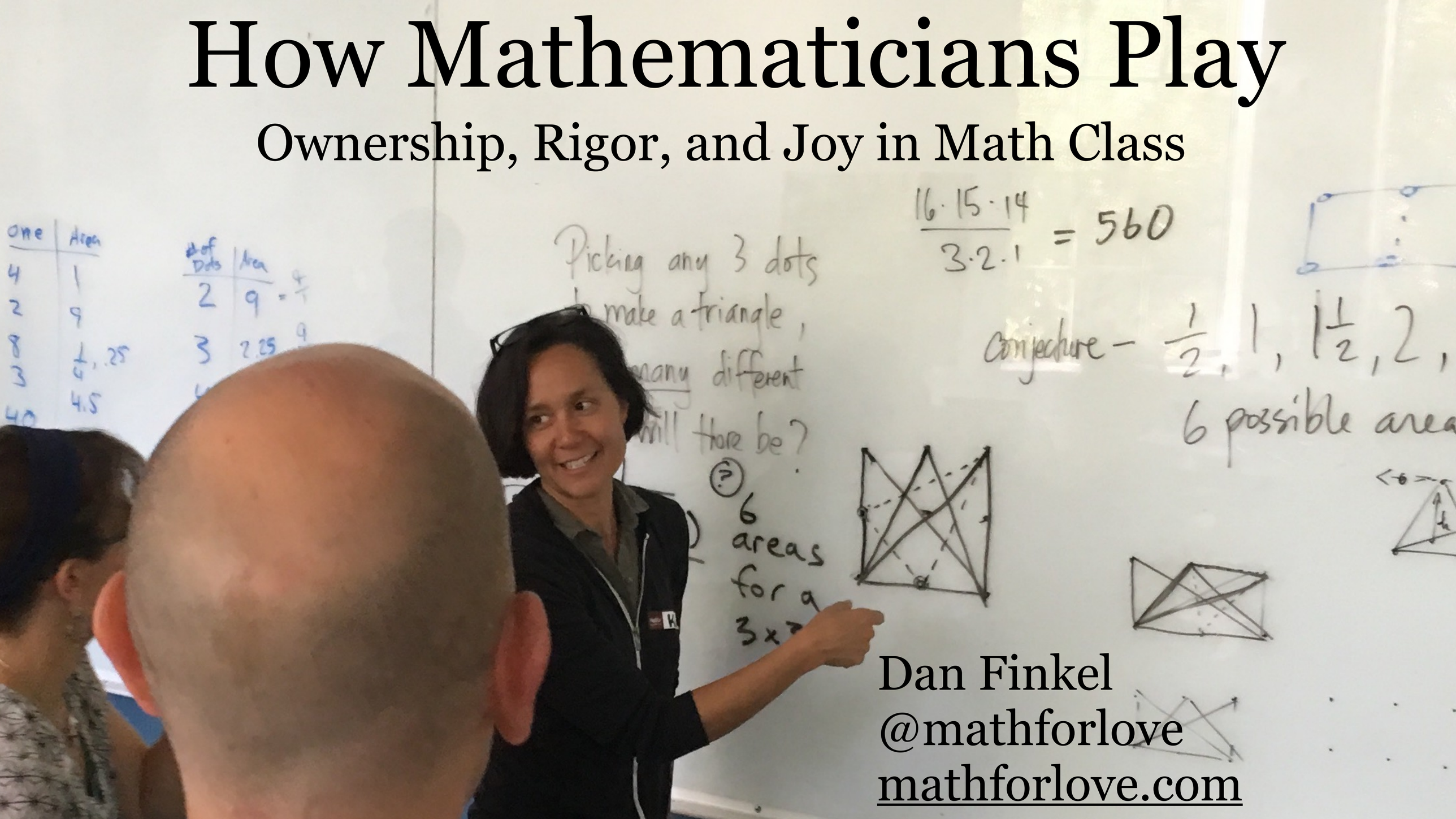
How Mathematicians Play

Ownership, Rigor, and Joy in Math Class



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Agreements

1. Nothing you do here reflects on your intelligence

Agreements

2. If something isn’t the right for you, make it right

How do we invite our students to take
ownership
of their mathematical learning?

Frame

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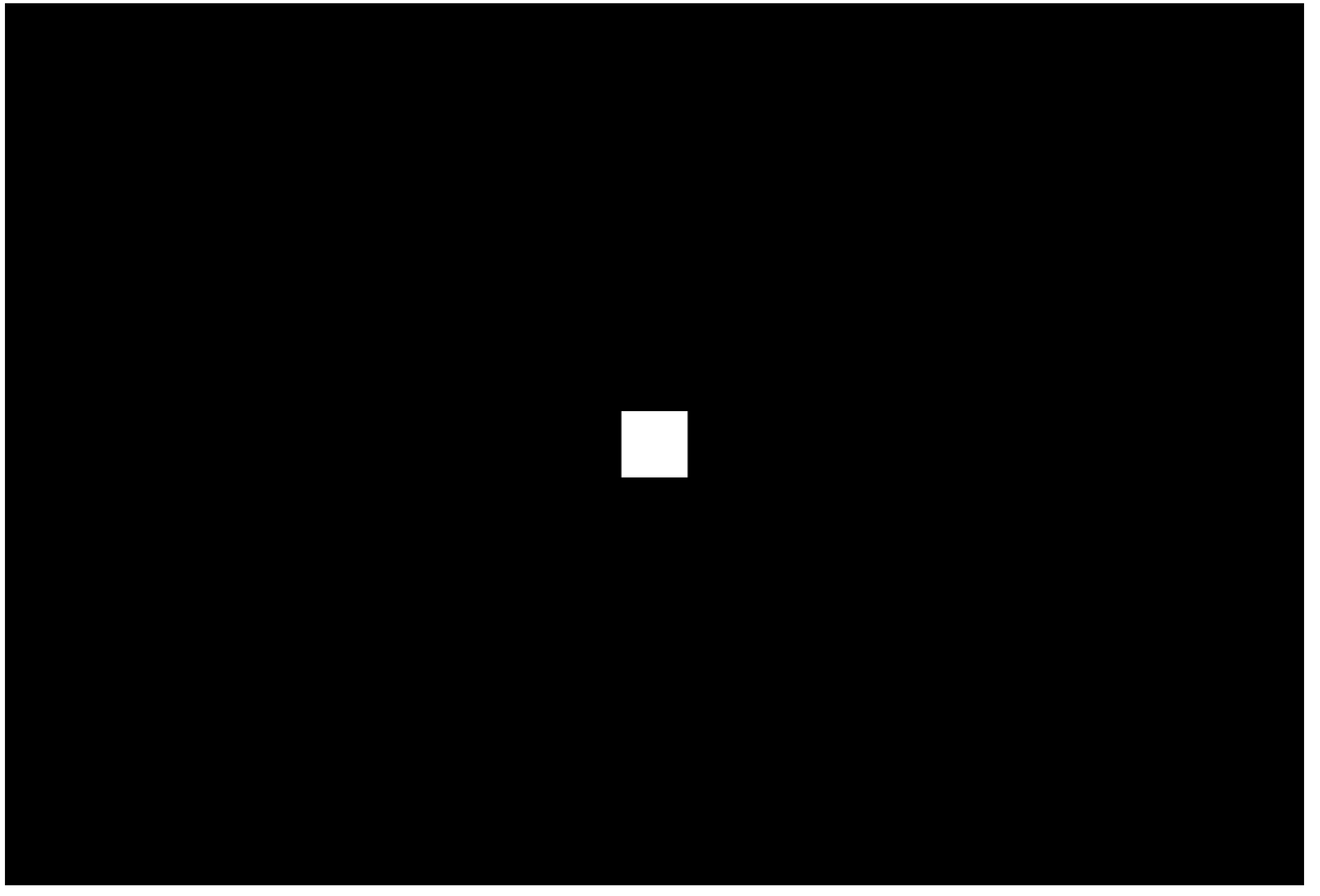

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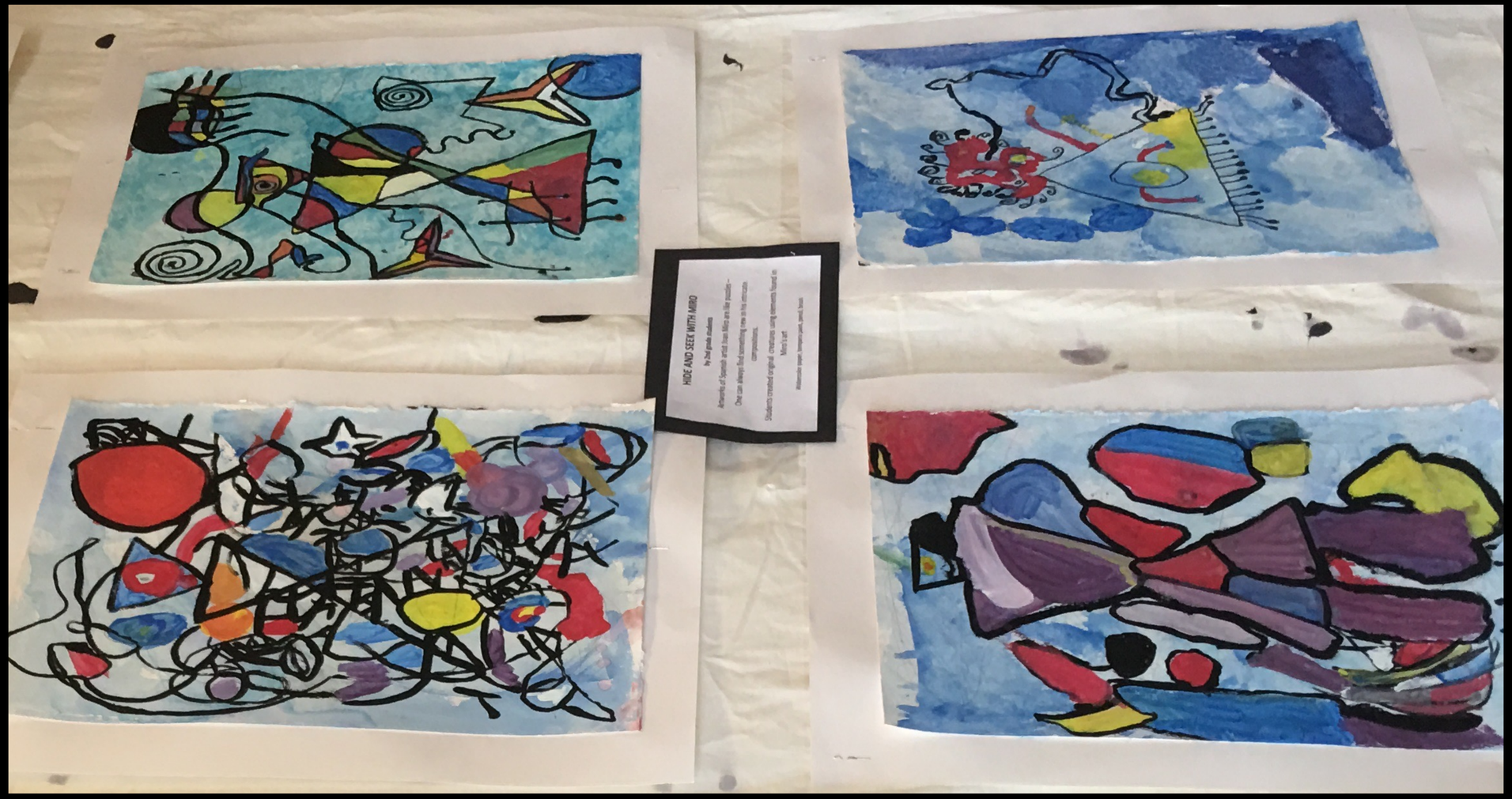
Art in the style of Miro

Art in the style of Miro



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Rules for moving pieces on a board

Rules for moving pieces on a board

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A ball, a field, a goal

You can’t use your hands

A ball, a field, a goal

You can’t use your hands

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A ball, a field, a goal

You can’t use your hands

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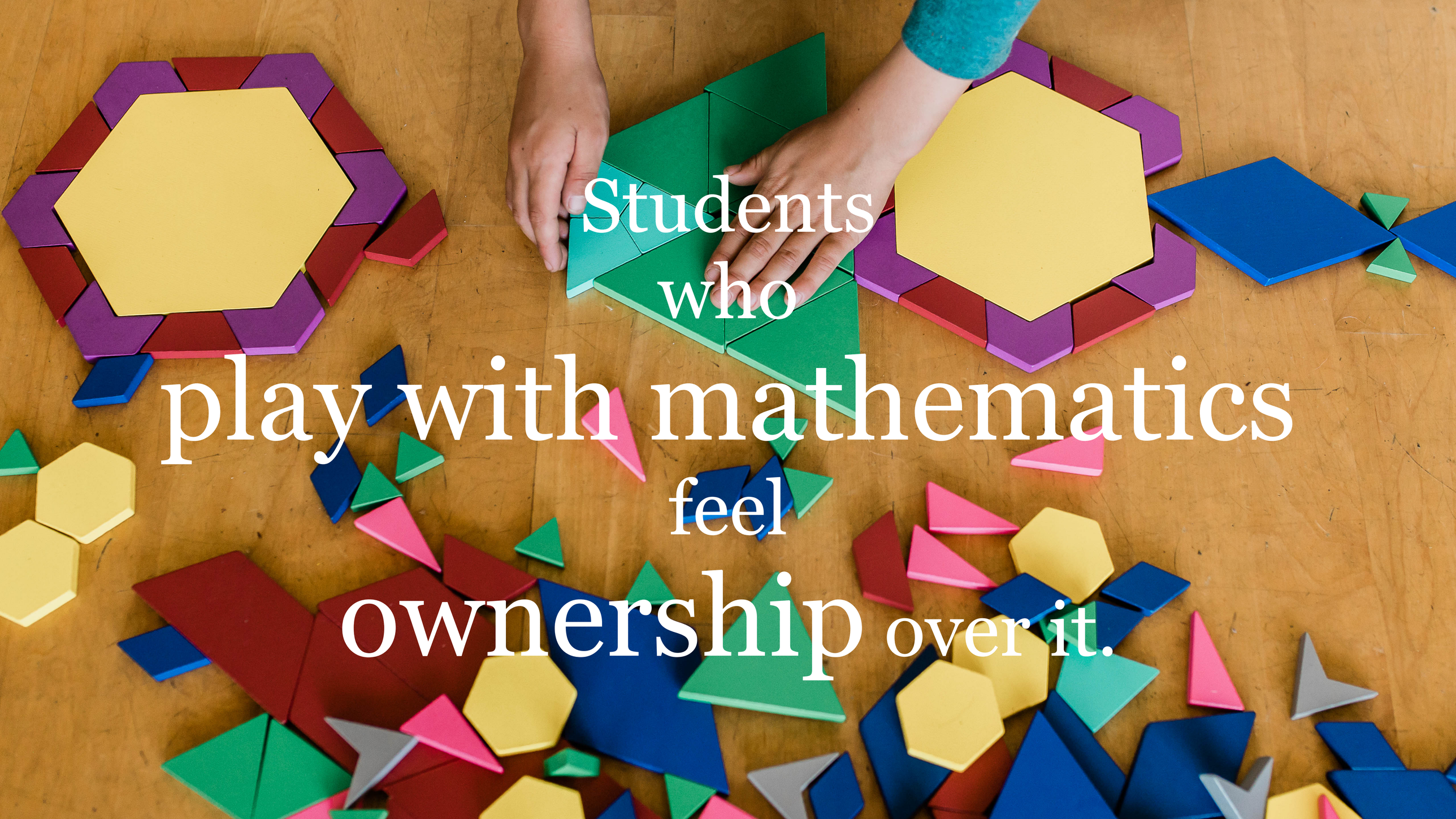
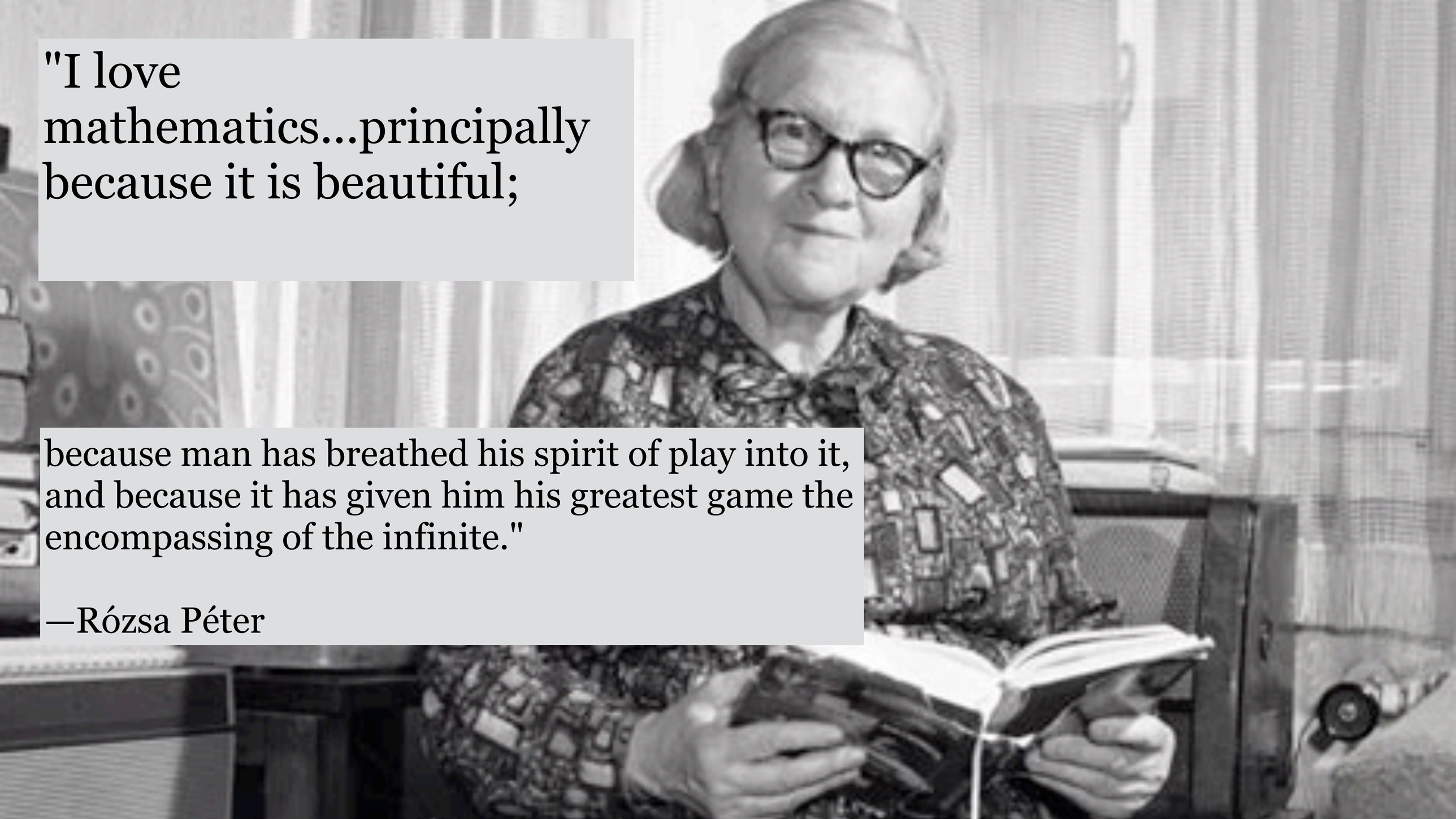

Students
who
play with mathematics
feel
ownership over it.



"I love mathematics...principally because it is beautiful;

because man has breathed his spirit of play into it, and because it has given him his greatest game the encompassing of the infinite."

—Rózsa Péter

What should the frame be…

To invite students to
play mathematically?

If the frame is too thin,
it’s hard to focus

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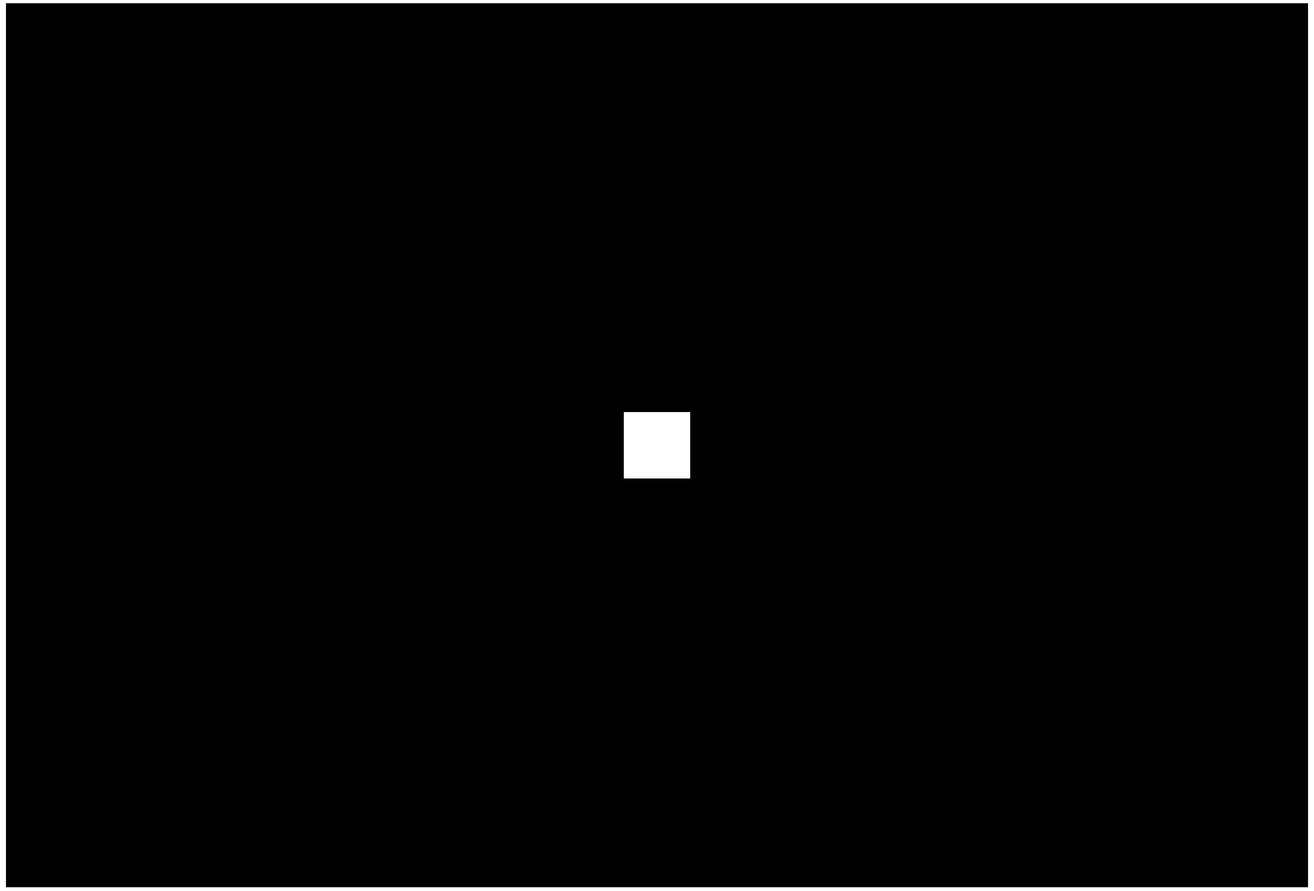

If the frame is too thick,
it’s not an invitation for students to contribute

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Did your teachers give you frames that were:

A) Too thick? (no room for student contribution)
B) Too thin? (not enough guidance)
C) Just right?

What do students learn in math class?

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Official Answer

Facts & algorithms

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Accidental Answer

"Math isn't for me."
"I can't do this."
"This is boring."
"When am I ever going to use this?"

Enlightened official answer

Numeracy and problem solving
Persistance and perseverance
Strategic, flexible, connected thinking
Argument and critique
Mathematical thinking

My Ideal World Answer

Courage
Curiosity
Kindness

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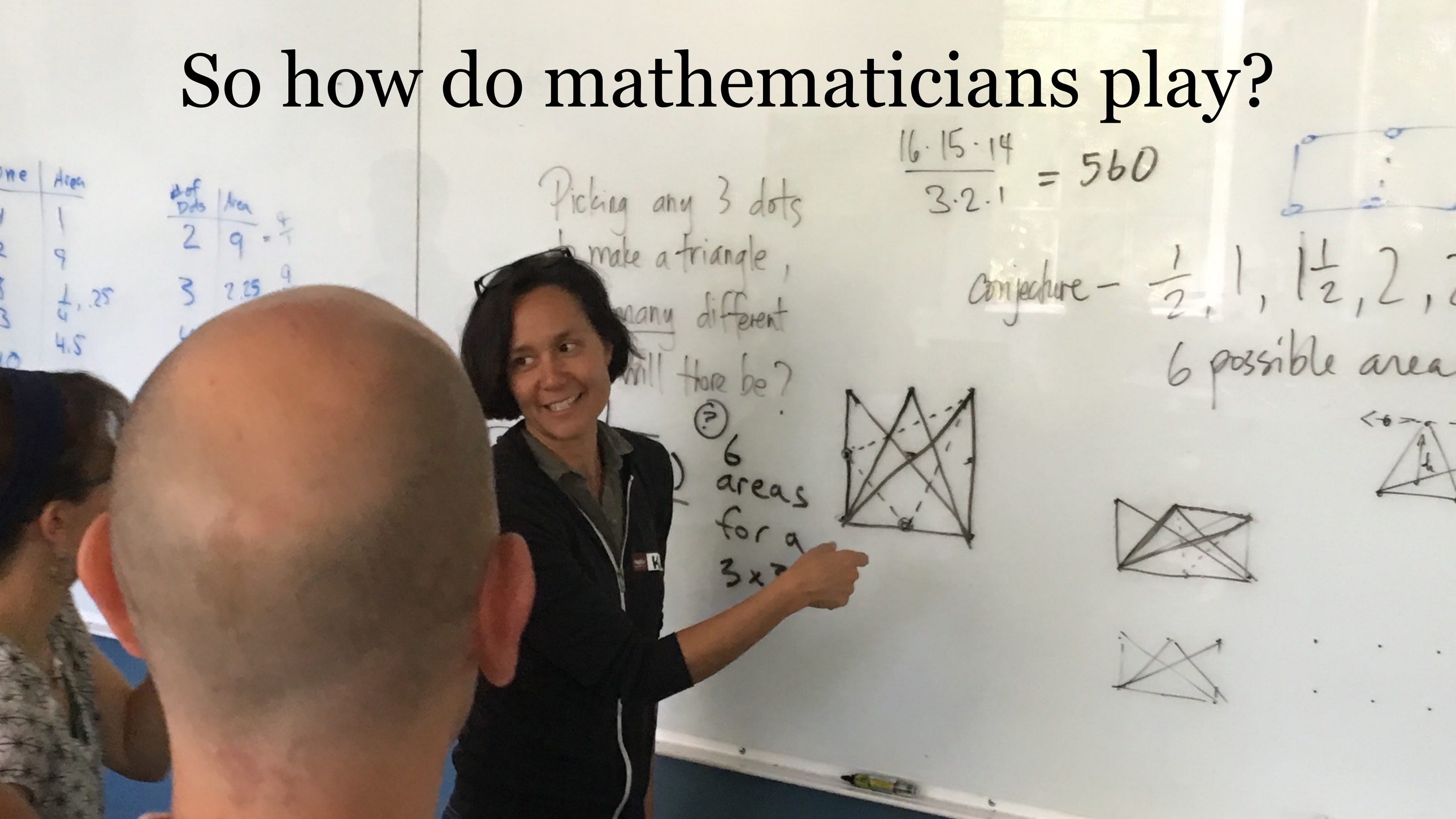
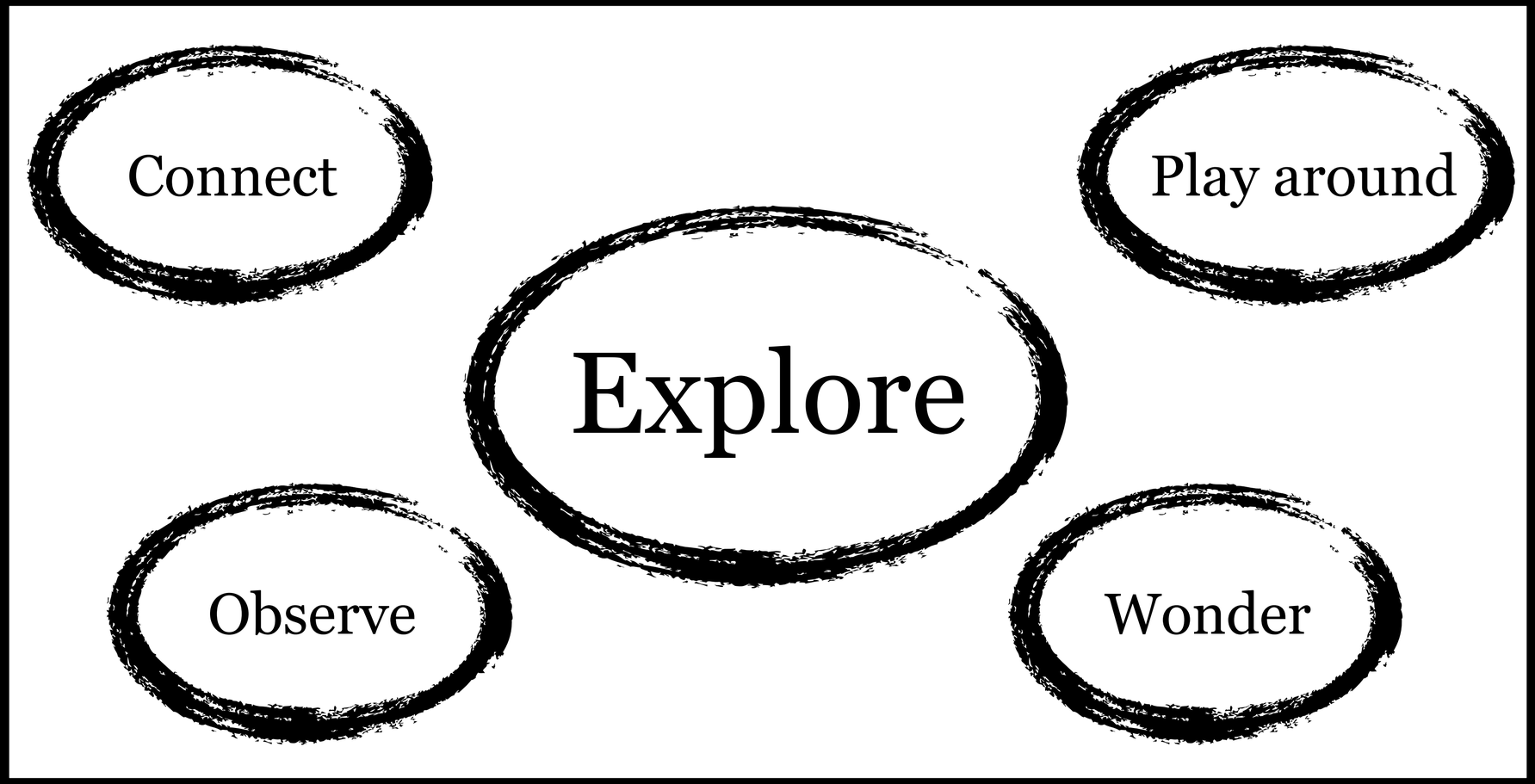
So how do mathematicians play?





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How Many?

16 Dice

2 Colors

8 Sixes

8?
24? Threes

4 rows of 6 Threes
$4 \times 6 \times 3$ Dots

24×3 Dots

4×18 Dots

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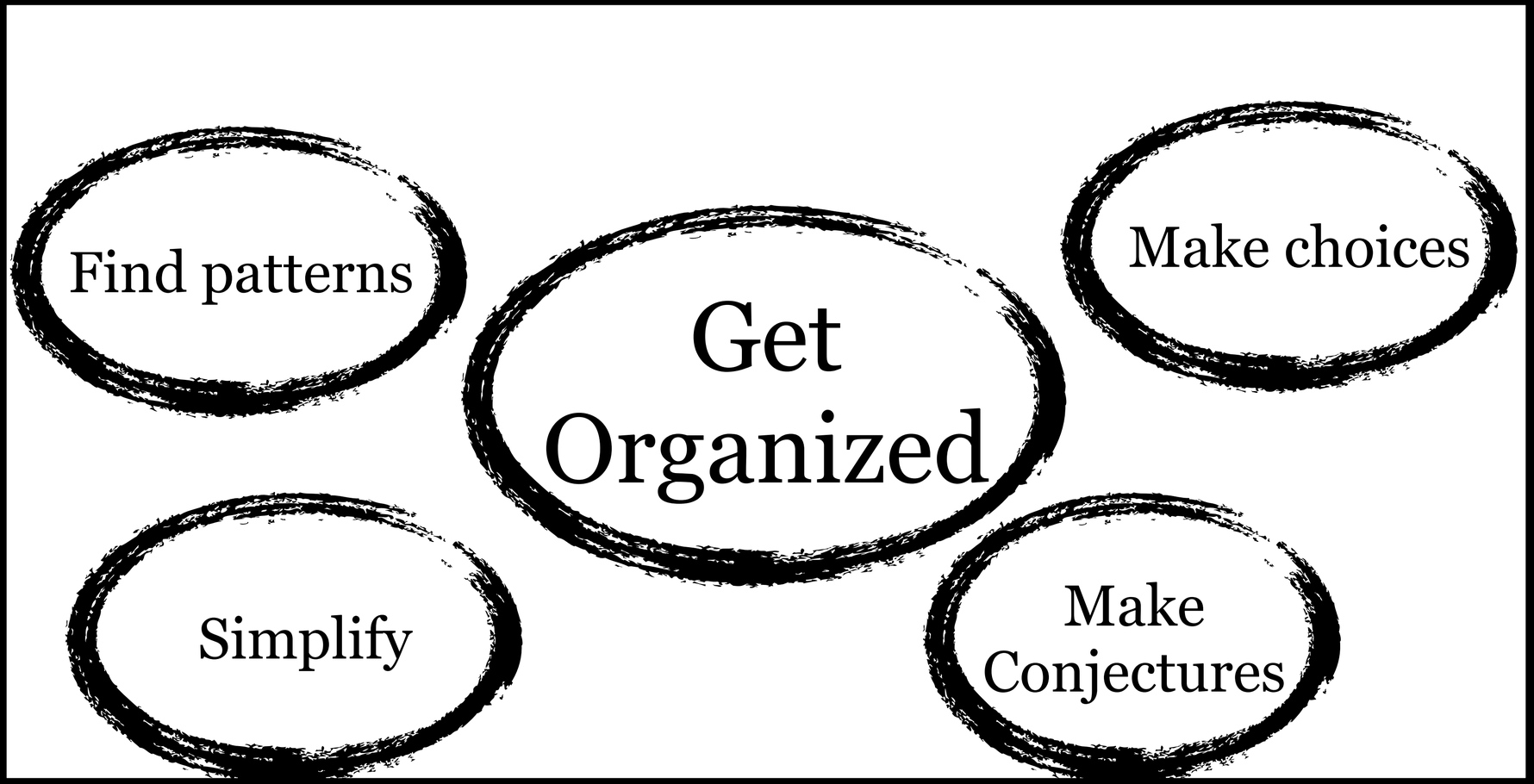

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$$8 = _____ + _____$$

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$$8 = _____ + _____$$

$$8 = 1 + 7$$
$$8 = 2 + 6$$
$$8 = 3 + 5$$
$$8 = 4 + 4$$
$$8 = 5 + 3$$
$$8 = 6 + 2$$
$$8 = 7 + 1$$

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$$8 = _____ + _____ + _____$$

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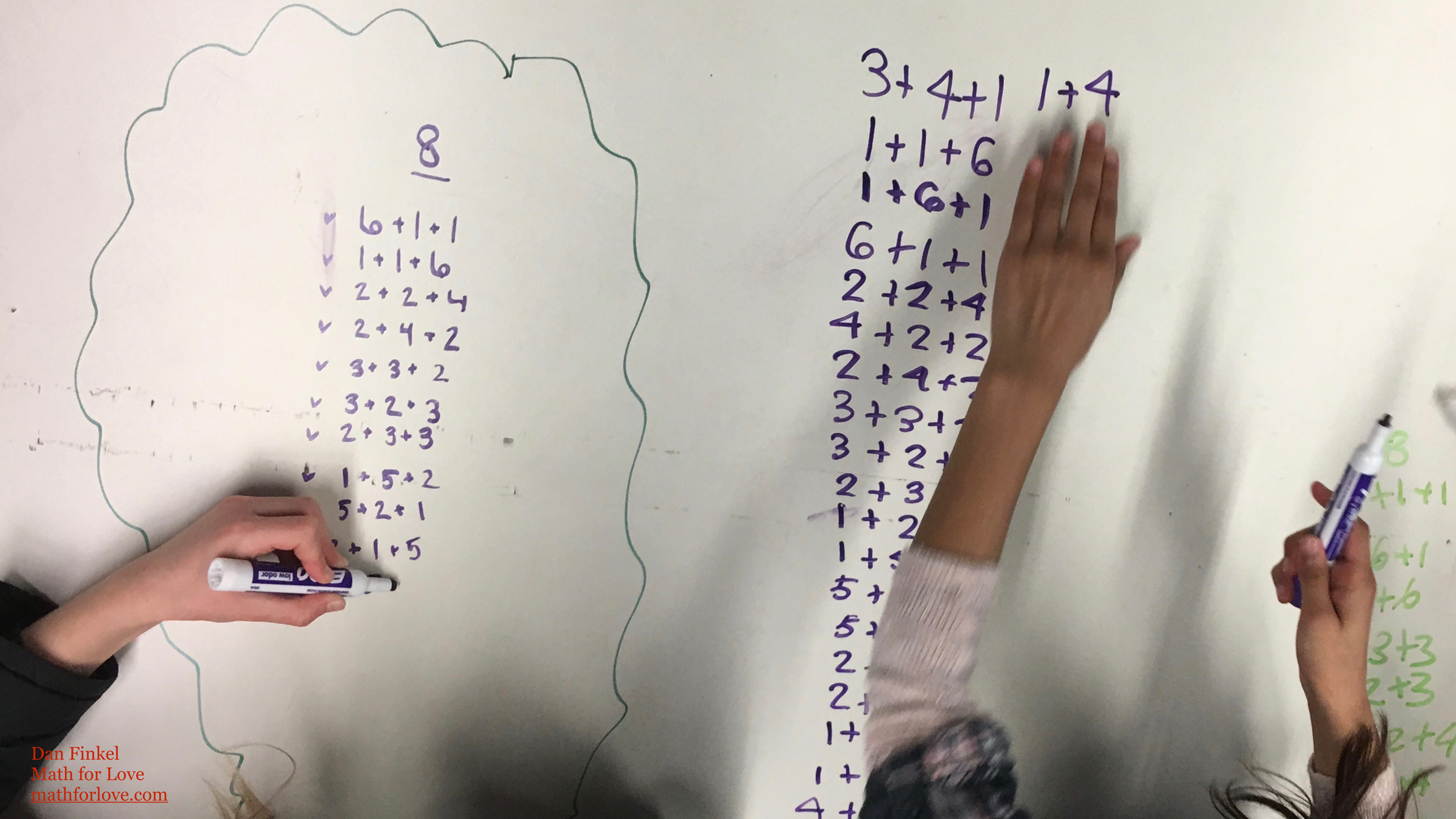

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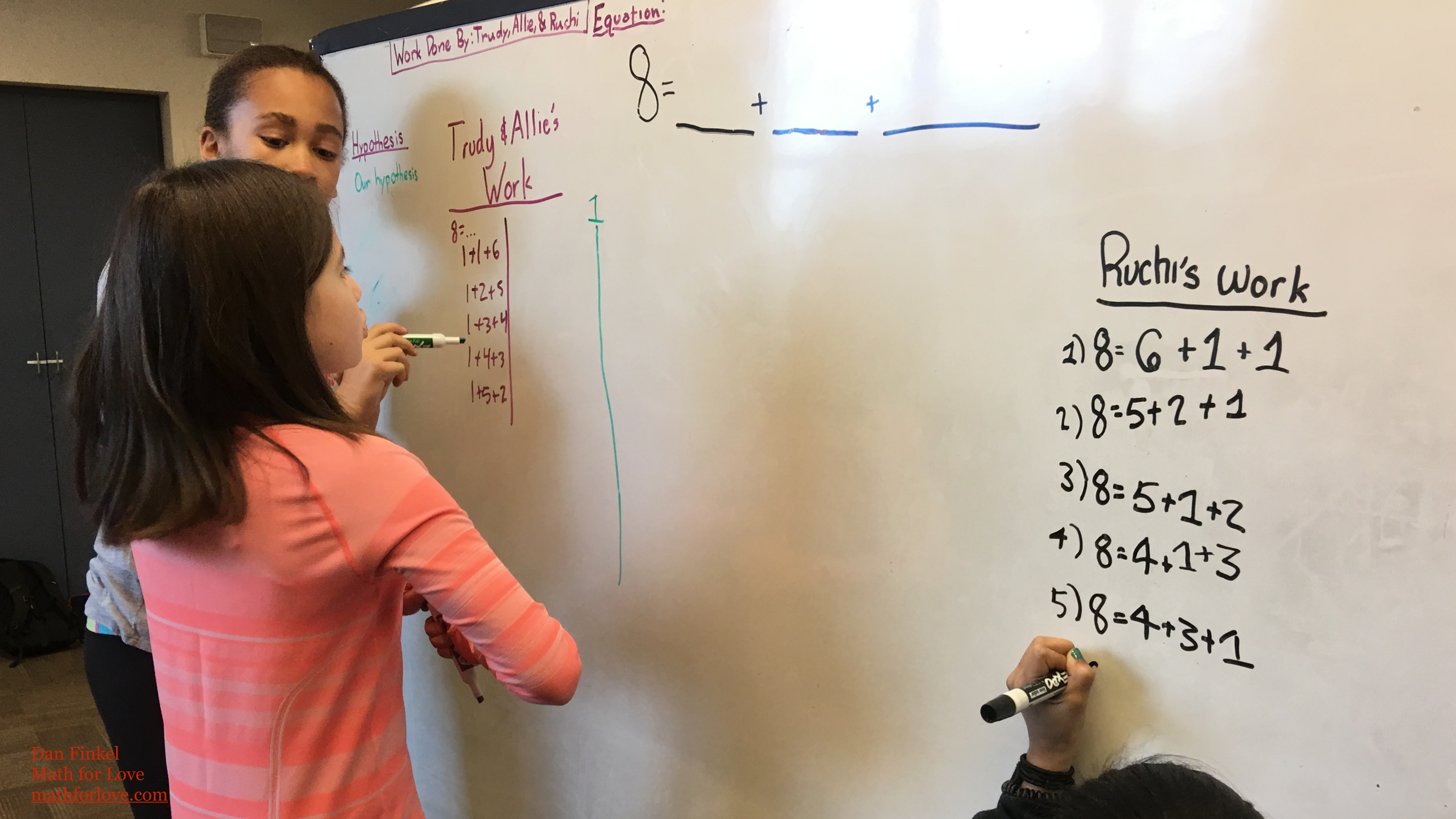

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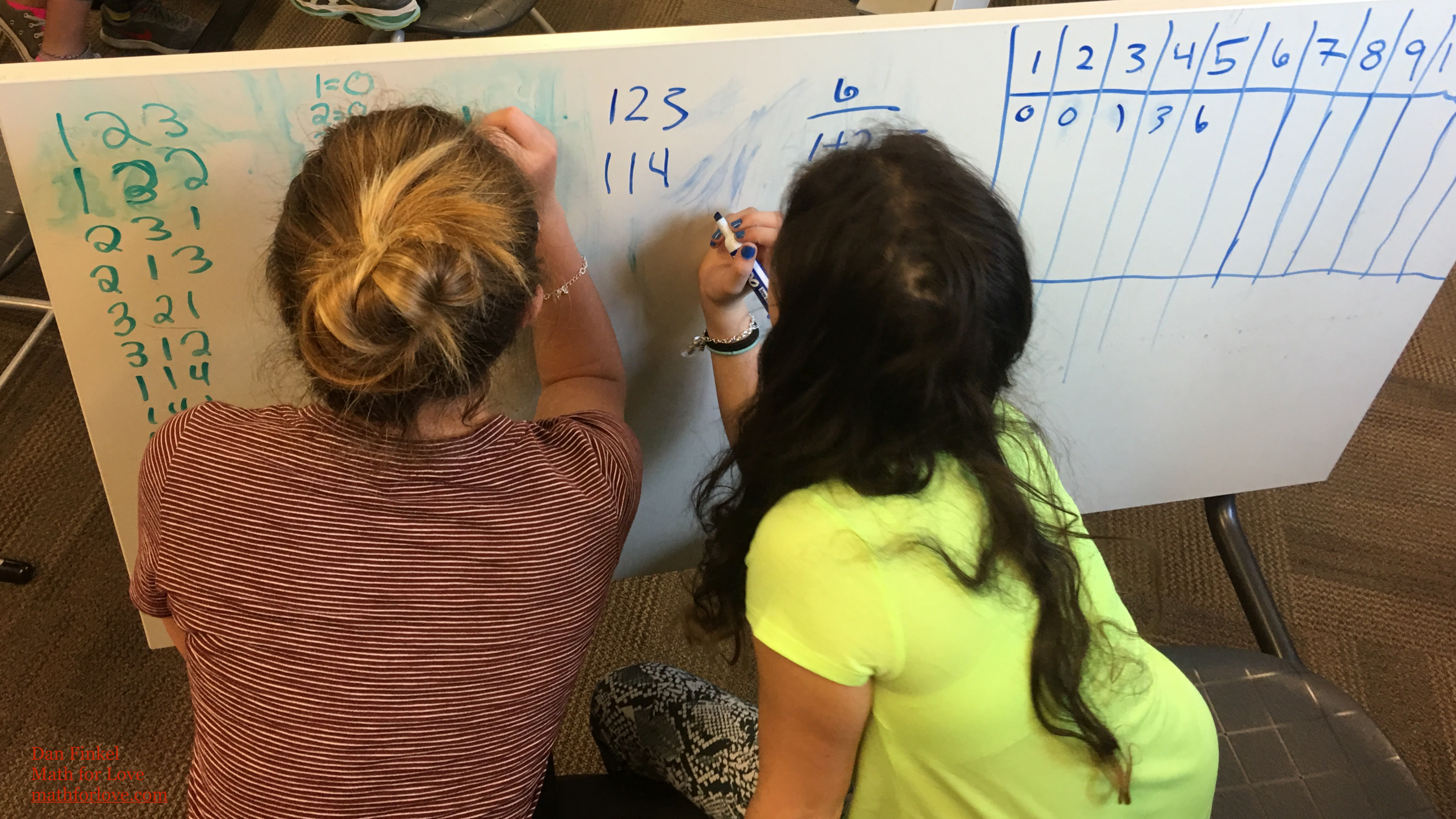

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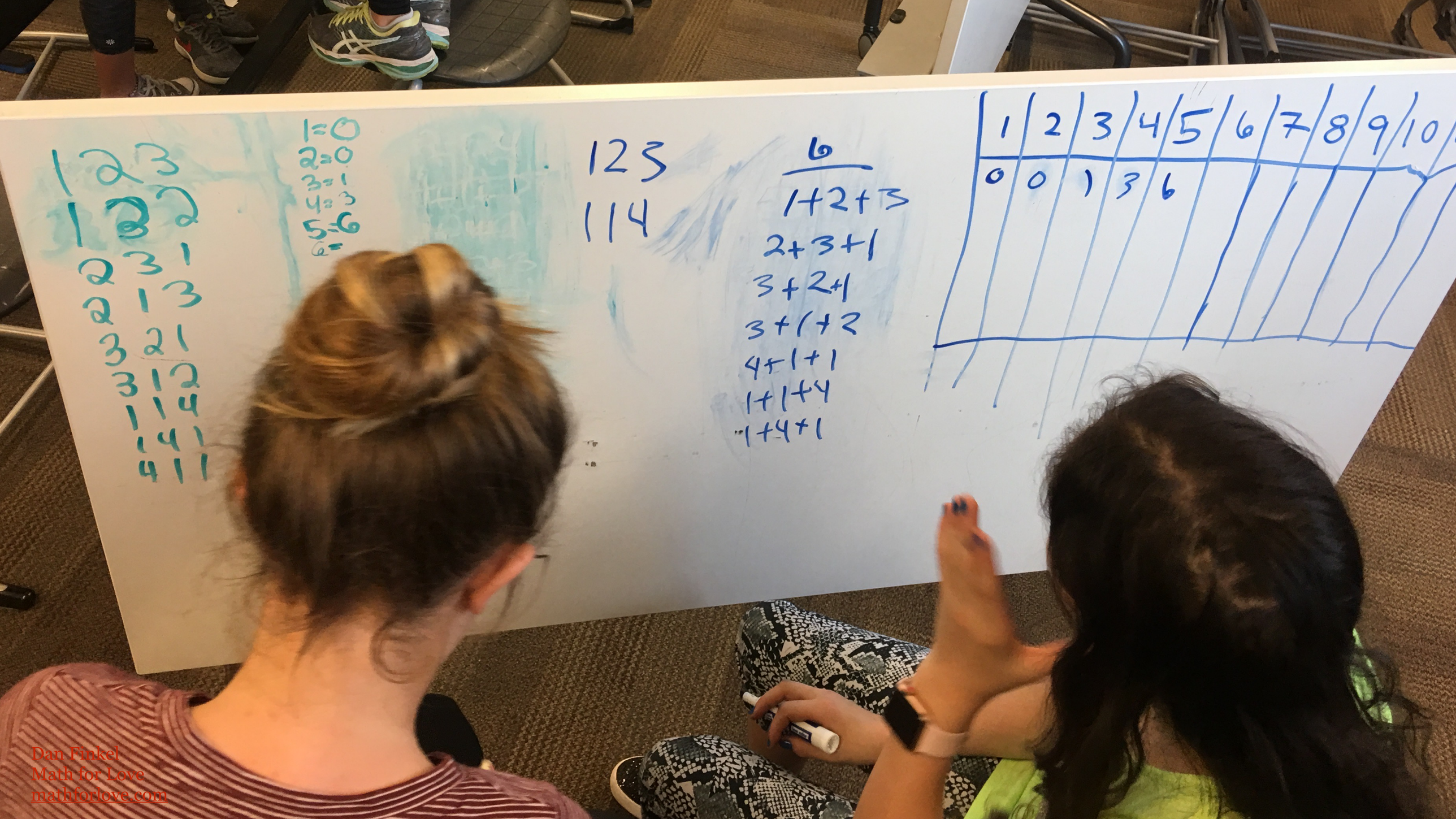

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5 + 1 = 5

5 + 2 + 1 = 6

4 + 2 + 2 = 3

3 + 3 + 2 = 3

4 + 3 + 1 = 6

21

Key:

If there is 2 of the same #, there will be 3 ways.

If there are 3 different #'s, than there will 6 ways

8. 1 + 6 + 1 = 3

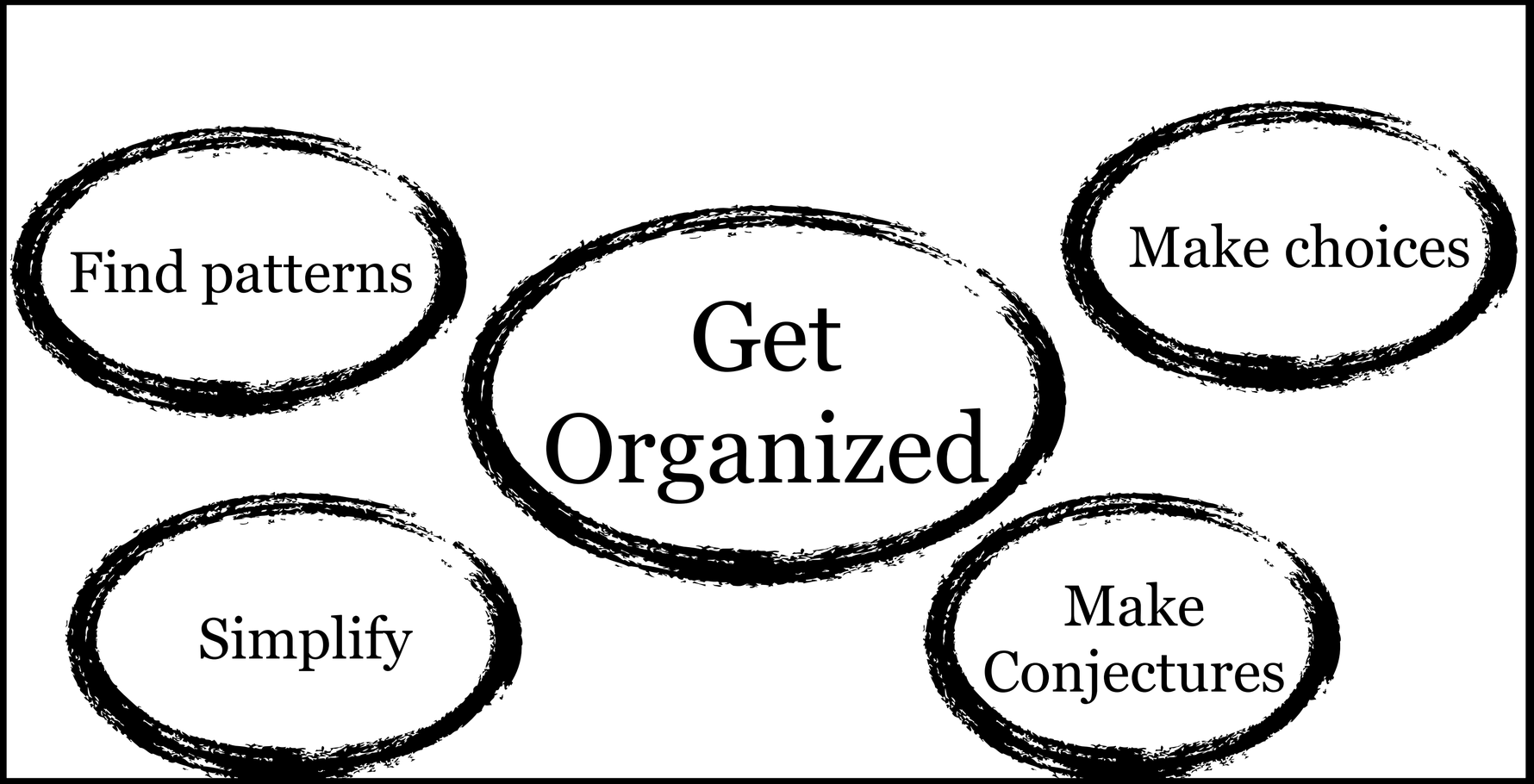
5 + 2 + 1 = 6

4 + 3 + 1 = 6

4 + 2 + 2 = 3

3 + 3 + 2 = 3

21



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Conjectures and Counterexamples!

Dogs have four legs.
Cats have four legs.

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Conjectures and Counterexamples!

Dogs have four legs.
Cats have four legs.

Conjecture.
All animals have four legs.

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Conjecture.
All animals have four legs.

Conjecture.
All animals have four legs.

Counterexample.
Ducks are animals with just two legs.

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Conjecture.
All animals have four legs.

Counterexample.
Ducks are animals with just two legs.

Revised Conjecture.
All animals have either **two or four** legs.

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Where do we find conjectures?

Everywhere

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Conjectures can come from common misunderstandings

- All fractions are less than 1
- If two rectangles have the same perimeter, they must be identical
- You can’t share 4 things with 3 people fairly

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Conjecture.
When you multiply numbers together, you end up with bigger numbers than you started with.

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Conjecture.
When you multiply numbers together, you end up with bigger numbers than you started with.

Counterexample.

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Conjecture.
When you multiply numbers together, you end up with bigger numbers than you started with.

Counterexample.
$1 \times 5 = 5$

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Conjecture.
When you multiply numbers together, you end up with bigger numbers than you started with.

Counterexample.
$1 \times 5 = 5$

Revised Conjecture.
When you multiply numbers together, you end up with numbers *greater than or equal to* the factors.

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Most rules, facts, and formulas can be rephrased as conjectures

Facts *end*

Conjectures *extend*

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Commutative Property of Multiplication

For all real numbers, $a \times b = b \times a$.

Facts *end*

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I noticed that $3 \times 7 = 7 \times 3$.

Conjecture

For any a and b, $a \times b = b \times a$.

Conjectures *extend*

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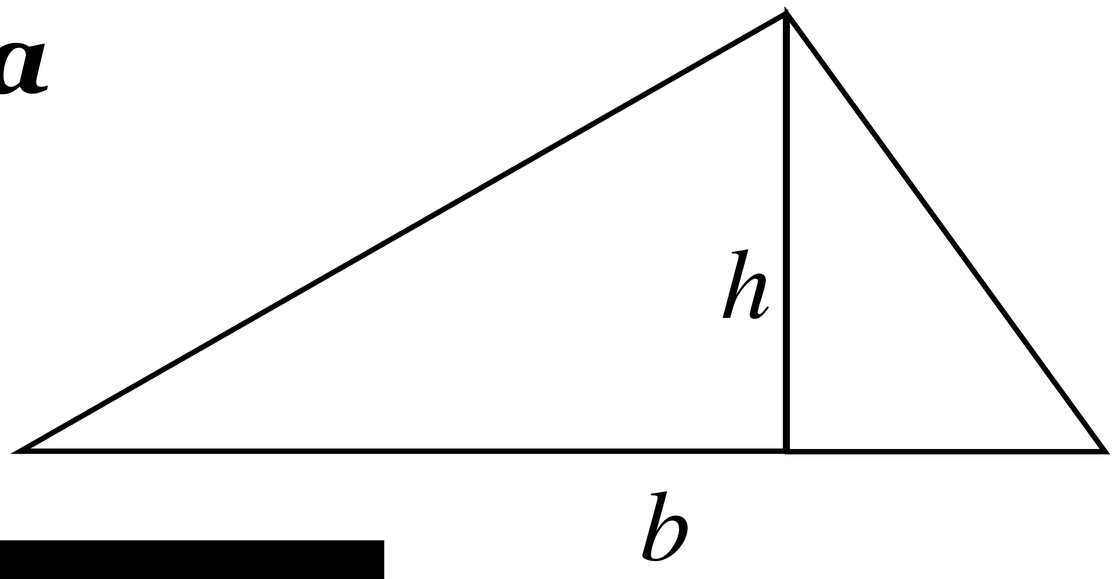
Triangle Area Formula

$$A = \frac{1}{2}bh$$



Facts *end*

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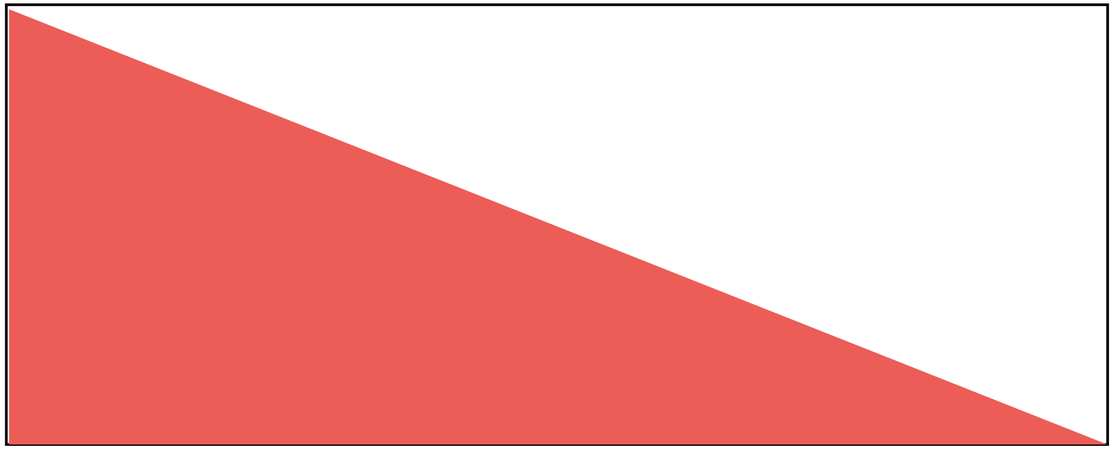
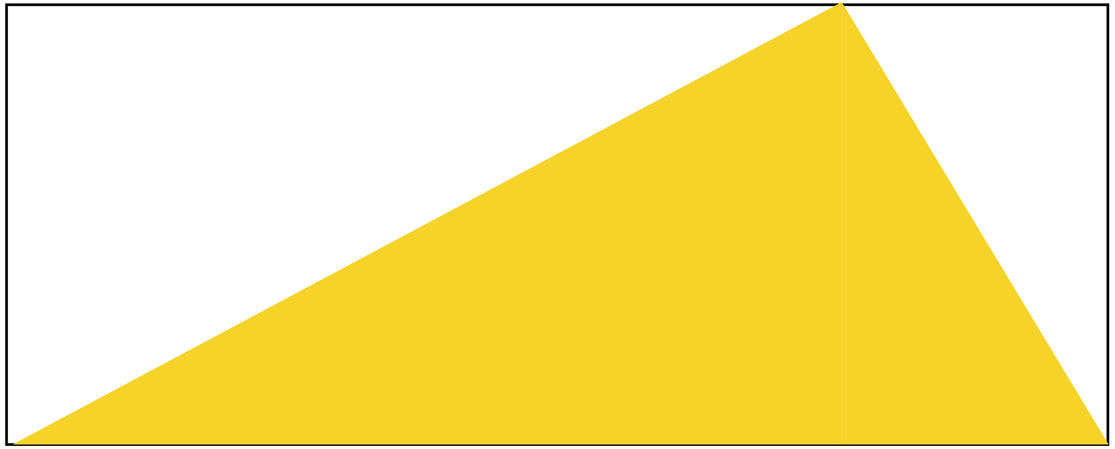
I noticed the red triangle takes up half the rectangle. Could the yellow one be half of its rectangle too?





Conjecture

A triangle always takes up half of the rectangle that contains it.

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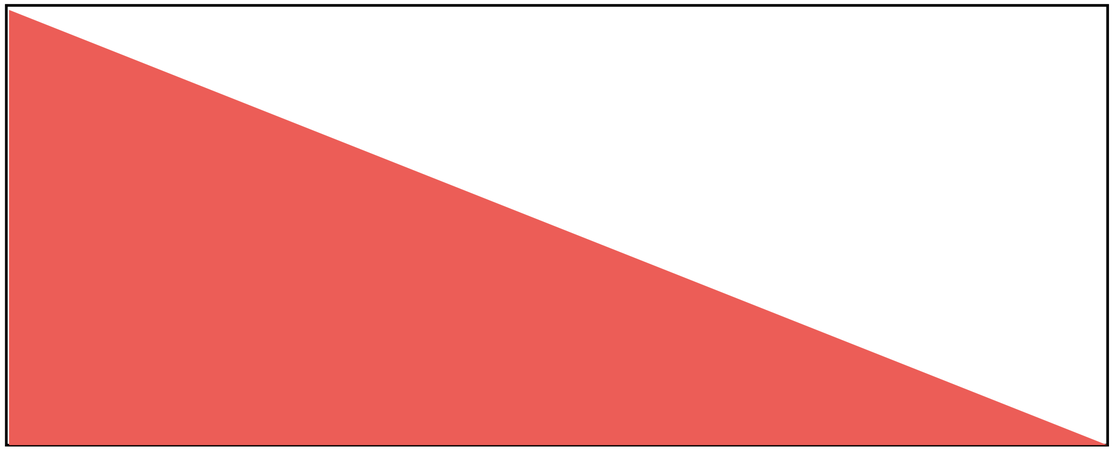

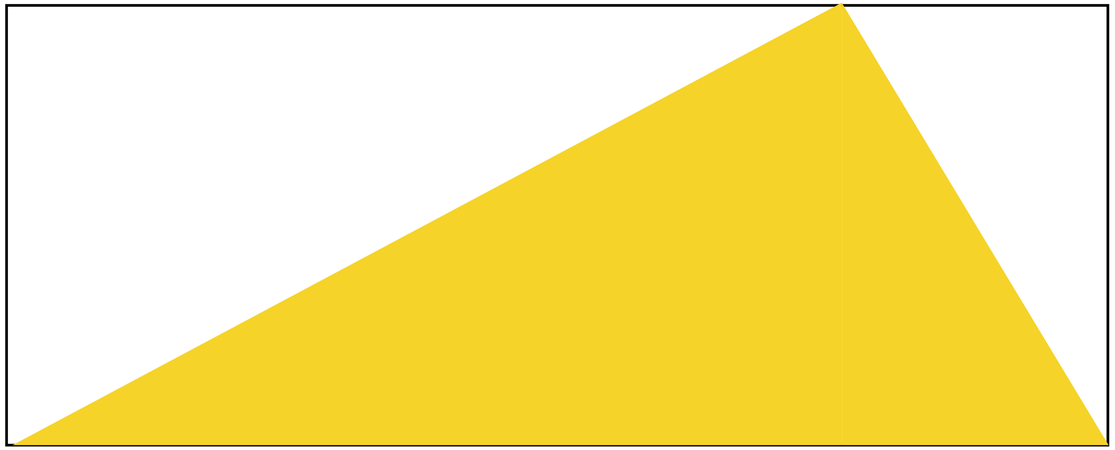

Conjecture

A triangle always takes up half of the rectangle that contains it.

Conjectures *extend*

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Anywhere we organize data we naturally find patterns and make conjectures

Skip counting by 3s…

3	6	9	12	15
18	21	24	27	30

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Skip counting by 3s...

3	6	9	12	15
18	21	24	27	30
33	36	39	42	45
48	51	54	57	60

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Skip counting by 3s...

3	6	9	12	15
18	21	24	27	30
33	36	39	42	45
48	51	54	57	60

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Skip counting by 3s...

3	6	9	12	15
18	21	24	27	30
33	36	39	42	45
48	51	54	57	60

Conjecture

Skip count by any number and you cycle through all the digits in the units place.

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Also, the teacher can offer a conjecture as a frame for a lesson!

Math Magic Trick

Pick a whole number between 1 and 10

- Add 2.
- Multiply by 2.
- Subtract 2.
- Divide by 2.

Then subtract your original number.

Your answer is…

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Math Magic Trick

Pick a whole number between 1 and 10 — 9

- Add 2. — 11
- Multiply by 2. — 22
- Subtract 2. — 20
- Divide by 2. — 10

Then subtract your original number. — 10 - 9

Your answer is... — 1

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Math Magic Trick

Pick a whole number between 1 and 10 — 9

- Add 2. — 11
- Multiply by 2. — 22
- Subtract 2. — 20
- Divide by 2. — 10

Then subtract your original number. — 10 - 9

Your answer is ALWAYS 1! — 1

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Math Magic Trick

Pick a whole number between 1 and 100 — 9

- Add 2. — 11
- Multiply by 2. — 22
- Subtract 2. — 20
- Divide by 2. — 10

Then subtract your original number. — 10 - 9

Your answer is ALWAYS 1! — 1

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Math Magic Trick

Pick a whole number between 1 and 100 — 45

- Add 2. — 47
- Multiply by 2. — 94
- Subtract 2. — 92
- Divide by 2. — 46

Then subtract your original number. — 46 - 45

Your answer is ALWAYS 1! — 1

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Math Magic Trick

Pick any number! 4122

- Add 2. 4124
- Multiply by 2. 8248
- Subtract 2. 8246
- Divide by 2. 4123

Then subtract your original number. 4123 - 4122

Your answer is ALWAYS 1! 1

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Math Magic Trick

Pick any number! 0

•Add 2. 2
•Multiply by 2. 4
•Subtract 2. 2
•Divide by 2. 1

Then subtract your original number. 1 - 0

Your answer is ALWAYS 1! 1

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Math Magic Trick

Pick any number! 5.7

- Add 2. 7.7
- Multiply by 2. 15.4
- Subtract 2. 13.4
- Divide by 2. 6.7

Then subtract your original number. 6.7 - 5.7

Your answer is ALWAYS 1! 1

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Math Magic Trick

Pick any number! -33

•Add 2. -31
•Multiply by 2. -62
•Subtract 2. -64
•Divide by 2. -32

Then subtract your original number. -32 - (-33)

Your answer is ALWAYS 1! 1

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Is it always 1?

We have proved NOTHING.

-47/99? 1/100 0.02113? -9,999,924,598.003?

π?

To prove the trick will **not** always work takes just a single counterexample.

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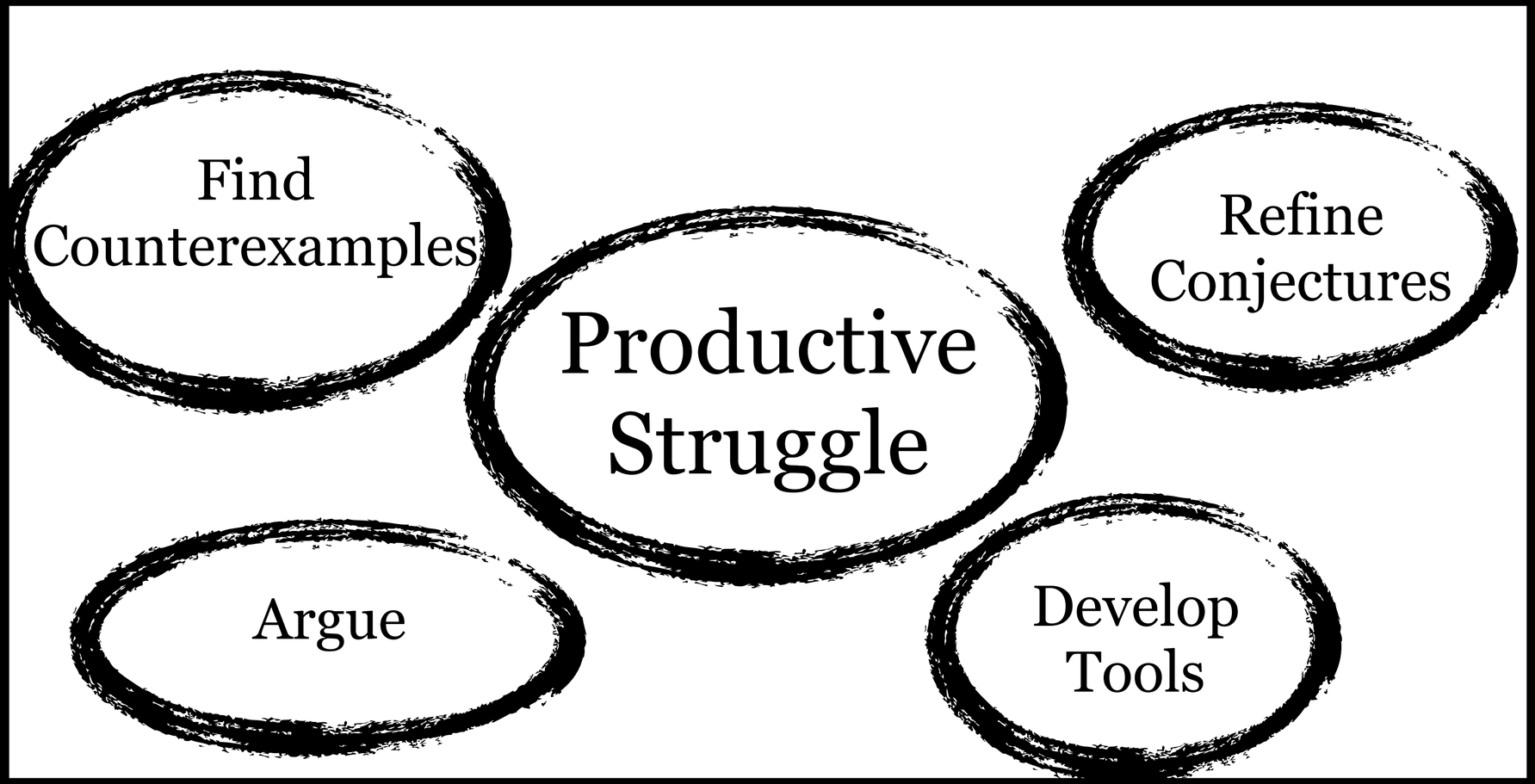

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Would a visualization help?

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Pick a number

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147

Pick a number

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147

Add 2

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147

Add 2

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147 : **Multiply by 2**

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Multiply by 2

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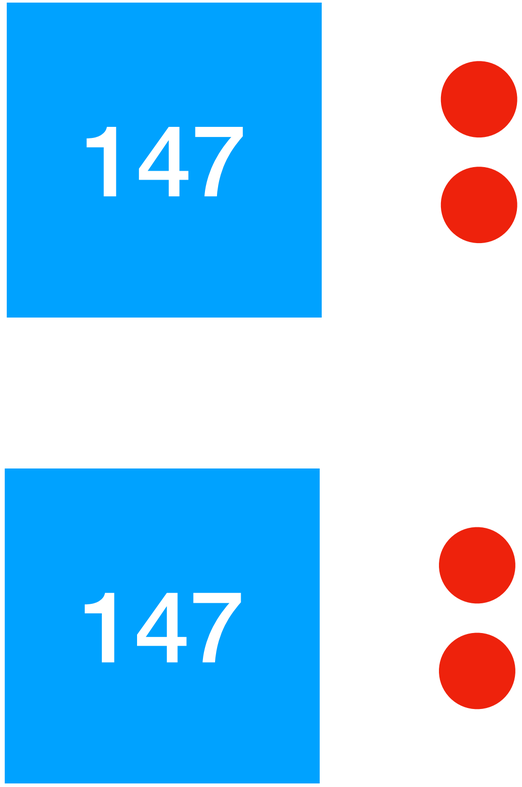

Subtract 2

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147

Subtract 2

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147

Divide by 2

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Divide by 2

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Divide by 2

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Subtract your original number

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Subtract your original number

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The answer is 1!

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Pick a number

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-68.23

Pick a number

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-68.23

Add 2

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-68.23

:

Add 2

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-68.23

Multiply by 2

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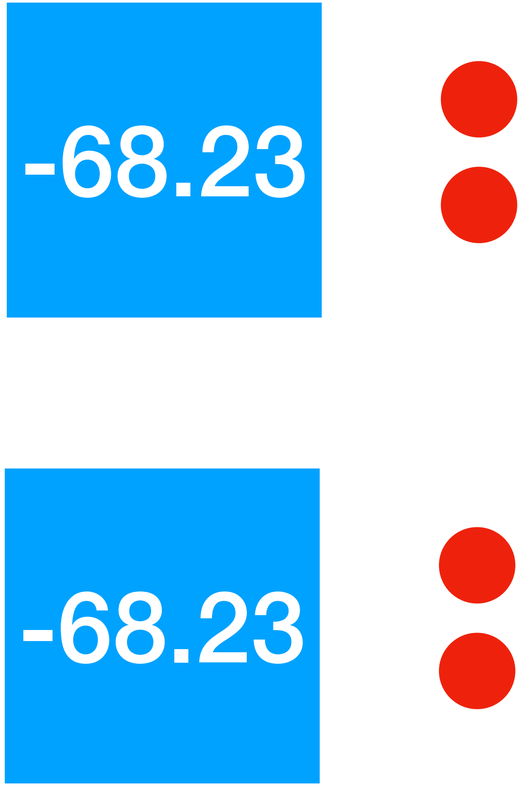

Multiply by 2

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-68.23

-68.23

Subtract 2

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-68.23

:

Subtract 2

-68.23

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-68.23

:

Divide by 2

-68.23

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-68.23

÷

Divide by 2

-68.23

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-68.23

Divide by 2

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-68.23

Subtract your original number

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Subtract your original number

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The answer is 1!

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Pick a number

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???

(Or not)

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???

Add 2

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???

Add 2

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???

Multiply by 2

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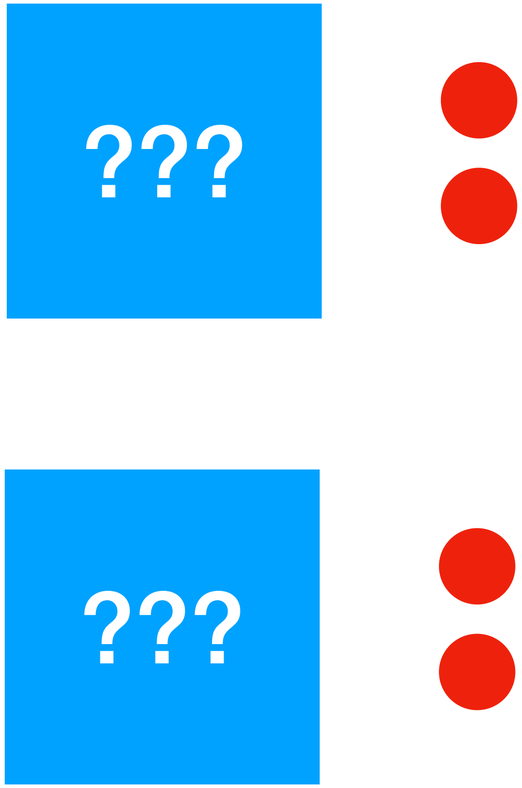

Multiply by 2

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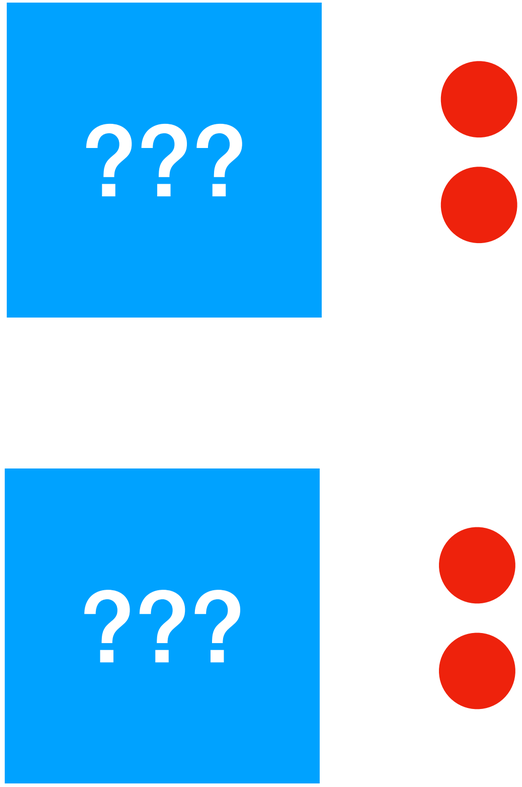

Subtract 2

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???

???

Subtract 2

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???

???

Divide by 2

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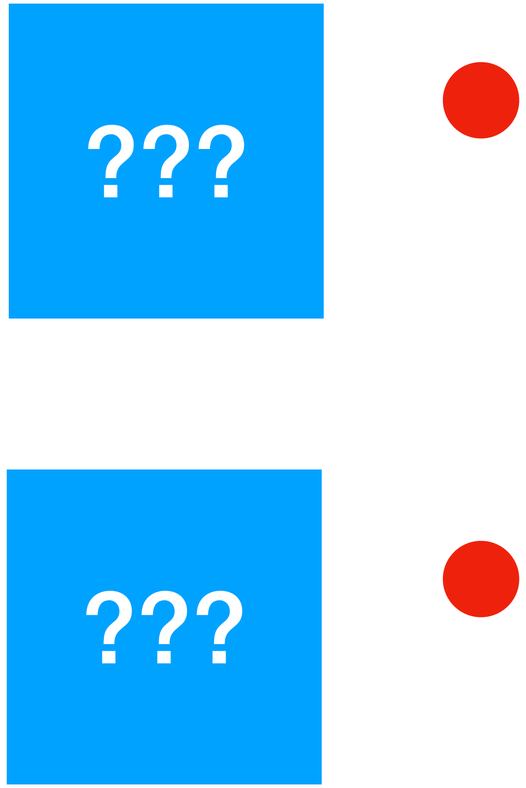

Divide by 2

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???

Divide by 2

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???

Subtract your original number

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Subtract your original number

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The answer is 1!

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Does this visualization convince you that the math magic trick ALWAYS works?

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“The human mind has never invented a labor-saving device equal to algebra.”
—J. Willard Gibbs

Was this mathematical experience…

a. 😀

b. 😐

c. 🙁

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To ruin the same experience:

To ruin the same experience:

$$\text{Show } ((x+2)\times 2-2)/2-x=1$$

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Facts *end*

Conjectures *extend*

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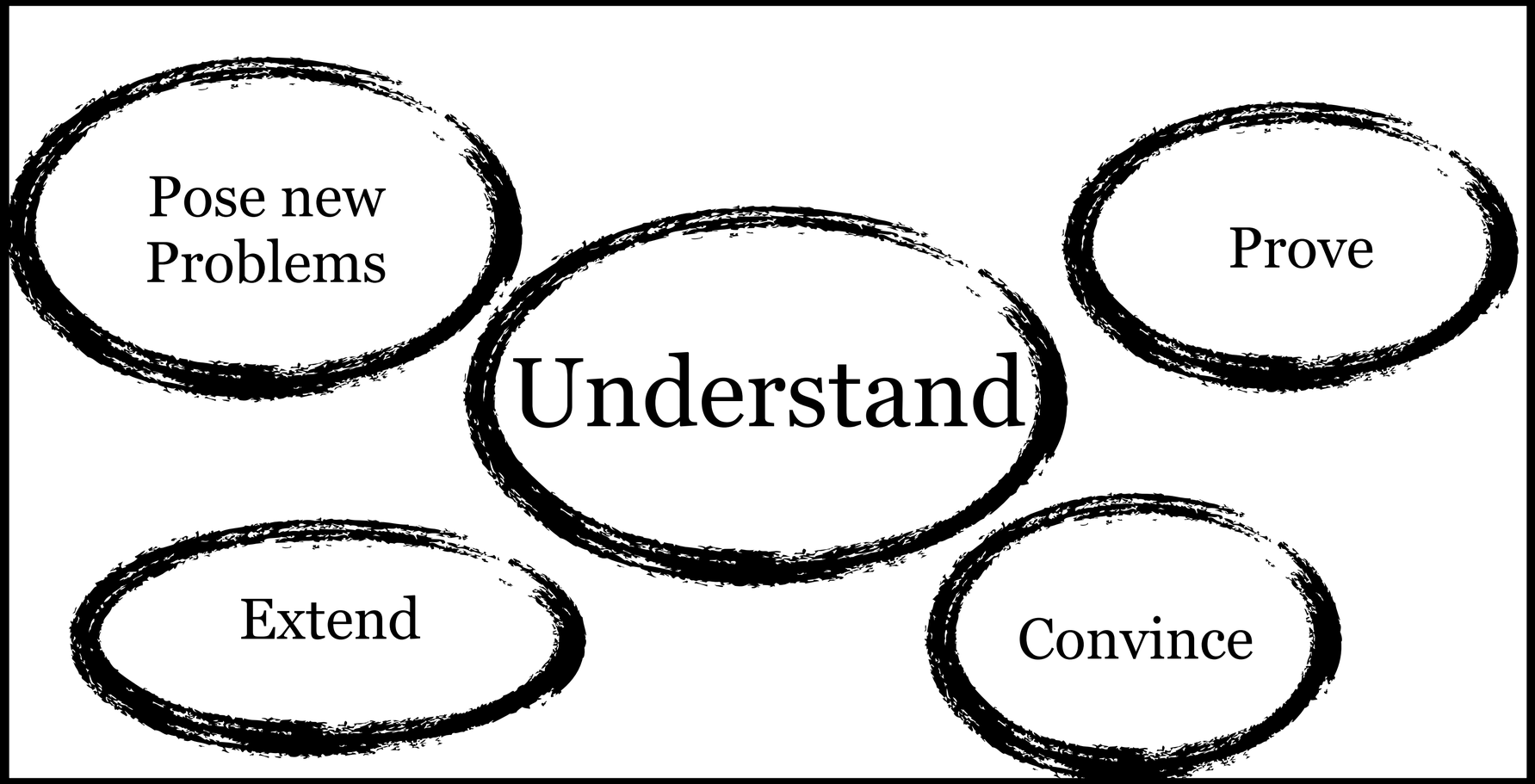

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Math Magic Trick

Pick any number.
•Add 2.
•Multiply by 2.
•Subtract 2.
•Divide by 2.

Then subtract your original number.

Your answer is ALWAYS 1!

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… and I can convince anyone of it, because…

But now I wonder…

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- What if all the 2s were 3s?

- What if all the 2s were 3s?
- Or 4s, or 10s, or 30s, or any number?

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- What if all the 2s were 3s?
- Or 4s, or 10s, or 30s, or any number?
- What if we changed the order of operations?

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“Don't just read it; fight it!

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"Don't just read it; fight it!

Ask your own questions,
look for your own examples,
discover your own proofs."

-Paul Halmos

What do students learn in math class?

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Official Answer

Facts & algorithms

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Develop
Tools

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Accidental Answer

"Math isn't for me."
"I can't do this."
"This is boring."
"When am I ever going to use this?"

What do students learn in math class?

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What do students learn in math class?

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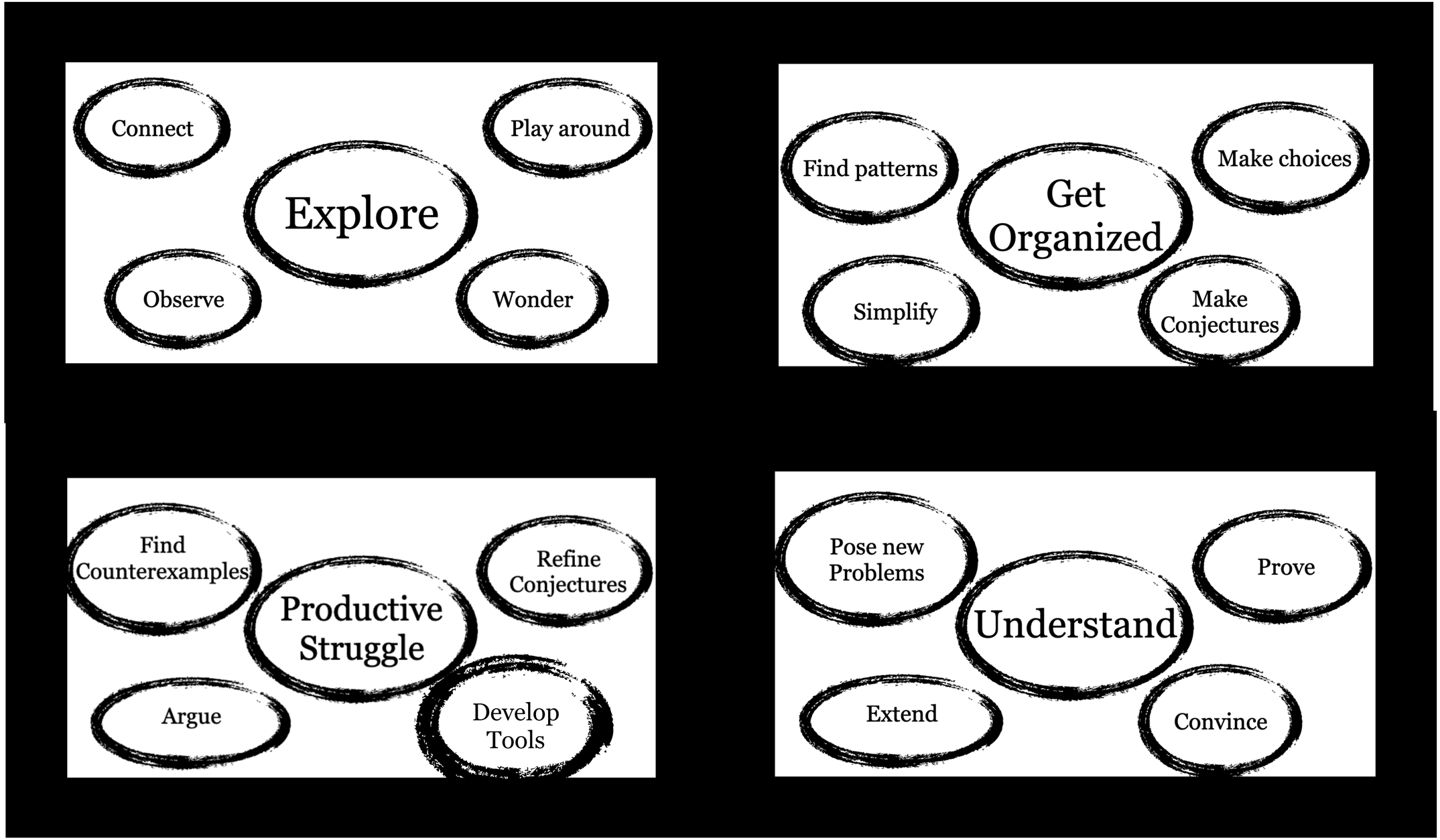

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We can invite students to play the way mathematicians play, and get:

Facts and algorithms

Courage

Joy

Kindness

Perseverance

Curiosity

Argument
& Critique

Ownership

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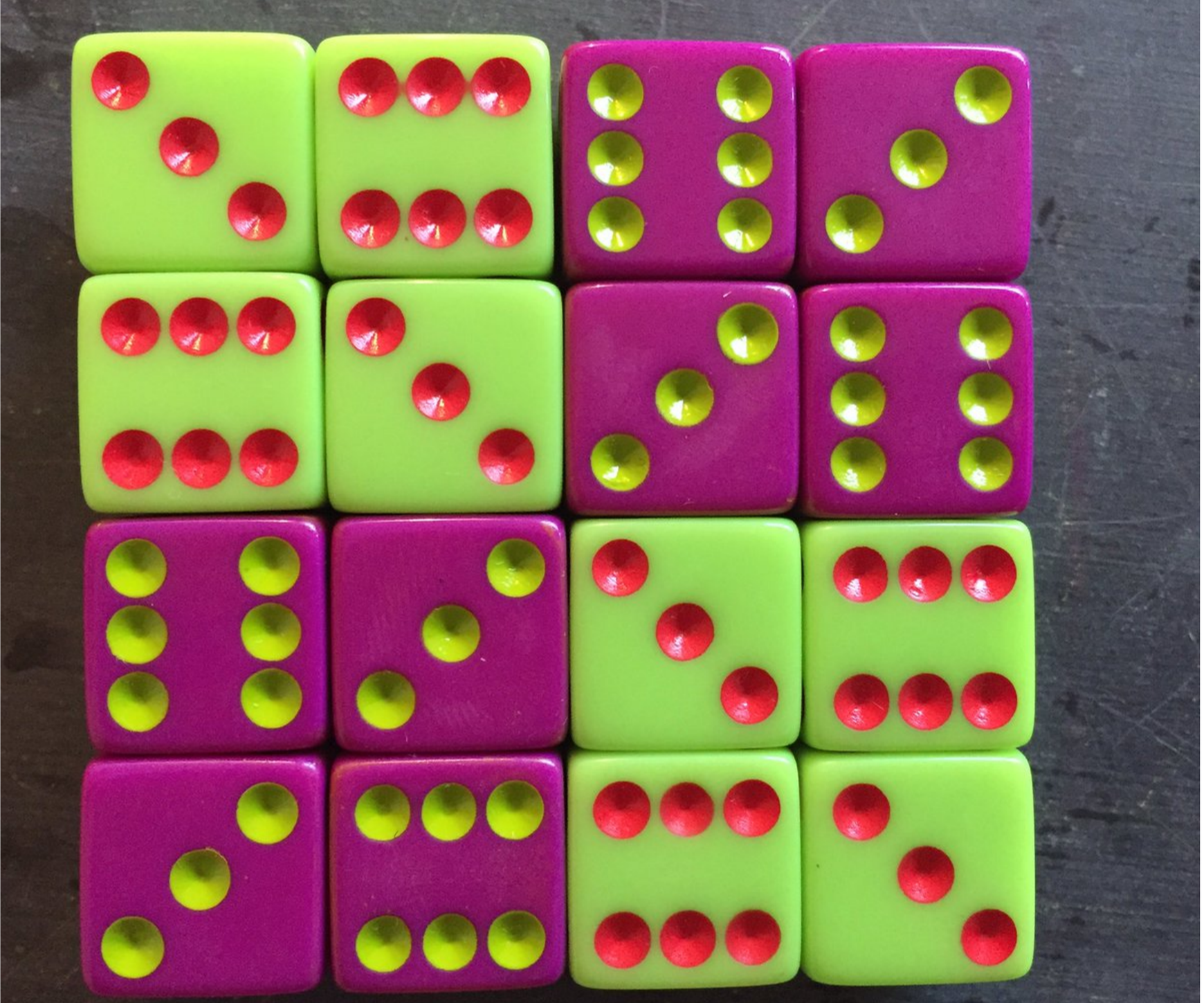

Openers

Games

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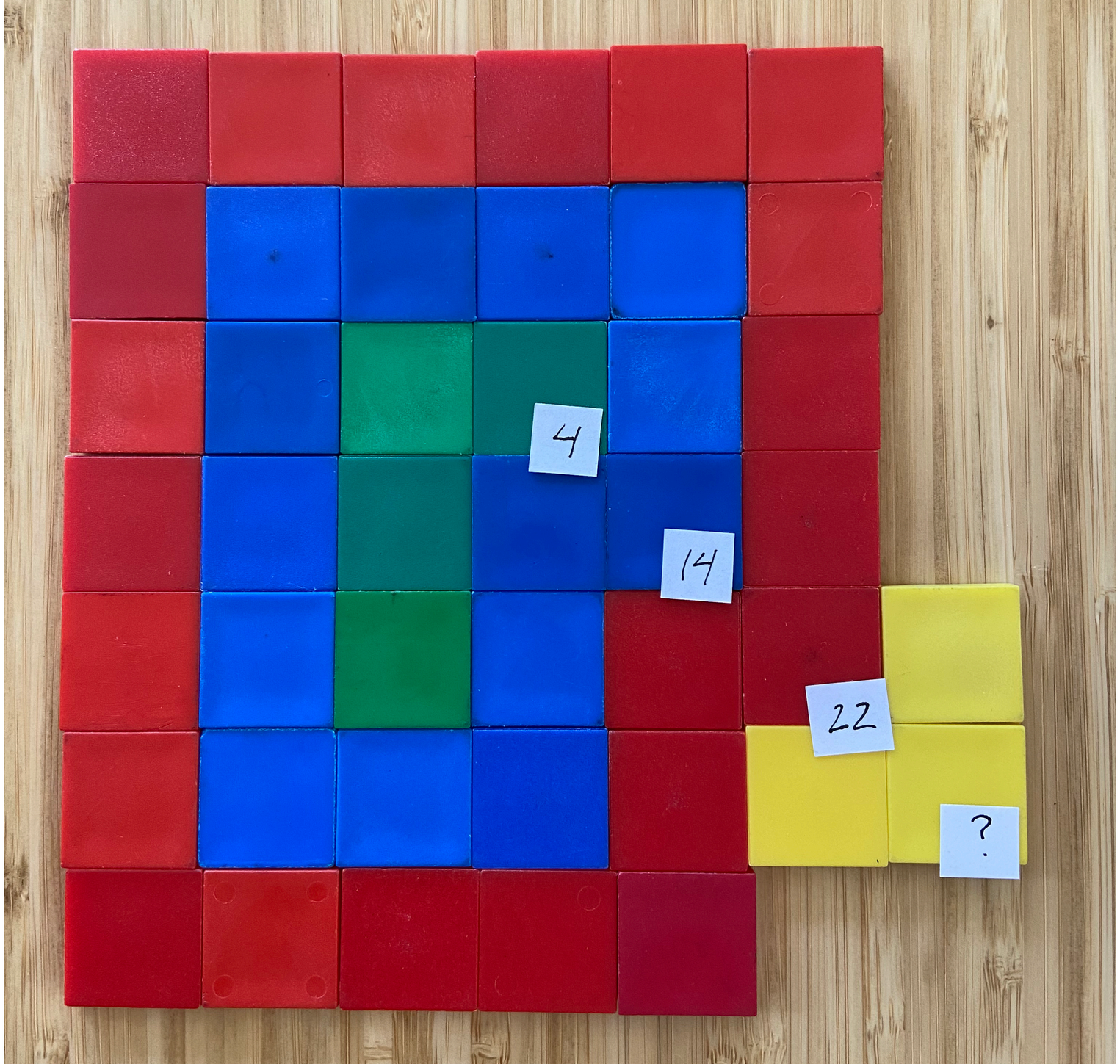
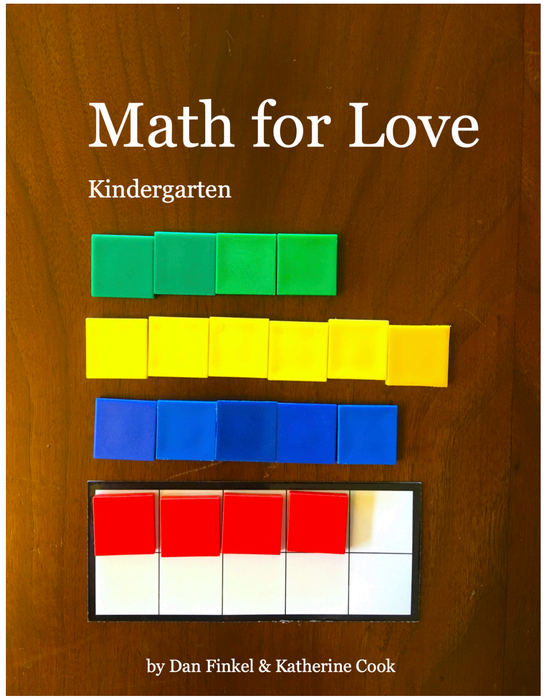
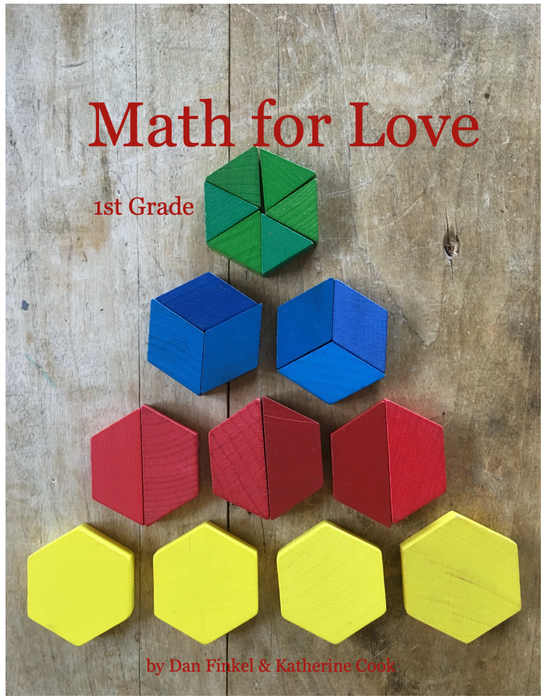
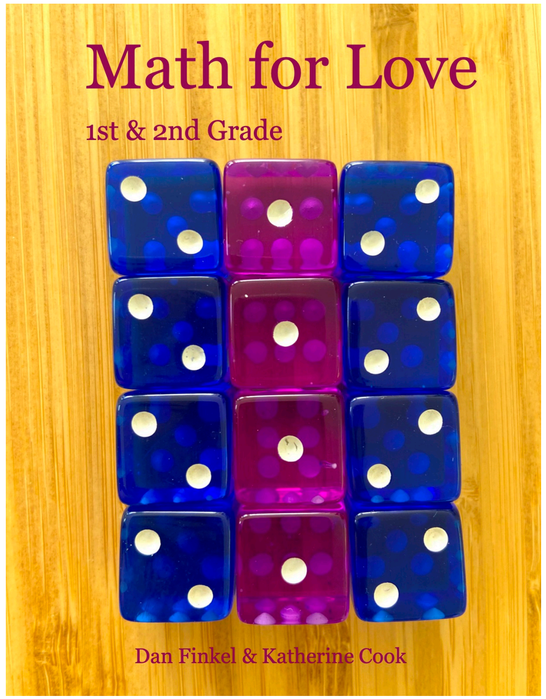
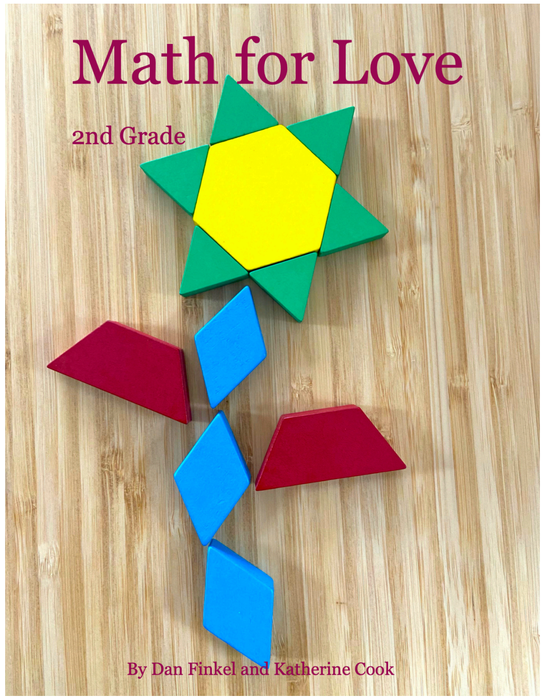


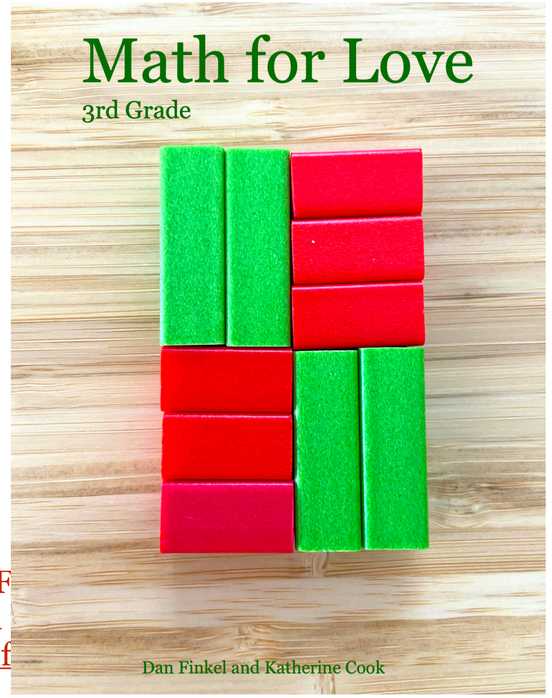
Rich tasks

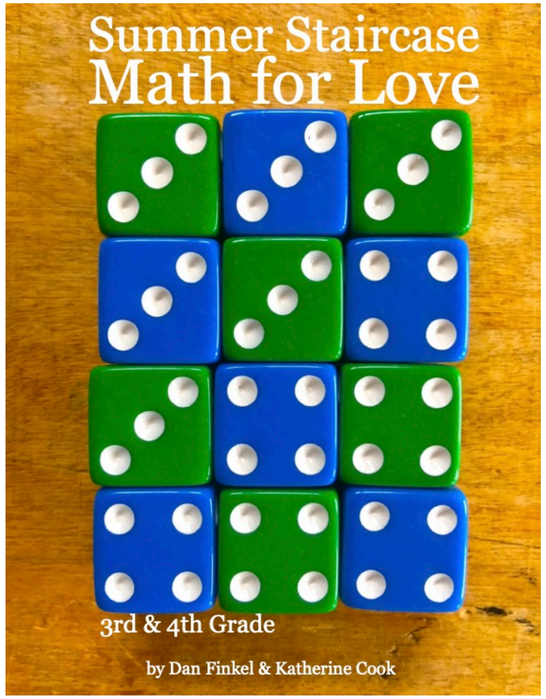


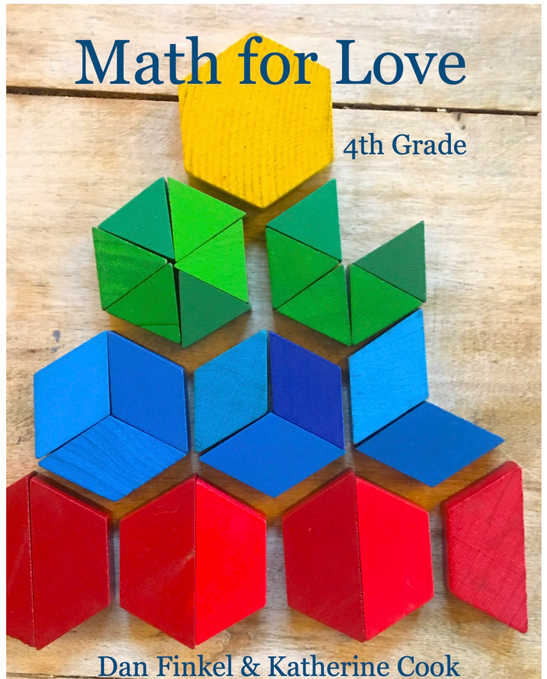












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Math
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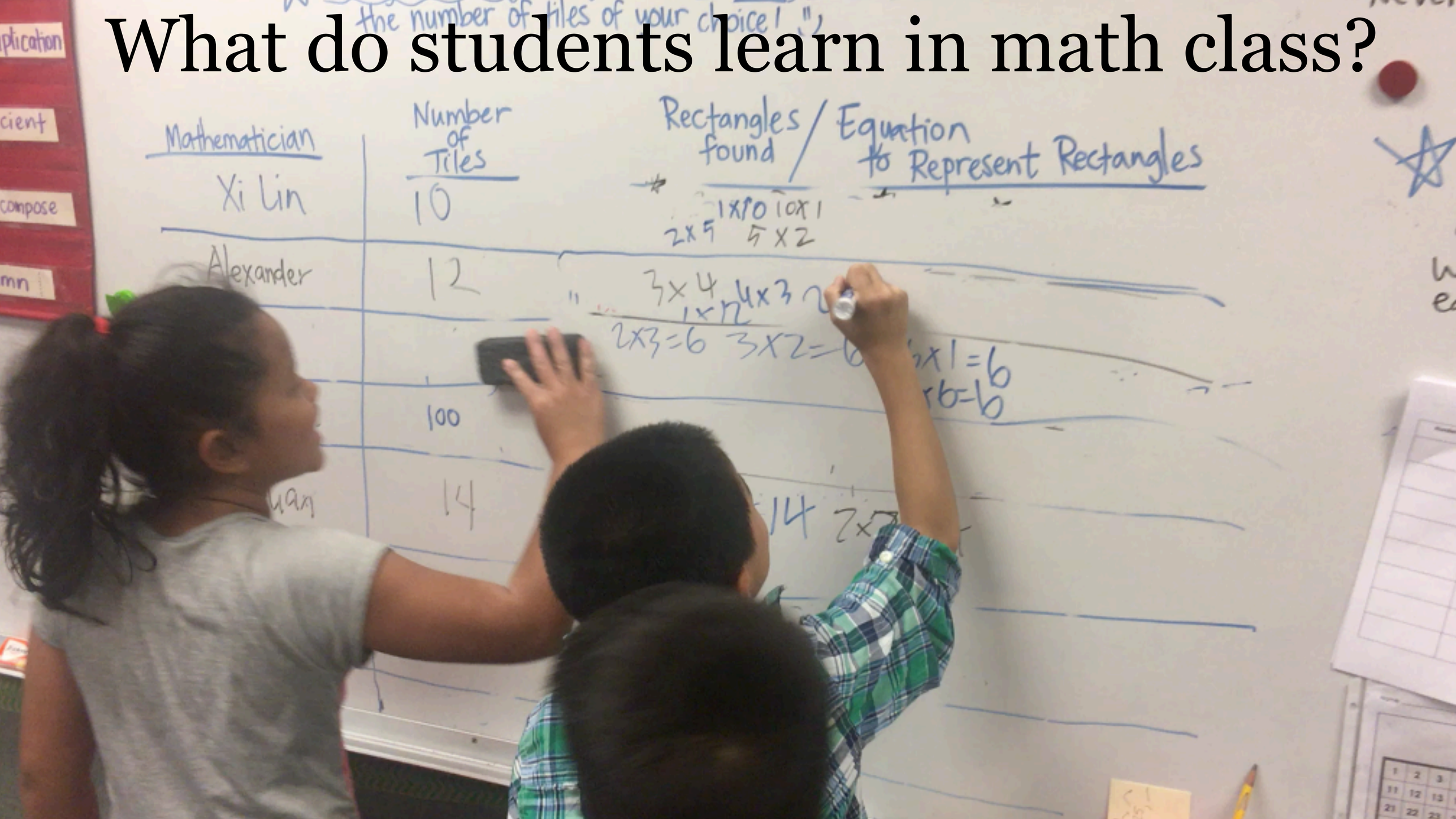
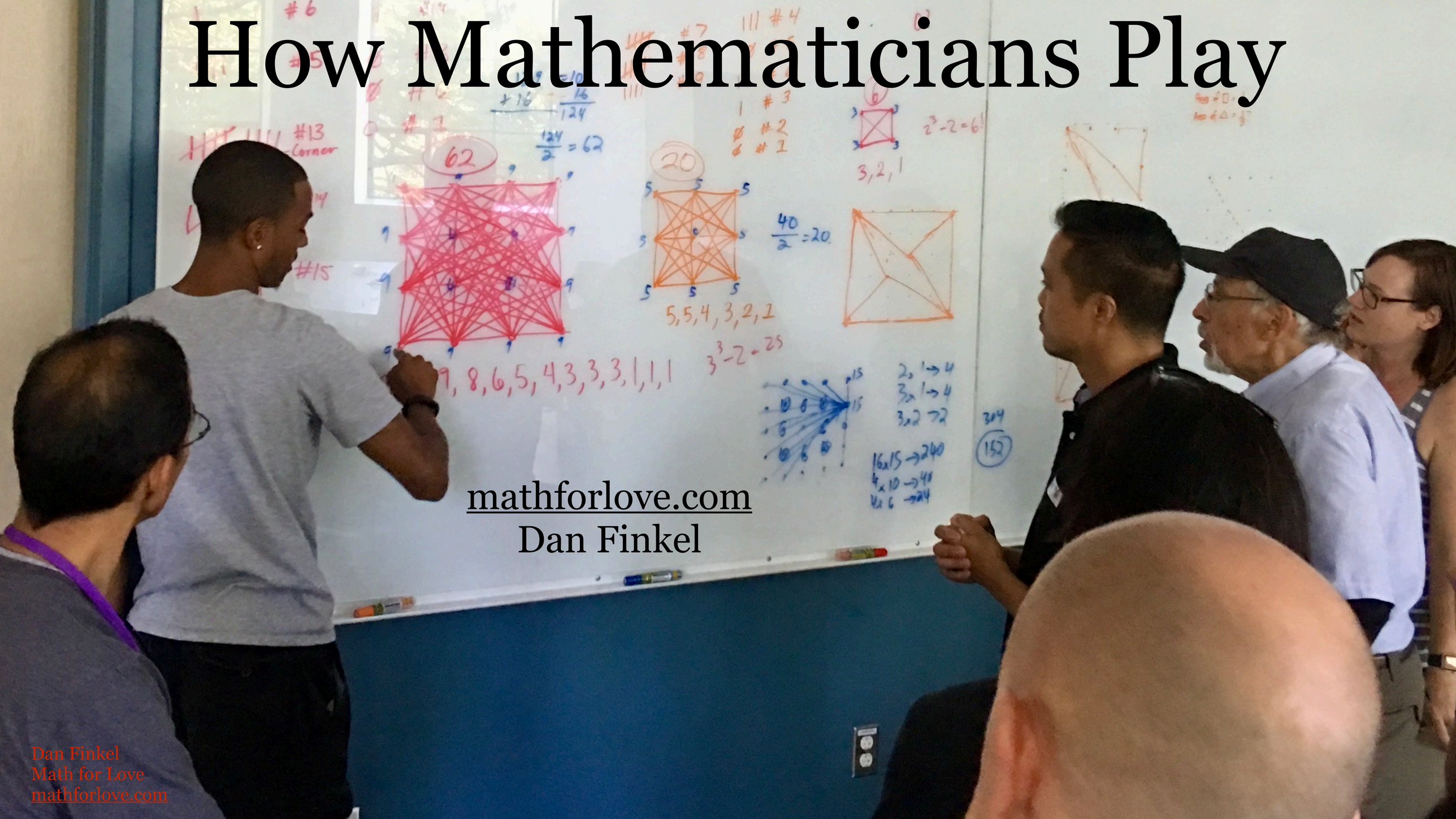
What do students learn in math class?



How Mathematicians Play



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