into tens.)

The cool thing is, we can actually cut into tens on both sides.



And now all we have to do is figure out all the pieces. What's 10 × 10? (100).
What's 10 × 7? (70)
What's 5 × 10? (50)
What's 5 × 7? (35)

So we can fill in our picture like this.

	10	7
10	100	70
5	50	35

Now we just add up all the pieces.

100 + 70 +50 + 35 = 255.
So 15 × 17 = 255.

Now try drawing a picture of this one on your own or with a partner: 13 × 14 = _____.
[Pause to let students solve and discuss]

Now that you're got a way to multiply two digit numbers visually, consider this challenge: using the digits 1, 2, 3, 4 once each, what's the large product you could make? For example, you could make 43 × 21 and get the product 800 + 60 + 40 + 3 = 903.

	40	3
20	800	60
1	40	3

Conjecture: that's the greatest product I can make using each of those digits once. Do you agree, or is there a counterexample?

Big Blockout

Topics: Multiplication, commutativity and associativity of multiplication
Materials: Three dice per game, board, colored pencils
Common Core: 5.OA.2, MP1

Roll three dice; add two and multiply by the third. How do you get the highest score?

Why We Love Big Blockout

Big Blockout is a quick and fun game for multiplication practice that poses a fascinating question at the same time. This Blockout adaptation connects the game to the array model of multiplication.

The Launch

Big Blockout can be played with 2-4, but fewer players is usually better.
Players take turns rolling three dice on their turn. On your turn, draw an array on the board. One side of the array is the sum of two dice of your choice; the third die gives the other side. In other words, you add two of your rolls together, and multiply by the third. That is your score for the turn.

Example. You roll 3, 5, 6 on your turn. You could add 6 + 3 to get 9, and multiply by 5 to score 45 points on the turn. But wait! If you add 5 + 3 to get 8, and multiply by 6 you can get 48 points! So scoring 48 points is actually the better option. This means drawing an 8 by 6 array (if there is space for it) would be your best move. (You could have gotten 33 points as well--do you see how?)

Prompts and Questions

- What's the best way to get the most points after you roll? Is there some rule for which numbers you should add and which you should multiply?
- Do some scores come up more often than others?

The Wrap

The fundamental choice in Big Blockout is which two numbers to add and which number to multiply by. Let's try a few more examples—see if you can figure out the best move (without looking at a specific board, we're really just looking for the largest product). Since we know from the last game that multiplication describes a rectangle, we can look build a rectangle for each of these problems to help us.

You roll 1, 4, 5. What's your best move? There are three options.

(1+4) × 5 = 25 **(1+5) × 4 = 24** **(4+5) × 1 = 9**

25 is the best move.

You can pose as many of these followup questions as you have time for. After each one, give the students a minute to solve one or more of the problem below and discuss amongst themselves.

You roll 2, 4, 5. What's your best move?
You roll 3, 4, 5. What's your best move?
You roll 4, 4, 5. What's your best move?
You roll 5, 4, 5. What's your best move?
You roll 6, 4, 5. What's your best move?

Variations

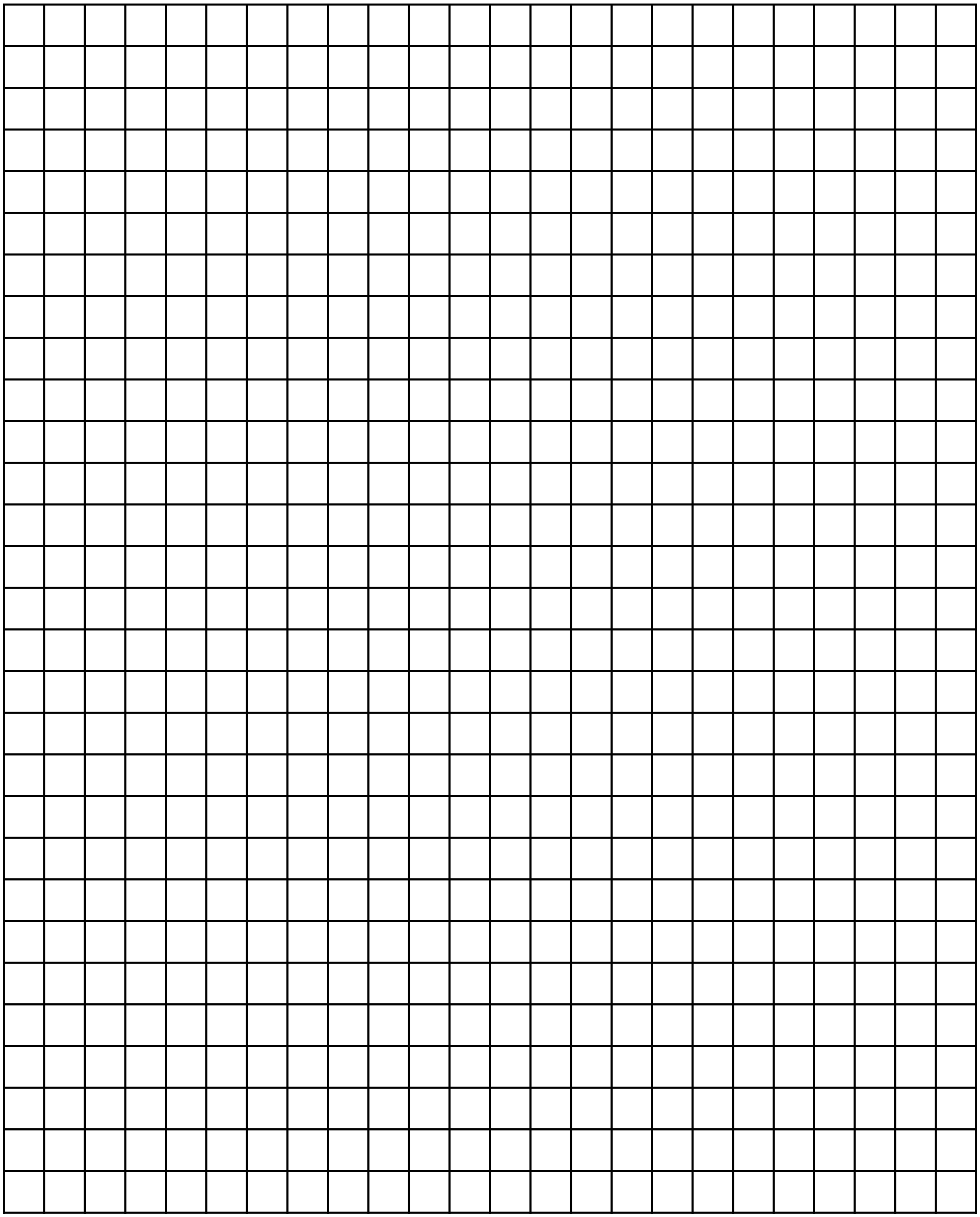
Big Blockout Place Value: Instead of adding two dice, make a 2-digit number out of them. So if you rolled 3, 4, and 2, you could get 34 × 2 = 68 points. Or better, you could get 32 × 4 = 128 points. (Play without the board - just tally scores every turn.)

Big Blockout Pro: Roll four dice instead of three. Add three dice of your choice together and multiply by the fourth.

Tips for the Classroom

6. As students get more accustomed to the game, increase the difficulty by adding in 8, 10, and 12 sided dice.
7. For students who are less confident, let them use their multiplication tables to help them with the game.
8. You can also play Big Blockout without the Board, and just keep track of the score. Play to 200, for example.

Big Blockout Board



Day 5

Goals

1. Explore a rich problem involving multiplication.
2. Learn Prime Climb, a staple for Choice Time and a powerhouse for practicing arithmetic.

Opener

Unit Chat - See Appendix 4 for images

Activity

The Power of 37

Game/Puzzle

Prime Climb

Note: Prime Climb can be an option for Choice Time every day, once students know the rules. It's also fine to play with abbreviated rules (1 pawn instead of two, e.g.) at first. Team play (teams of 2) is also a good option.

Video instructions available at mathforlove.com/games/prime-climb/how-to-play

---

Choice Time

Prime Climb
Big Blockout
Challenge Problems - see Appendix 3

Closer

Ask students to share anything they noticed after playing Prime Climb. This could be something about the numbers or the colors, something about strategy, or just an observation.