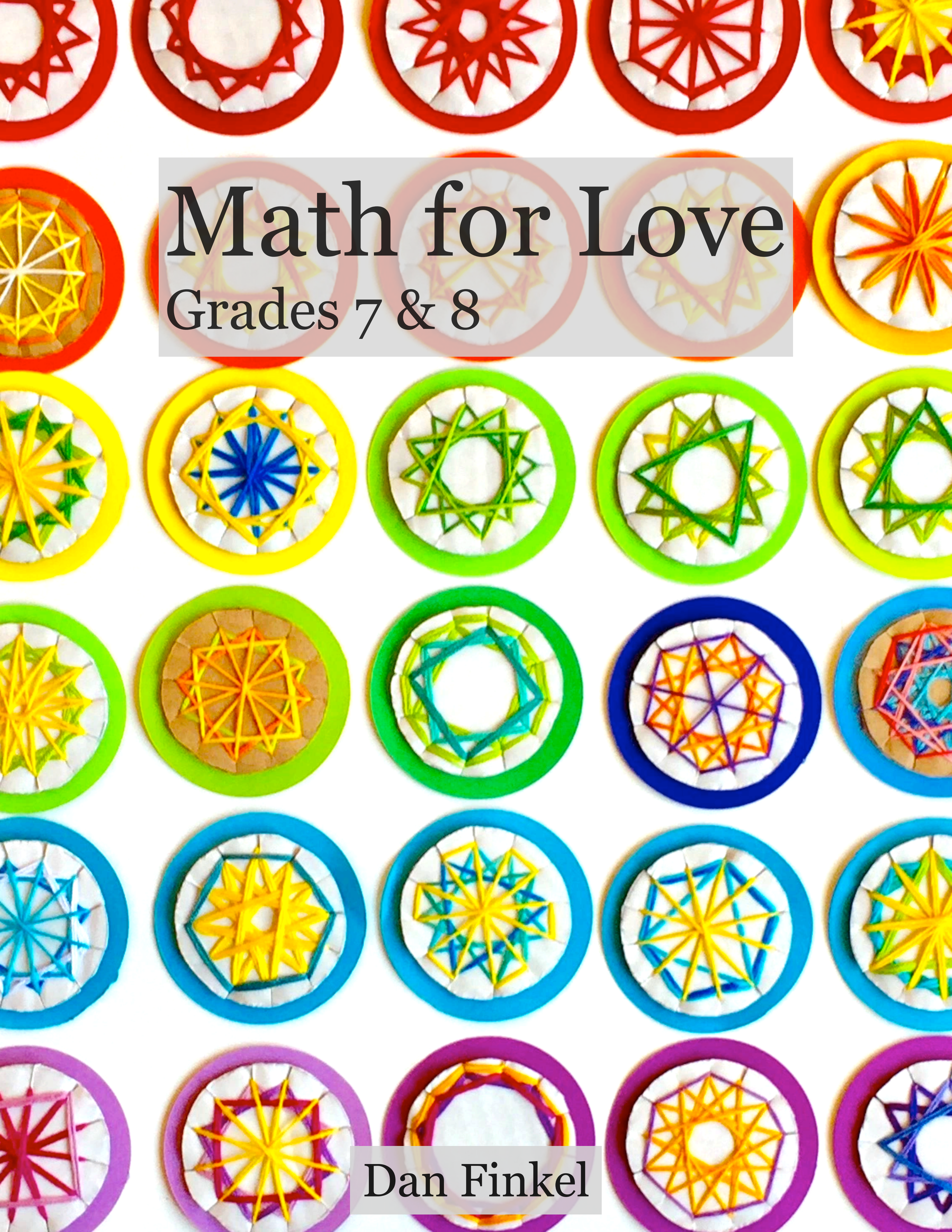

Math for Love

Grades 7 & 8

Dan Finkel

Cover image by Malke Rosenfeld

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Source: mathinunexpectedspaces.wordpress.com/2017/12/10/star-o-rama-and-how-to-make-them

A word about using this book

This book was designed to support a summer math program lasting sixteen 75 - 90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. The activities are good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this approach to rich learning and deeper math tasks, available under "Rich Learning" at mathforlove.com/pd.

Enjoy!

A word about the copyright

We want this book to be used by teachers to help students explore math in a positive way. Feel free to make photocopies, share ideas with parents and colleagues, and use this as a resource draw on. In general, we support this kind of fair use of our materials. Please don't post elements from this book online without citing the source, share large chunks of the book electronically, or sell any part of the book to anyone.

Table of Activities

Where to find each activity the FIRST time it's used in this curriculum

Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

Goals of the Math For Love Curriculum

We wrote this program to be both *play-based* and *rigorous*. The goals of the program are:

- Improve conceptual understanding of and fluency in mathematics
- Build students into more powerful thinkers and problem-solvers
- Give everyone an opportunity to have fun and enjoy math

Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement**. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation**. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.
3. **Thinking**. Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. **Positive Environment**. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make conjectures and break them with counterexamples, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Be ready with questions (and hints!)**
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value *their* thinking and contributions, not just the answer.

As students take on rich problems, you can also help with the right kind of hint. Suggest using a table or list to track their results and look for patterns. Get them started on a simpler version of a problem that might be too complicated. If they're truly confounded, make sure they have a success, then get them working on the next challenge.

★ **Model how to play games, and teach how to win and lose**
Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with openers. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Build a *thinking classroom***
If you can, get students standing at whiteboards, not just sitting at desks. (See *Building Thinking Classrooms in Mathematics,* by Peter Liljedahl for more on physical changes you can make to the classroom.) Communicate that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you *learn* to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Encourage making and breaking conjectures**
Establish a habit of supporting students' conjectures, hypotheses and predictions, and students will learn more and commit to the thinking process. Help them break and improve conjectures (using **counterexamples** especially—see Day 1), and

they'll begin behaving like true mathematicians. Making conjectures normalizes mistakes as part of the learning process, and gives students a practical way to learn from them. It also makes doing and thinking mathematics the central activity of your class. It's also a core element of this curriculum.

★ **Give your students *time* to think and explore**
Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**
We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

★ **Value play**
It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less, you can end sooner. If you need something longer, you should find many of the activities extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- *Goals*
- *Opener*
- *Activity*
- *Game/Choice Time*
- *Closer*

Goals

These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students, though some will be labeled as good options to mention to students if you like.

Opener

The Opener is the first activity of math class. The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

The Openers in this curriculum are Counterexamples or Would You Rather prompts. See Day 1 and Day 3 for more.

In general, the Opener should last about 5 - 10 minutes.

Activity
Following the opener, there is a suggestion for an activity. These activities are almost always *rich tasks*, and are designed to generate student curiosity and interest, help them explore a new question or environment, collect data, make conjectures or predictions, and ideally, arrive at a new understanding of the mathematics. This is where the bulk of class time will be spent.

You can find video discussions of specific rich tasks under the "Rich Task" label at mathforlove.com/pd.

These tasks are designed to give students the maximum opportunity to think & engage, practice skills, explore questions, and have fun.

Game/Choice Time
In the remaining time following the activity, we often have a new math game for students. Students can also use this time as Choice Time, to play games they've learned earlier in the class, or to return to tasks or questions they'd like to think about more.

This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

Closer
The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer generally takes 5 minutes or less.

Day 1

Goals

1. Establish classroom values and community.
2. Learn to make conjectures and produce counterexamples.

Opener

Counterexamples

The ideas of conjectures and counterexamples will be absolutely central to this class. This is an opener that you will return to again and again. The language and routine can guide how you approach virtually all the activities you do with students.

Activity

Preassessment (optional)
1-2 Nim

See the appendix for the preassessment.

Much more discussion about launching 1-2 Nim is at the video guide here:
mathforlove.com/video/rich-task-1-2-nim-lesson-plan-with-dan-finkel

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Game/Choice Time

Pig

Later, we'll take a full day to explore the probability behind Pig. For today, you can just introduce and play the game, and leave students to mull over some of the central questions discussed in the lesson plan, like how to determine a good (or optimal) strategy for Pig, and how to even discuss strategy in a situation where chance determines so much.

Closer

Ask students to make a list (with a partner, a trio, or on their own) of traits that you need to do math. When they're done, discuss their lists and their ideas.

In the discussion, ask them to consider the activities of the day. There is a lot of math in 1-2 Nim and Pig, but maybe not the kind of math students are used to. They're also both games, and hopefully are fun to play! How does this fit in with their conception of what math is, and what you need in order to do it well?

Close by letting them know that this program is designed to have them playing and exploring a lot, and also thinking deeply. The most important thing they'll need to know is that getting frustrated sometimes is part of the process, and if they can keep engaged and playing and thinking, they'll learn what they need to learn, and get better at what they're doing.

Counterexamples

Topics: logic, deduction, mathematical argument, communication
Materials: None
Common Core: Variable, but especially MP1 and MP3.

Prove the teacher wrong. Rigorously.

Why We Love Counterexamples

Every kid loves to prove the teacher wrong. With Counterexamples, they get to do this in a productive way, and learn appropriate mathematical skepticism and communication skills at the same time.

It is possible to play Counterexamples with kids as young as kindergarteners as a kind of reverse “I Spy” (“I claim are no squares in this classroom. Who can find a counterexample?”). What’s great, though, is that you can transition to substantial math concepts, and address common misconceptions. Counterexamples is a perfect way to disprove claims like “doubling a number always makes it larger” (not true for negative numbers or 0) or sorting out why every square is a rectangle, but not every rectangle is a square. For older kids, you can even go into much deeper topics, like: “every point on the number line is a rational number.”

The language of counterexamples is crucial to distinguish true and false claims in mathematics; this game makes it natural, fun, and plants the skills to be used later. Counterexamples is also a great way to practice constructing viable arguments and critiquing the reasoning of others.

How it works

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher, though it can be a student) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form “All _______s are ________” or “No _______s are ________.” You can also play around with statements like “If it has _______, then it can ________.” For instance:

It’s often best to start with non-mathematical examples.

- All birds can fly. (Counterexample: penguins)
- If something produces light, then it is a light bulb.
- If something has stripes, then it is a zebra.

Once students have the hang of it, make the examples more mathematical.

- Doubling any number makes it bigger. (Counterexample: -1 doubled is -2, which is smaller. 0 doubled is 0, which is the same size.)
- Multiplying two numbers gives a product that's larger than either of the starting numbers.
- Multiplying two numbers never gives the same answer as adding them. (Counterexample: $2 + 2 = 2 \times 2$. Or $3 + 1.5 = 3 \times 1.5$.)
- Fractions are always between 0 and 1.
- If shape A has a larger area than shape B, it has a larger perimeter also .
- If a shape has all its sides the same, then it's a square. (Counterexample: a rhombus. Squares need four equal sides AND four equal angles.)

Example

Teacher: I claim all animals have four legs. Who can think of a counterexample?
Student 1: A chicken!
Teacher: Why is a chicken a counterexample?
Student 2: Because it has two legs.
Teacher: Right. I said every animal has four legs, but a chicken is an animal with just two legs. So I must have been wrong. Let me try to refine my conjecture then. I should have said that animal must have 2 or 4 legs. That feels right.
Student 3: What about a fish?
Teacher: Aha. A fish is an animal with no legs. Thank you for showing me the error of my ways. What I should have said is that animals have *at most* four legs.
Student: 4: What about insects?
And so on.

Tips for the Classroom

1. It's good to make up false conjectures that are right for your students. But start simple.
2. Kids can think of their own false claims, but sometimes these aren't the right kind, and they often have to be vetted.
3. Once you introduce the language of counterexamples, look for places to use it in the rest of your math discussions.
4. You can also use Counterexamples to motivate a normal math question. Instead of saying "draw a triangle with the same area as this square," you can say, "I claim there is no triangle with the same area as this square." If students know to look for counterexamples, this will set them to work trying to disprove the claim right away.

1-2 Nim

Topics: logic, patterns, addition, counting, subtraction, multiplication, division
Materials: Counters (tiles, beans, pennies, etc.) and/or paper and pencil
Common Core: MP1, MP2, MP3, MP5, MP6, MP7, MP8

You can take one or two counters from the pile. How do you get the last one?

Why We Love 1-2 Nim

Nim is fun, challenging, and rewarding for a wide range of students. Completely unlocking the game is an exciting and powerful achievement for a student. Extensions for the game abound.

The Launch

It's good to highlight a few things when you launch 1-2 Nim. First, students will win and they will lose, and it's important to remember to do both gracefully. But second, losing is better than winning, in a way, because every time you lose, you can see what strategy your opponent used to beat you, and then learn that strategy. That's how you become a N*im Master*.

Launch the game by playing a few demonstration games with students.

Instructions
Nim is a two-player game. Start with a pile of 10 counters. On your turn, remove one or two counters from the pile. You must take at least one counter on your turn, but you may not take more than two. Whoever takes the last counter wins.

Example Game
Start with 10 counters in the pile.
Player A takes 2 counters, leaving 8.
Player B takes one counter, leaving 7.
Player A takes two counters, leaving 5.
Player B takes one counter, leaving 4.
Player A takes one counter, leaving 3.
Player B takes one counter, leaving 2.
Player A takes two counters, leaving 0 and winning the game.

Play several demonstration games as needed. When the students understand the rules, have them play against each other in pairs. Students can try to challenge the teacher if they think they have a strategy that can win.

The Work

As students play, the teacher can move among them and challenge them to play, or ask them what they've noticed so far. The teacher should be able to beat the student unless the student plays perfectly. When students have played for 10 - 15 minutes or so, bring them together again and discuss what they've noticed so far. Students may have noticed that when they can give their opponent 3 counters, for example, they win. (We call this the *3-trap*, since if you can give your opponent 3 counters, you have effectively trapped them, and can win the game no matter what they do.)

Pose the central question and discuss: how can you win at 1-2 Nim?

Students may have philosophical questions related to game-playing: what does it mean to have a winning position or losing position in a game? Is there luck in the game? Where does the game go from feeling "random" to feeling like you can control it?

These are productive conversations. Before students go back to work, there are two points to underline to make the exploration productive:

1) Making the game simpler makes it easier!
   How can you make the game simpler? By shrinking the pile. Challenge the students to play you with a pile of 1 counter. Do they want to go 1st or 2nd? What about with 2 counters? It's so easy it feels like a joke, but this is what serious mathematicians do, and we get valuable information here.

2) Make a table!
   This is how the data you collect by making the game easier can actually help you. The beginning of a table might look like this.

Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second
4	...
5	...

If students want to master the game, all they have to do is extend the table. It tells them what to do.

Prompts and Questions

The Central Question: how can you win 1-2 Nim?

Good questions for the teacher to ask students:

- What move should I (the teacher) make?
- How did you/they/I win that game?
- What do you think your/my opponent will do if you/I take two counters?
- Would you like to take back your move?
- What have you noticed about this game?

Possible student conjectures (all interesting, all false or incomplete) that may arise:

- Whoever goes first wins.
- Whoever goes second wins.
- Odd vs. even numbers of beginning counters determines your strategy.
- It matters/doesn't matter what you do until there are less than six counters in the pile.
- Whoever can give their opponent four open counters wins.

The Wrap

It may be wise to end class without a conclusion, depending on where students are, and discuss again on a subsequent day. Send them home to try out the game against friends and family, and refine their strategy.

When students are ready, discuss strategy—do students have any ideas of how to win, regardless of the size of the pile? Once they share, ask if anyone would like to try another game against you. Let them get advice from their peers (students can quietly raise one or two fingers to suggest what they think should happen). Can they beat you?

The major breakthrough is that the table actually tells you what to do. Have a student share a table and discuss its contents. What patterns do students see? Students should be able to defend these results.

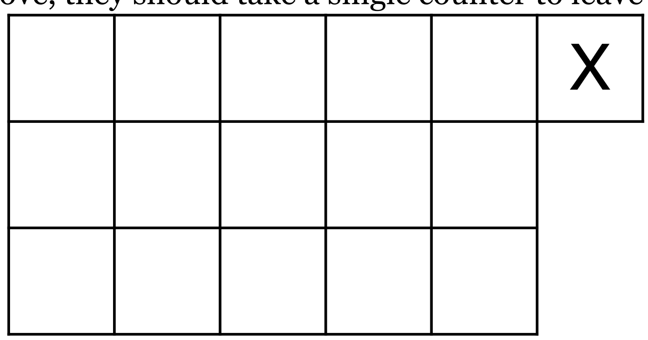
Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second.
4	Go first. Take 1.
5	Go first. Take 2.
6	Go second.
7	Go first. Take 1.

You can play students with the table visible. On your turn, look at the table and talk out loud what you should do, i.e., "There are 7 counters, and its my turn. So the table says, take 1." The takeaway here is that the table is literally instructions for winning.

Students will also, hopefully, notice the pattern in the table. (Go first, take 1. Go first, take 2. Go second.) It looks like the "3-trap" actually extends to a "6-trap" and a "9-trap," and so on. In other words, the winning strategy might be as simple as: on your turn, give your opponent a multiple of three. Does that really work? Challenge the class to a game with 25 counters. Let students discuss their strategy, and then choose a student to play you. Can they win?

Finally, when students can articulate a winning strategy and successfully beat you, there's still a question of *why* this "give your opponent a multiple of three" strategy succeeds. Here's a sketch of an argument that shows why it does.

Arrange, let's say, 16 counters in a 3 by 5 array, with one extra counter. When the first player makes their move, they should take a single counter to leave a 3 by 5 array.



Whatever their opponent does, they'll always be able to give back a 3 by *something* array. Try it out! So the choice of arrangement of tiles actually makes the argument clear.

There's a moral to this exploration: the way to *become* a master of nim, or any game, is to apply mathematical rigor and organization. In other words, thinking and working mathematically makes you powerful.

To end, you can challenge students with a couple of potential extensions.

Variations

1) What's the right first move if you're playing 1-2 Nim with 150 counters? What about 542 counters?
2) Try 1-2-3 Nim: players may take one, two, or three counters per turn. How do you win this game?
3) Try 1-2-3 Poison: Whoever takes the last counter <u>loses</u>.
4) What about 1-3-4 Nim? Players may take one, three, or four counters, but NOT 2. You can launch this version with the first half of the video here: <u>mathforlove.com/puzzle/can-you-solve-the-rogue-ai-riddle</u>

Tips for the Classroom

1. Demonstrate the game with student volunteers for at least three games (or many more!), until you are certain everyone understands it and is excited to play.

2. When demonstrating 1-2 Nim, narrate the game out loud, using mathematical language, and leaving empty space for students to chime in: "My opponent just took 2 leaving… [wait for students] 5 in the pile. Who has advice for what I should do next?"

3. Remind students that they will lose many games as they play, and that every loss is an opportunity to learn. Can they steal the strategy of the person who just beat them? Point out how students are trying out new strategies as they play you in demonstration games.

4. As students play each other, circulate to see what strategies they are developing. Challenge them to play you, and see if they can beat you.

5. Encourage student conjectures, but do not call them as true or false. Challenge students to break their own conjectures.

6. A big moment in taking ownership of the game is to change the size of the pile. Making the pile smaller makes it easier to understanding and win. Making it bigger makes it more challenging.

7. We use the term "3-trap" to describe how you trap your opponent by giving them a pile of three counters. Understanding how to win boils down to understanding what pile sizes you want to leave your opponent with.

8. There are two incredibly powerful approaches to solving Nim. The first is to simplify. How could the game be easier? What if the pile had only one counter? From this place of almost absurd simplicity, we slowly raise the difficulty. What about two counters? Three counters?

9. The second approach is to organize the data in a coherent way. A table does this very nicely.

10. If student want to play three-player, keep in mind that we discourage it. Normally trying out different numbers of players is a great impulse. In Nim, it leads to spoilers, who can't win, but can choose who does win, which diffuses the mathematical tensions in the game.

11. Optional homework: have students teach 1-2 Nim to a friend or family member.

Pig

Topics: Probability, strategy, addition, estimation (optional: fractions and probability)
Materials: One 6-sided die, pencil and paper
Common Core: MP1, MP7

Roll the dice and collect points. You can go as long as you want, but roll the wrong number and you lose all your points from that turn!

Why We Love Pig
Pig is easy to learn and gives students practice with addition (and multiplication, with Odd Pig Out). But Pig is mathematically rich. Students must articulate and defend strategies relating to handling chance and probability.

The Launch
Before you start, remind students of the importance of winning and losing gracefully. They'll do a lot of both with Pig, and it's important to take it lightly, since it's so easy to have setbacks.

Pig is a game that will sometimes punish a good decision and reward a bad one, which presents a real challenge for students: how can you tell if you made a good decision or a bad decision? Does strategy even matter in a game of luck like Pig? The teacher can bring these questions out over the course of the lesson, and let students grapple with them. In the long term, taking a scientific approach by running experiments and collecting data is one excellent way to handle the problem.

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students give you "thumbs up/down" if they think you should keep rolling.

How to Play
Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a 2, 3, 4, 5, or 6, the player adds that many points to their score for the turn. A player may choose to end their turn at any time and "bank" their points. If a player rolls a 1, they lose all their unbanked points and their turn is over.

Beginner Game: The first player to score 50 or more points wins.
Advanced Game: The first player to score 100 or more points wins.

Demonstrate enough turns so that students can see how rolling a 1 will lose them unbanked points, and that points in their bank will be safe even when a 1 is rolled.

The Work

Students can play Pig for fun anytime. Games can be quick and light.

The deeper work of Pig comes when we start to examine strategy. As students play, ask them to notice reflect on what strategy they're using as they as they play. Are they taking big risks, or is their play more conservative? After students have had enough time to play, discuss strategy for Pig as a class. What strategies did students use? Does strategy matter? How do you know? Pig is clearly a game of chance, but does that mean strategy makes no difference?

The stage is being set to actually run an experiment. How can we determine for sure whether strategy matters or not? We could pit two strategies against each other, and see which one wins; the more extreme the strategies, the more clearly we could see the difference. And what are the most extreme strategies? The most conservative we call *Better Safe than Sorry*: roll once and immediately bank your points. The most extreme we call *Let It Ride*: keep rolling until you get 50 points and win, or roll a 1 and lose all your points.

So if one person uses the *Better Safe than Sorry* strategy, and the other plays the *Let It Ride* strategy, who is more likely to win? Let students vote, and collect the number of votes from students as to which strategy they think will win. It might look like this:

Better Safe than Sorry	*Let It Ride*	No Difference
13	5	4

Now you can run an actual experiment. Have students play in pairs, each playing one of the opposing strategies. They should play to 50 points, and keep track of how many times each strategy wins. If students have 10-15 minutes to collect data, you'll likely have a good number of finished games. Collect all the data together on the board, and add up how many games were won for each strategy. In my experience, and quite surprisingly, *Let It Ride* tends to win about 75% of the time.

There's a big discussion here about probability, statistics, certainty, and uncertainty. Is the classroom data convincing to students? Has it settled the questions about whether strategy matters and which strategy is better? What would be a better strategy to pit against *Let It Ride* in a future game (for example, roll 3 times and then bank your points)? It's possible to run successive experiments, or to have students program a computer to run experiments for them.

And of course, you can always let students just play the game for fun when you have some extra time in class.

Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?

- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?

The Wrap

In addition to the experimental approach described above, we can wrap up playing the game by discussing the probabilities of outcomes, and how they can help us make predictions for good moves. Consider these questions:

- What is the probability that you roll a 1 on a given roll? (Answer: 1/6)
- What is the probability you won't roll a 1? (Answer: 5/6)
- If you don't roll a 1, what is your average point gain? (Answer: 4)

Considering these values, you can reframe the question for each roll in the following way: is it worth risk losing the points you haven't banked (1/6 chance) yet in order to have a of gaining about 4 points (5/6 chance). That means your chances of gaining an average of 4 points on a given roll is 5 times greater than your chance of losing all your points. When is this worth it?

If you have 10 points unbanked, should you risk them at 5:1 odds in order to gain 4 more? That actually seems like a good bet. In fact, anything up to 20 points ($20 = 5 \times 4$) seems like a good bet when you're considering the problem in this framework. This gives a mathematical rule of thumb for how you might want to proceed. Another experiment to try, then, is to play to 100, but have some people try the "Bank when you have 20 points or more" strategy, and others play some other strategy of their choice. Who will tend to win? (The difference may be subtle.) Still, this is another example of how mathematical analysis can give us some control over a situation, even when there's a great element of chance involved.

Tips for the classroom

1. Demonstrate the game a couple times with the whole group. Solicit advice about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!
3. As students play each other, circulate through the room and ask them about their strategies. It's ok for students simply to play, but there's an opportunity to probe deeper into the workings of chance and the strategy of the game too.

Name________________________________

Pig

Rolls	Rolls

Bank	Bank

Day 2

Goals

1. Explore conjectures and counterexamples for Star Polygons.
2. A goal for students can be to make and/or break at least one conjecture today.

Opener

Counterexamples

Activity

Star Polygons

You can also continue the exploration of 1-2 Nim and variations if students remain engaged with those questions.

Game/Choice Time

Odd Pig Out

Again, you can just introduce the game today, and delve into the exploration around it later.

Closer

Ask students to reflect on the goal for the day: did they make and/or break a conjecture? What did they learn from the process? Give them a few minutes to write, then discuss their reflections in trios, and then briefly as a whole class.

One takeaway: when you do math with any depth, it's usually impossible to know the whole story of what's going on right away. It requires making some attempts to make sense of what's going on, and

Star Polygons

Topics: Patterns, factors & multiples, primes, ratios & proportion,
Materials: Worksheets with equally spaced dots, scratch paper, Geogebra or other computer geometry program
Common Core: 4.OA.5, 4.G.1, 5.OA.3, 5.G.4, 6.RP.1, 6.RP.1, 7.RP.2, 7.RP.3, 7.G.2, MP1, MP2, MP3, MP5, MP6, MP7, MP8

A rich environment to explore, and a perfect introduction to the art of making and breaking counterexamples.

Why We Love Star Polygons

These geometric designs are beautiful and fascinating to study. Students can get involved almost immediately, and there's plenty to discover as they dig in.

The Launch

Show the accompanying video at mathforlove.com/2020/03/star-polygons.

If you launch without the video, we recommend Counterexamples as a warm up.

Start with the eight-pointed star, and ask for a number from the class. (Three is a good number to demonstrate with.) Starting at the top, connect a dot to the dot "three away," and then repeat until every dot is connected. That's an interesting discovery!

Conjecture. No matter what number we picked, we would have hit every dot.

Students will probably offer 0, 2, or 4 as a counterexample. Indeed, all these work. After a little more work of seeing which numbers lead to all the dots being hit, we might arrive at the following:

Conjecture. Odd numbers will hit every dot. Even numbers won't.

This is true (for 8 dots). A piece of vocabulary here: if the "connection rule" leads to us connecting up all the dots, we call the finished design a "star polygon." You can take a minute to ask students if they see anything wrong with this name. In particular, star polygons aren't *polygons*, technically, since they involve crossed lines. Still, that's what they're called!

Students should, hopefully, have the hang of how the connection rules work, though it's easy to make mistakes. (Demonstrating mistakes is a good idea, in fact.)

You can pose: will this conjecture hold for different numbers of dots? Let's try to break it! If we see other patterns, we can start to pose new conjectures too. And indeed, that will be the main work for today. Pass out the sheets and let student explore.

The Work

As the students work, you'll likely need to handle the following issues that may come up:

1. A few students may not understand the process, or the "connection rule." Find them and help them get sorted out.
2. Once everyone is productively searching, you'll want to call the class together occasional to
   i. Let students share conjectures (and counterexamples), and
   ii. Share a good way to organize data to look for patterns.

Probably the best structure is the simple chart:

Number of dots	Connection rules that lead to a star polygon	Connection rules that don't lead to star polygons
6	1, 5	2, 3, 4, 6
7	1, 2, 3, 4, 5, 6	7 (sidenote: should we call this a connection rule?)
8	1, 3, 5, 7	2, 4, 6, 8
9	1,2,4,5,7,8	3,6,9
10		
11		

Once students use this kind of organization, their ability to make coherent conjectures and find counterexamples will make a leap forward.

Prompts and Questions

- What conjectures have you written down so far?
- What conjectures have you disproved?
- How could you organize your data so it is easier to read?
- Would your idea work on 11 dots? Try it!
- So 5 dots with connection rule 2 made a 5 -pointed star, and so did 10 dots with connection rule 4. Do you think you can find other dot/connection rule combos that will produce the same 5-pointed star?

The Wrap (and extensions)

This lesson can go 1-3 days. If you just spend a single day, don't expect students to come to a perfect conclusion. However, you can bring them together and let them discuss what conjectures are still standing, and what their best current guesses are for what's really going on. Some conjectures that may come up on the first day:

Conjecture. If the number of dots is odd, every even connection rule will produce a star polygon.

Note: This is false. Letting the number of dots D = 15 and the connection rule C = 6 produces a counterexample.

Conjecture. If D is even, and C is odd, then you get a star polygon.
Note: Also false. D = 10 and C = 5 gives a counterexample.

Conjecture. If the number of dots D is prime, any connection rule from 1 to D-1 will hit every dot, producing a star polygon.
Note: This is true.

Conjecture. There's a symmetry in connection rules: connecting k and D-k dots both produce the same picture (i.e., with 5 dots, connection rules 2 and 3 both produce a 5-pointed star).
Note: Also true.

Conjecture. If C/D is an equivalent fraction to E/F, then they produce the same star polygon.
This is true, and is a great conjecture to explore if you want to go deeper into ratio and proportion. See Extension 1 below.

Conjecture. If C/D is a fraction in lowest terms, then they produce a star polygon.
Note: This is true, but subtle to prove. You'll need to get into the extensions for students to see why this is true, which will probably take addition days. But they'll be rich, mathematically!

Several extensions are possible, depending on students' insights.

Extension 1: When do you get a square? A triangle? A five-pointed star?
Looking at particular shapes that emerge from dots and skip rules is a great way to connect to issues of ratio and proportionality. When, for example, do we end up drawing a five-pointed star? Certainly with five dots (D = 5) and connection rule 2 or 3 (C = 2,3). But also D = 10 and C = 4, 6. And D = 15 and C = 6, 9. Indeed, when the proportion D:C = 5:2 or 5:3, we get the 5-pointed star. Exploring this connection will likely allow students to delve into concrete and abstract understandings of ratio and proportion, especially as they test their more and more general conjectures of this nature. They can continue to make conjectures until, ideally, they can articulate why this ratio perspective makes sense.

Extension 2: A complete accounting for when star polygons occur and when they don't. When do we hit every point? If we apply our understanding from extension 1, we can see that when D and C have any common factor (greater than 1), there must be dots we miss. But what about when they don't have a factor in common? In this case, it turns out we do hit every point. Proving that this is indeed the case is an excellent project for older or more advanced students. For even more advanced students, counting the number of C that create star polygons for a given D is a very interesting and challenging project. (The answer is known as the *totient* or *Euler phi function*.)

Extension 3. What if C is larger than D? Trying connection rules greater than the number of dots leads to a potential exploration of modular arithmetic.

Extension 4. What are the angles of a star polygon? Exploring the angles leads to an entirely different, beautiful theory.

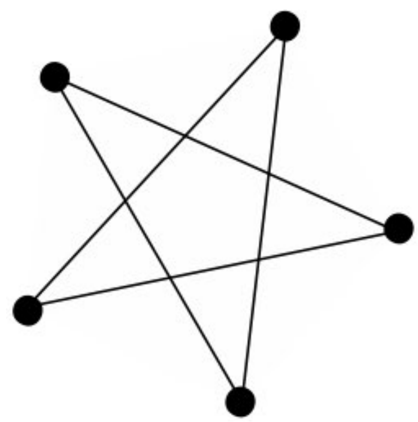
Tips for the Classroom

1. Use Geogebra, Desmos, or another computer program to draw equally spaced dots and the star polygons that result.
2. Helping students organize their search and record their data in a table will be one of the most fundamental ways to help them come up with conjectures.
3. Searching for counterexamples is a great job for students who might be temporarily aimless.
4. Even if your students are capable of using algebra or abstraction, make sure they defend their thinking with concrete examples.

Star Polygons

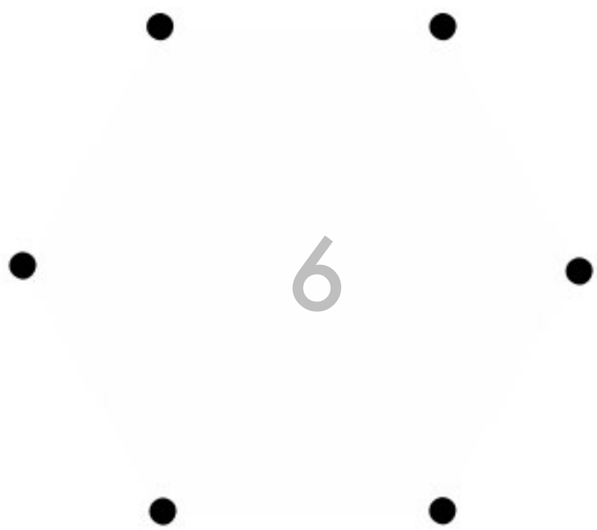
The five-pointed star below was made by starting with 5 evenly spaced points and connecting every 2nd point. We say we've used the "connection rule of 2."

It's called a *star polygon* when you hit **all the points** you started with in one continuous loop.



Working with a partner, experiment with building star polygons or other shapes by connecting regularly spaced points.

Write conjectures as you go. Which conjectures can you break? Which seem to hold?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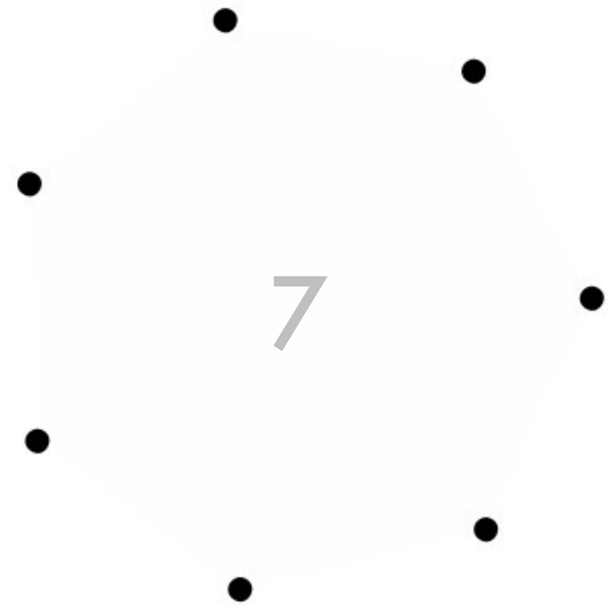
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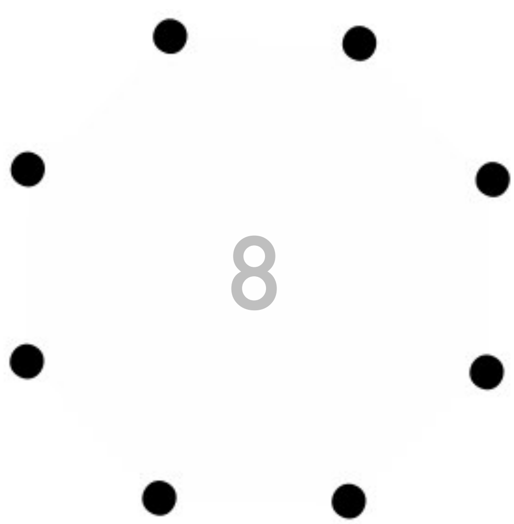
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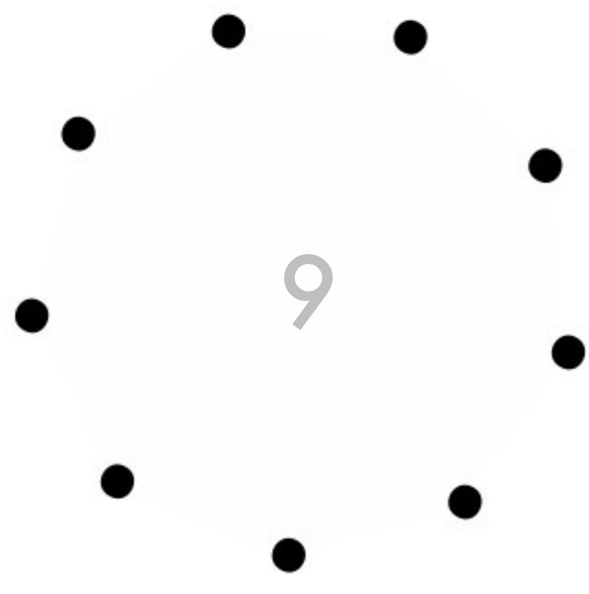
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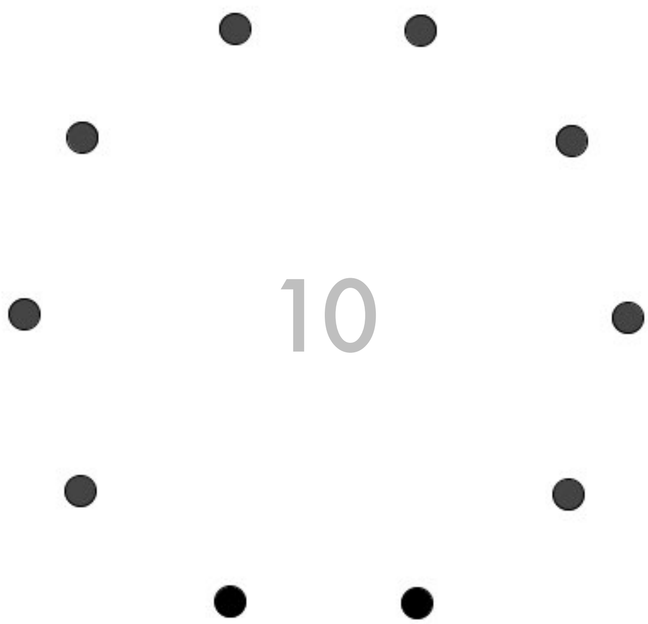
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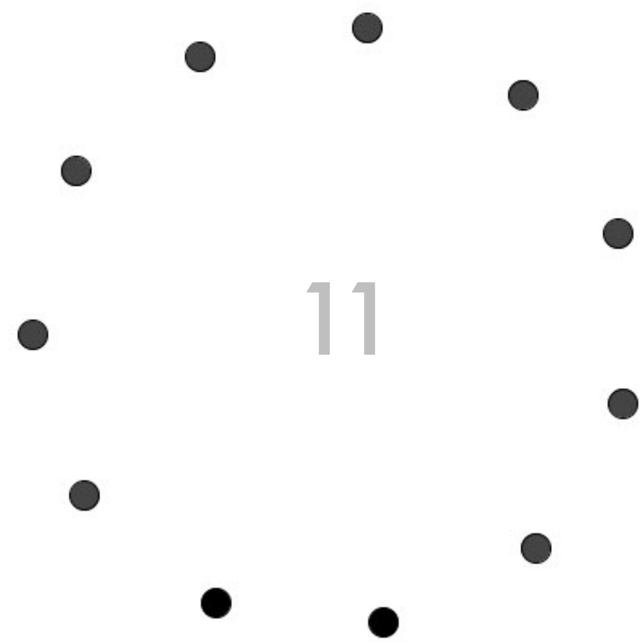
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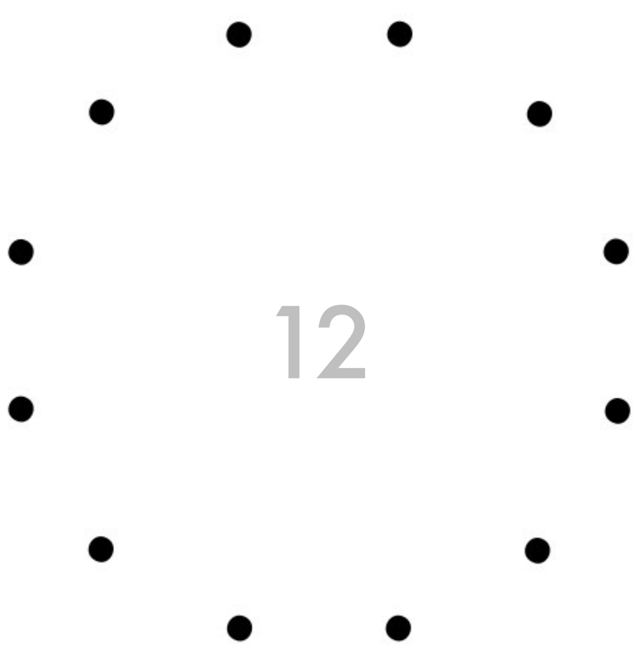
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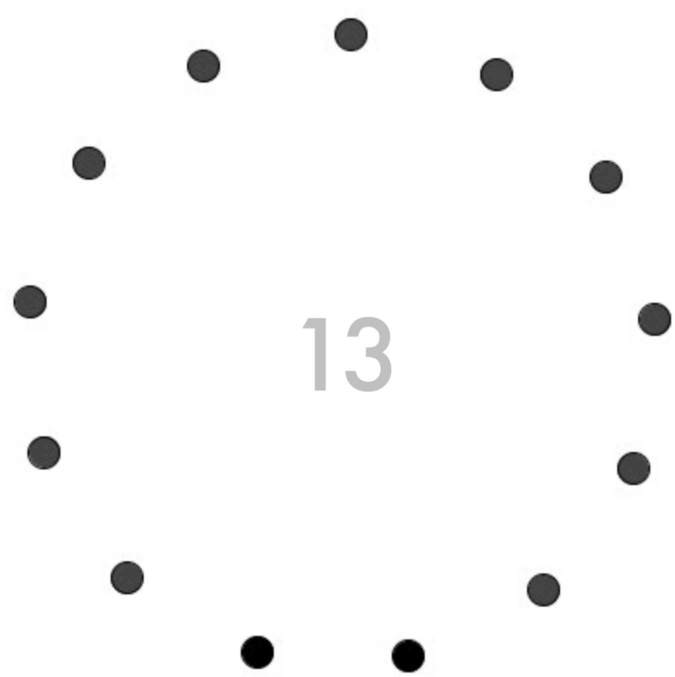
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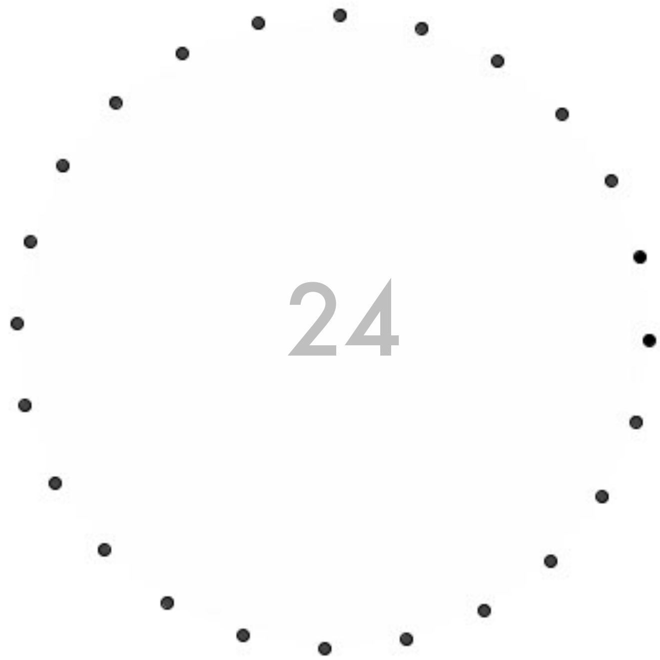
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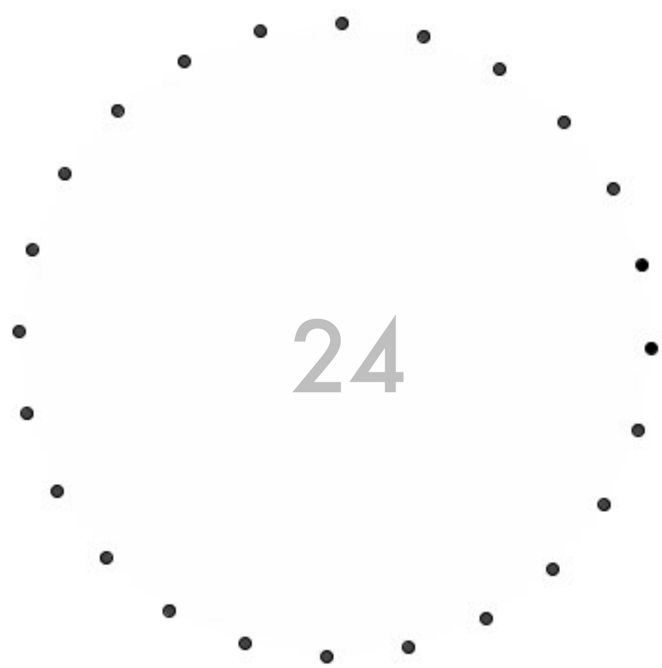
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Odd Pig Out

Topics: probability, strategy, multiplication, addition
Materials: Two 6-sided dice, pencil and paper, two 10-sided dice (optional)
Common Core: MP1, MP5, MP6, MP7

Roll the dice and multiply. You can go as long as you want, but roll an odd number and you lose all your points from that turn!

Why We Love Odd Pig Out

Odd Pig Out is a natural extension of Pig to multiplication. It is great practice for multiplication and addition in a fast-moving, fun game.

The Launch

The teacher chooses a volunteer, explains the rules, and plays a demonstration game. Because students already know Pig, this game should be relatively intuitive to learn.

Players take turns rolling the dice as many times as they like. After each roll, they multiply the numbers they rolled together. If the product is even, they add that number to their current points for the turn. If the product is odd, players lose all their points from that turn and their turn is over. A player may choose to end their turn at any time and "bank" their points.

Play to 300.

Prompts and Questions

- Is there an easier way to add up all those numbers?
- How many points to you have for this turn so far?
- Who's ahead?
- Are you sure that's the product of those two numbers? What does your multiplication table say?
- What strategy are you using?

The Wrap

Ask students whether they're more likely to roll odd products or even products. How many odd numbers are there on the multiplication table (up to 6 by 6)? How many even numbers? How are they distributed? Do students see any patterns?

(Optional) If you'd like to dig into the probabilities, the same mathematical approach you took with Pig can give a good rule of thumb for a strategy for Odd Pig Out. Students will need to do the harder mathematical work of figuring out:

- What is the probability that you roll an odd product on a given roll? (Answer: 1/4)
- What is the probability that you roll an even product? (Answer: 3/4)

- If you don't roll an odd product what is your average point gain?
  (Answer: the sum of the even numbers on a multiplication table of the appropriate size, divided by the number of even products. This is, in fact, a fascinating mathematical counting problem. For a 6 by 6 multiplication table, the sum of the even numbers is $21^2 - 9^2 = 441 - 81 = 360$.
  There are 36 - 9 =27 even products, so the average is $360/27 = 13^1/_3$.)

If we use the same argument as with Pig, you should be willing to risk up to 40 points if you have 3 to 1 odds of winning $13^1/_3$ so that's a good estimate of how risky you should be willing to be in Odd Pig Out.

Tips for the classroom

1. Demonstrate the game a couple times with the whole class (or in a station). Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing.
3. Note that the deeper mathematics discussed in the Wrap are optional, or can be returned to when students are ready. Just playing the game is great for multiplication fact practice.

Odd Pig Out

Roll two dice and write down their product. You may choose to continue rolling as long as the products are even. End your turn to bank your points.
If you roll an odd product, end your turn and lose all unbanked points.

Products	Products
Bank	**Bank**

Day 3

Goals

1. Collect data to test hypotheses.
2. Explore probability with Pig and Odd Pig out.
3. Learn Prime Climb, a staple for Choice Time and a powerhouse for practicing arithmetic.

Opener

Would you Rather

WYR: Have $10,000 OR roll a die and get a million dollars if 6 comes up.

Activity

Pig & Odd Pig Out - probability exploration

Students know the game. Now they're ready to explore the probability by considering what strategies will let them win the game most often. Use the full lesson plan for Pig. If you finish early, challenge students to find strategies for Odd Pig Out as an extension.

Game/Choice Time

Prime Climb

Note: Prime Climb can be an option for Choice Time every day, once students know the rules. It's also fine to play with abbreviated rules (1 pawn instead of two, e.g.) at first. Team play (teams of 2) is also a good option.

Video instructions available at mathforlove.com/games/prime-climb/how-to-play.
If you dont' have Prime Climb, you can substitute Big Blockout or another game from a later day's plan.

Closer

Ask students to reflect on and discuss who in our society uses the kind of probability knowledge we explored with Pig, and how they use it. Specifically, how does knowing that you can use strategy even in situations with chance or randomness affect what you might do?

Students may have lots of ideas, and there are lots of potential examples. Casinos are an obvious one: they calculate the odds, and reserve the best strategies for themselves, so they always win in the long run. Insurance companies are another interesting example of a group that is looking at uncertainty and finding ways to approach it strategically to make (a lot of) money.

Would You Rather

Topics: logic, deduction, mathematical argument, communication
Materials: Image/slide (optional)
Common Core: Variable, but especially MP3, MP4

Justify your decision with mathematics.

Why We Love Would You Rather

Would You Rather questions provide a very simple routine for an openers. They serve as a warm up, get mathematical conversations started, and offer a quick and dirty approach to mathematical modeling. They are simple to ask and can be relevant and instantly motivating.

You can make up your own *Would You Rather* questions to fit your class, or even have your students bring in their own. There are also some great resources for where to find them online, including, most notably, wouldyourathermath.com.

The Launch

Would You Rather questions offer a choice between two options. The job of the students is to decide which of the two options they would rather have, and convince their classmates of the wisdom of their choice.

The choice is usually real-world in nature, and may require students to make reasonable guesses to fill in missing information. As a result, *Would You Rather* provides a protocol that is quick and easy to use as a warm up, and gets students to model with mathematics (Math Practice 4) and have mathematical conversations (Math Practice 3).

Example
WOULD YOU RATHER…

Option 1		*Option 2*
Buy 1 banana for 29¢	OR	Buy 1 pound of bananas for $1.89 ?

[Students have 1 minute to consider which they would rather do. Afterward students talk in groups of 3 about which they would rather do.]

Student 1: I would rather buy 1 banana because I only want 1 banana.
Student 2: But if it is cheaper to buy bananas by the pound, you should do that. You could still buy just 1 banana.
Student 3: I think it is more expensive to buy 1 banana for $1.89 a pound.
Student 1: You think so?
Student 3: Well, what does one banana weigh?
Student 2: Probably it takes 5 bananas to make one pound.

Student 1: 5? I'd say 3.
Student 3: In any case, let's say it is 5. Then 5 bananas would either cost you 5 × 29¢. Or you could buy them for $1.89.
Student 2: Let's see… 5 × 29¢ is $1.45. That is cheaper!
Student 1: But what if it were just three bananas in a pound?
Student 2: That would make option 1 even better! It would only cost you 3 × 29¢ for a pound of bananas.
Student 3: I'd like to know exactly how much a banana weighs. But unless these are tiny bananas, I think option 2 is going to be better.
Student 2: I bet we could figure out how much the bananas would need to weigh for the two options to be the same.
and so on…
[After students converse as a group, there is a brief class discussion where groups can put forward what they found.]

Students from group: … so we figured out that as long as the banana weighs more than about two and a half ounces, you should pick option 1.

Student 4: I have a banana in my lunch. Can we weigh it?
Student 5: There's a scale in the science room.
[The student gets permission to go get the scale, returns]
Students: What does it weigh?
Student 4: Four ounces!
Student 6: How many ounces in a pound?
Student 7: 16. That means it is four bananas to a pound.
Student: It's definitely better to pick option 1 then.
Teacher: I wonder how cheap they'd have to sell them per pound to make option 2 more appealing. In any case, we're all warmed up, so let's get to today's lesson…

Tips for the Classroom

1. Start simple. Questions that involve a lot of extra assumptions (like, how heavy is a banana) can also lead to sprawling conversations. Depending on what you're trying to get out of the activity, you might prefer to keep the questions relatively simple.

2. Keep things immediate. Don't ask *Would You Rather* questions about mortgages to 3rd graders.

3. Pictures can be used to excellent effect with *Would You Rather*.

4. Have the question up when students walk in and you'll lose no time at the beginning of the class. Kids immediately have something to think about.

5. As with all openers, be ready to move on, even if not all the questions have been resolved.

Would You Rather…

Have $10,000

OR

Have one chance to win $1,000,000 if you roll a 6 on a fair die.

Prime Climb

Topics: Multiplication, division, addition, subtraction, multi-step problems, factoring
Materials: Prime Climb game
Common Core: 3.OA.C.7, 3.OA.D.8, 3.OA.D.9, 4.OA.A.3, 4.OA.B.4, MP1, MP7

How quickly can you move your pawns to 101?

Why We Love Prime Climb

We invented Prime Climb to give students a more playful way to explore complex arithmetic problems and understand factoring. It's a reliable hit in the classroom!

Launch

Show students the color scheme of the board and multiplication table, and ask them what they notice. In particular, what's happening with the color scheme?

Let students discuss their thoughts. A specific point to underline, especially with respect to the multiplication table: if you look at two numbers that multiply together, (i.e. 7 × 8), the answer has exactly the same colors of each of the factors, just put together. (7 is purple, 8 is three orange; 56 is purple and three orange.)

Divide your small group into teams (individuals or pairs). Each team chooses a color to play. Each team gets two pawns, and place them on 0. The goal is to get a pawn to 101. (This is a quick version of the game. In the full game, the goal is to get both pawns to 101.) Explain the rules to students by demonstrating a few example moves.

Quick Start Rules
During a turn, there are four phases.

1. *Roll*. Roll the dice. You get two numbers from 1 to 10 to use for moving. In you roll doubles, you get that number four times instead of two. (The 0 on the die stands for 10.)

2. *Move*. Move your pawn(s). Apply your dice rolls one at a time to the number your pawn(s) is on, using your choice of +, -, x, or ÷. You can also use Keeper cards if you have them.

3. *Bump*. If you end your Move phase on the same space as another pawn, send it back to start. You may bump your own pawn.

4. *Draw*. If you end your Move Phase on an entirely red space (i.e., a prime greater than 10), draw a Prime card. If it is a Keeper card, save it for a future turn. Otherwise, apply the card now.

When someone lands a pawn exactly on 101, they win the game. You're never allowed to move to numbers off the board.

Example
With pawns on 4 and 26, you roll a 3 and a 9. You could:

- Add 3 to 4 to move your pawn to 7, then multiply by 9 to move your pawn to 63.
- Multiply 26 by 3 to move your pawn to 78, then add 9 to move it to 87.
- Add 9 to 4 to move one pawn to 13, and multiply 26 by 3 to add the other to 78. Since 13 is completely red, you would them draw a card.

You CANNOT add the 3 and 9 first and use a 12 for anything. You have to apply the numbers on the dice one by one.

The Work

Once they understand the rules, let students play the game. They may have questions that come up during the course of play. You can consult the full rules of the game, or just have students respond by deciding on what seems like the best way to settle the question and keep play going.

Prompts and Questions

- Can you get either pawn to a red circle with that roll?
- Can you bump anyone with that roll?
- You rolled a 3 and a 5. What if you added the 3 to your pawn first, then multiplied by 5?
- If you subtract, you could land on a red circle and draw a Prime card!

The Wrap

Settle any remaining questions about the rules, if there are any. Ask what strategy students have found to be useful in the game. For example, does it make sense to add and go past fifty? If you do, you won't be able to multiply again. How important are the cards for your strategy? How quickly can you reach 101 if you get a good roll?

Tips for the Classroom

1. Have students roll in the box lid to prevent them from knocking over pawns during the game.
2. Students can use the multiplication table or scratch paper to help themselves with hard multiplication problems. The board's color scheme can help too.
3. Students may dislike getting knocked back to start. However, they'll quickly learn that they can make fast progress if they get a good roll, especially when they roll doubles.
4. Encourage students to try to get cards on their turn by landing on red circles. That's a good hint for success in the game.

Day 4

Goals

1. Learn and use the area model to solve 2-digit by 2-digit multiplication problems.
2. Play arithmetic games.

Opener

Would you Rather

WYR: work a summer job that pays $15 per hour, for 21 hours a week, or one that pays $19 per hour, for 16 hours per week.

Activities

1. Mini-lesson on the Area Model
2. Pose to students: what is the largest product you can make with the digits 1, 2, 3, 4?

Note: the mini-lesson gives a visual model for understanding and solving 2-digit by 2-digit multiplication problems. The question following it gives a fascinating challenge to solve, which should provide plenty of practice with the area model. (The answer is 41 × 32 = 1312, though it's not obvious that this is largest without doing much more work.) This question extends naturally to: what is the largest product you can make with the digits 2, 4, 6, 7 (or any collection of four digits).

Extend to five digits (i.e., 1, 2, 3, 4, 5) if students can solve and explain the 4-digit solution.

Game/Choice Time

Big Blockout

As the program continues, students may opt to play games they've already learned, like Prime Climb or Odd Pig Out.

Closer

Ask students what the largest product they can make with the digits 2, 3, 4, 5 is. Does having solved the same question for 1, 2, 3, 4 help?

Would You Rather…

work a summer job that paid

Photo by Liam Martens on Unsplash

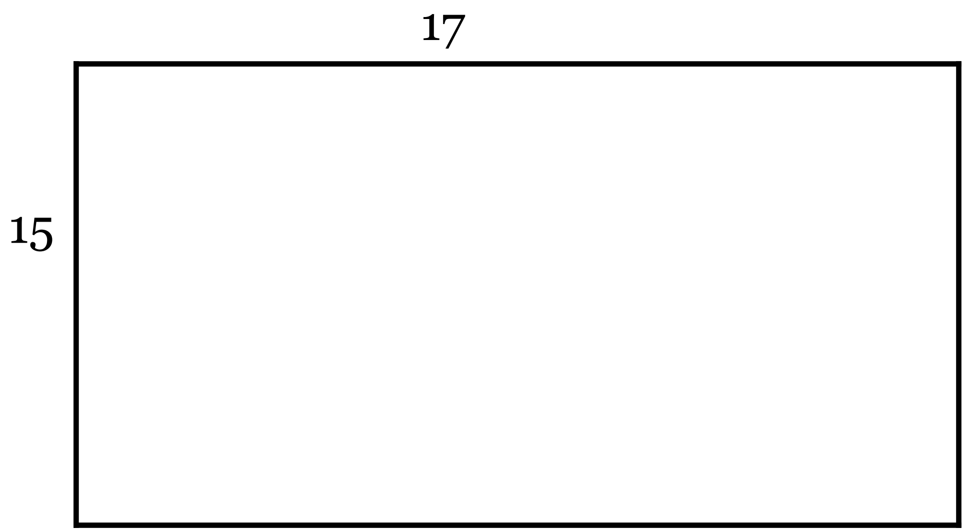
$15 per hour, for 21 hours a week?

OR

$19 per hour, for 16 hours a week?

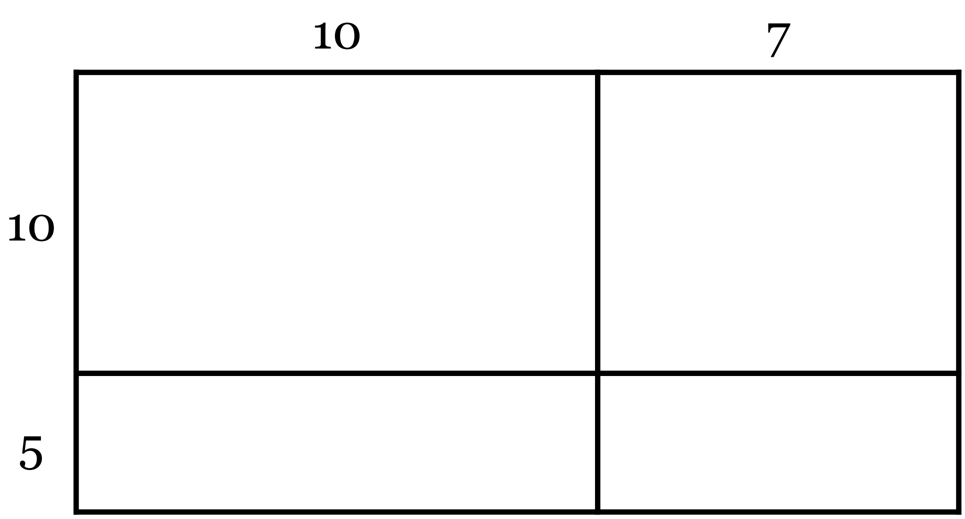
Mini-lesson: Area Model for 2-digit multiplication

Example Mini-lesson
Teacher: Here's an equation to solve: 17 × 15 = _____. Now this seems tricky, but I know how to draw a picture of it.



But this doesn't help quite enough. Really, we need to cut it up into pieces that are easy to work with. What's an easy piece to work with? (Ideally, students respond by suggesting cutting into tens.)

The cool thing is, we can actually cut into tens on both sides.



And now all we have to do is figure out all the pieces. What's 10 × 10? (100).
What's 10 × 7? (70)
What's 5 × 10? (50)
What's 5 × 7? (35)

So we can fill in our picture like this.

	10	7
10	100	70
5	50	35

Now we just add up all the pieces.

100 + 70 +50 + 35 = 255.
So 15 × 17 = 255.

Now try drawing a picture of this one on your own or with a partner: 13 × 14 = _____.
[Pause to let students solve and discuss]

Now that you're got a way to multiply two digit numbers visually, consider this challenge: using the digits 1, 2, 3, 4 once each, what's the large product you could make? For example, you could make 43 × 21 and get the product 800 + 60 + 40 + 3 = 903.

	40	3
20	800	60
1	40	3

Conjecture: that's the greatest product I can make using each of those digits once. Do you agree, or is there a counterexample?

Big Blockout

Topics: Multiplication, commutativity and associativity of multiplication
Materials: Three dice per game, board, colored pencils
Common Core: 5.OA.2, MP1

Roll three dice; add two and multiply by the third. How do you get the highest score?

Why We Love Big Blockout

Big Blockout is a quick and fun game for multiplication practice that poses a fascinating question at the same time. This Blockout adaptation connects the game to the array model of multiplication.

The Launch

Big Blockout can be played with 2-4, but fewer players is usually better.
Players take turns rolling three dice on their turn. On your turn, draw an array on the board. One side of the array is the sum of two dice of your choice; the third die gives the other side. In other words, you add two of your rolls together, and multiply by the third. That is your score for the turn.

Example. You roll 3, 5, 6 on your turn. You could add 6 + 3 to get 9, and multiply by 5 to score 45 points on the turn. But wait! If you add 5 + 3 to get 8, and multiply by 6 you can get 48 points! So scoring 48 points is actually the better option. This means drawing an 8 by 6 array (if there is space for it) would be your best move. (You could have gotten 33 points as well--do you see how?)

Prompts and Questions

- What's the best way to get the most points after you roll? Is there some rule for which numbers you should add and which you should multiply?
- Do some scores come up more often than others?

The Wrap

The fundamental choice in Big Blockout is which two numbers to add and which number to multiply by. Let's try a few more examples—see if you can figure out the best move (without looking at a specific board, we're really just looking for the largest product). Since we know from the last game that multiplication describes a rectangle, we can look build a rectangle for each of these problems to help us.

You roll 1, 4, 5. What's your best move? There are three options.

(1+4) × 5 = 25 **(1+5) × 4 = 24** **(4+5) × 1 = 9**

25 is the best move.

You can pose as many of these followup questions as you have time for. After each one, give the students a minute to solve one or more of the problem below and discuss amongst themselves.

You roll 2, 4, 5. What's your best move?
You roll 3, 4, 5. What's your best move?
You roll 4, 4, 5. What's your best move?
You roll 5, 4, 5. What's your best move?
You roll 6, 4, 5. What's your best move?

Variations

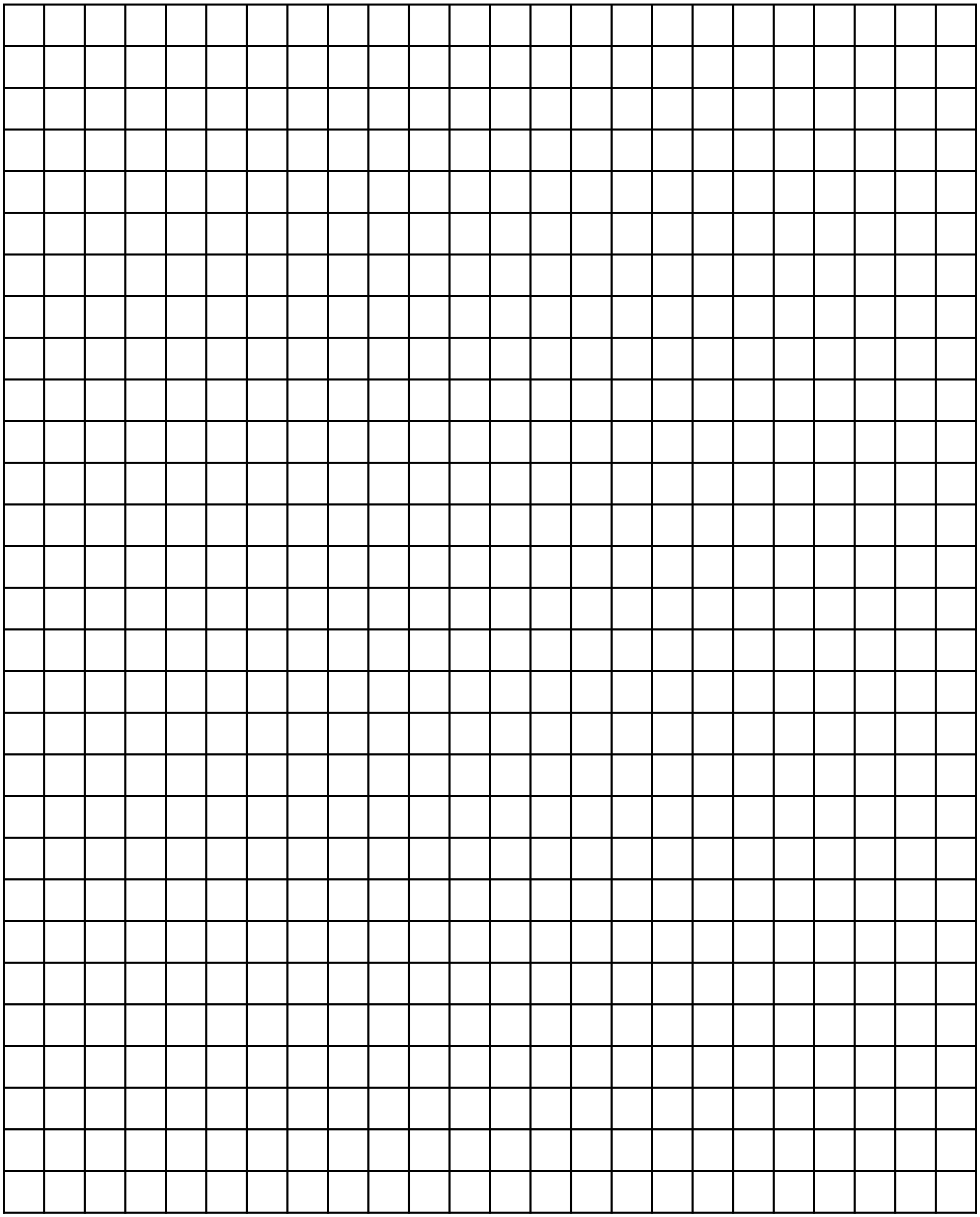
Big Blockout Place Value: Instead of adding two dice, make a 2-digit number out of them. So if you rolled 3, 4, and 2, you could get 34 × 2 = 68 points. Or better, you could get 32 × 4 = 128 points. (Play without the board - just tally scores every turn.)

Big Blockout Pro: Roll four dice instead of three. Add three dice of your choice together and multiply by the fourth.

Tips for the Classroom

3. As students get more accustomed to the game, increase the difficulty by adding in 8, 10, and 12 sided dice.
4. For students who are less confident, let them use their multiplication tables to help them with the game.
5. You can also play Big Blockout without the Board, and just keep track of the score. Play to 200, for example.

Big Blockout Board



Day 5

Goals

1. Explore a rich problem involving area and multiplication or decimals and fractions.

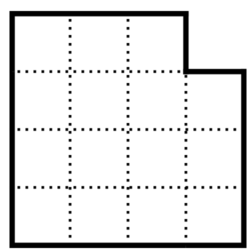
Opener

Counterexamples

A 3 by 5 rectangle has area 15 square units and perimeter 16 units.

Conjecture: there is no other shape that has the same area and perimeter as a 3 by 5 rectangle.

(This conjecture is true if we change “shape" to “rectangle.” But for all shapes, we have plenty of counterexamples. For example, the hexagon below:



Activity

Strange Rectangles

Game/Choice Time

Don’t Break the Bank

Both the original and extension version with decimals to tenths are below. Choose whichever would be appropriate to begin with for your students.

Closer

Ask students to consider the claim that “in math, there's only one right answer.” Do they think that's true? Is it true in this class? Let students discuss in small groups, then as a class.