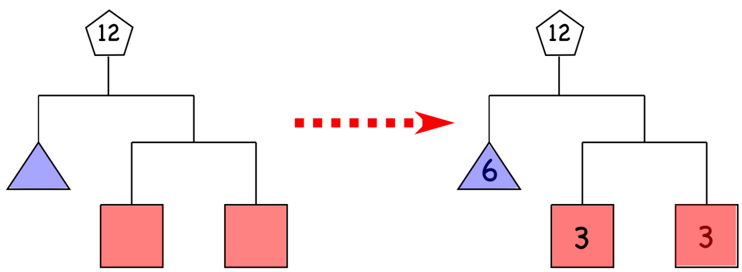
Puzzle of the Week

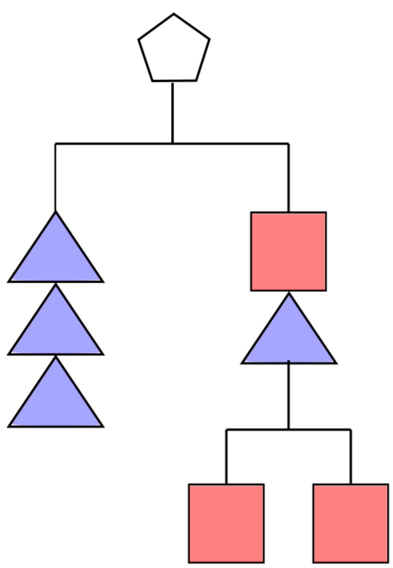
Balance Beam – 1

To balance, weights must be the same on opposite sides of a horizontal balance beam. The total weight is given above the balance beam.

In a given puzzle, figures of the same shape must have the same weight. However, it is allowed for different shapes to have the same weight.



THE CHALLENGE: The squares each have weight 2. Find the weight of each of the triangles and the total weight of all the figures.



EXPLORATION: Create balance beams for others to solve. Make sure there is enough information that they can be figured out.

Puzzle of the Week

Balance Beam – 1 - Notes

THE CHALLENGE: Replace the squares with 2’s. This means that the three triangles on the left side balance with a triangle plus 6 (three 2’s) on the right. For these two sides to be equal, two triangles must balance with the 6. So, each triangle has a weight of 3.

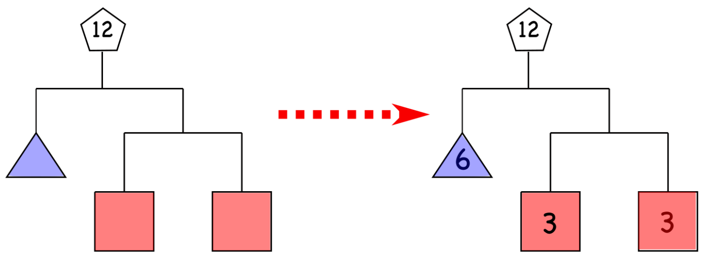
Replacing the triangles with 3’s means that there are a total of 9 on each side, which gives a grand total of 18 for the whole balance beam.

Puzzle of the Week

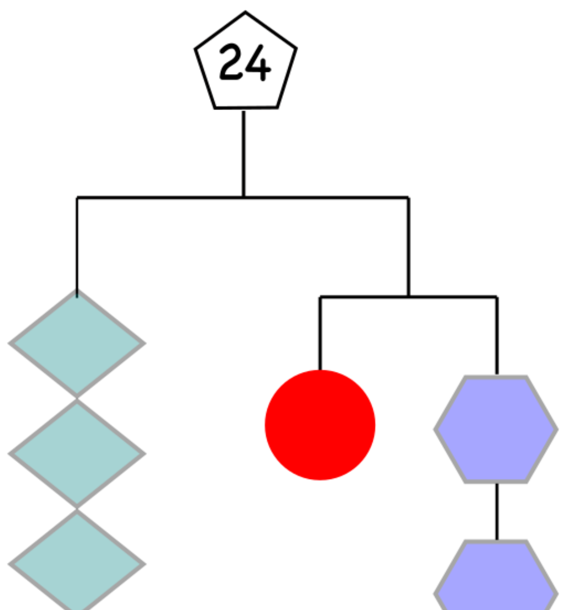
Balance Beam – 2

To balance, weights must be the same on opposite sides of a horizontal balance beam. The total weight is given above the balance beam.

For these puzzles, figures of the same shape must have the same weight. However, it is allowed for different shapes to have the same weight.



THE CHALLENGE: Find the weight of each of the diamonds, hexagons, and circles.



EXPLORATION: Create balance beams with at least three shapes for others to solve. Make sure there is enough information that they can be figured out.

Puzzle of the Week

Balance Beam – 2 - Notes

THE CHALLENGE: Because the two sides of the big beam are equal and add up to 24, each side must be 12.

The three diamonds on the left side add up to 12, so each diamond is 4.

The two sides of the smaller beam are equal and add up to 12. This means the red circle is 6 and the two hexagons add up to 6. Consequently, each hexagon is 3.

To summarize: a diamond is 4, a red circle is 6, and a hexagon is 3.

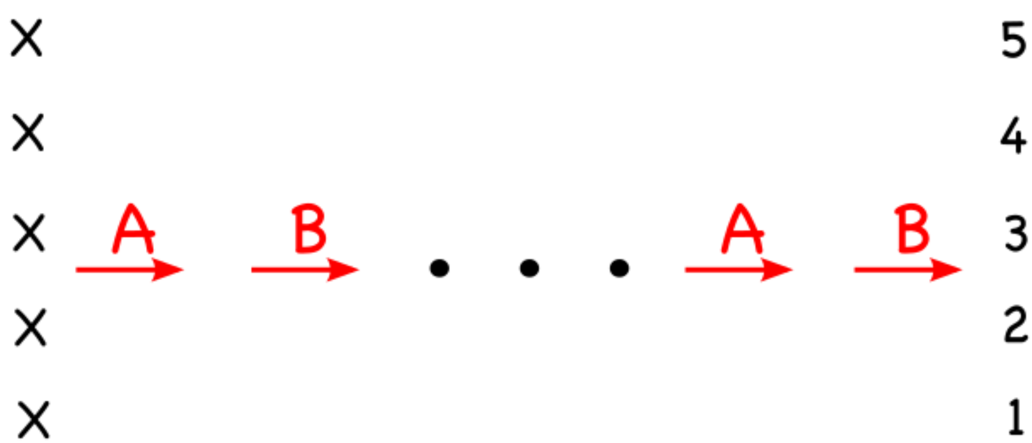
Puzzle of the Week

Card Deck Ordering

Two Steps: **Step A**: Remove the top card from a stack of cards and place it on the top of a discard pile. **Step B**: Move the new top card of the stack to the bottom of the stack.

If you start with a stack of cards ordered 1 - 3 - 2, repeatedly doing the Two Step process will result in a discard pile of cards in order from largest to smallest: 3 - 2 - 1.

THE CHALLENGE: Take cards numbered 1 to 5 and stack them so that if you repeat the Two Steps over and over with that stack, you will end up with a discard pile of cards in order from 5 down to 1.



EXPLORATION: Can you do this for decks with cards from 1 to 6, 1 to 7, or even higher? What patterns do you notice?

Puzzle of the Week

Card Deck Ordering – Notes

THE CHALLENGE & EXPLORATION: The difficulty with this puzzle is being systematic. For any size deck of cards, you can play around with it and eventually come up with the answer, and that is perfectly fine for a young child.

Let’s look for interesting patterns that make it easier.

Suppose you lay out the cards in order on the table. Here are the solutions for the first seven cases. Let’s see what we can learn from them. The numbers listed after the arrow give the order of the remaining cards after the first pass through the cards – that is, after each card has been touched just once.

1

1 2 -> 2

1 3 2 -> 3

1 3 2 4 -> 3 4

1 5 2 4 3 -> 5 4

1 4 2 6 3 5 -> 4 6 5

1 6 2 5 3 7 4 -> 6 5 7

If there are an even number of cards (say 6), then the odd positions are filled with the first half of the cards in order (3 in this case), and the other spots are filled using the solution for half as many cards only bumped up in value. In the example for 6, the odd spots are filled with 1, 2, 3, and the even spots are filled with 4, 6, 5 - the values 1, 3, 2 (the solution for a three-card deck) each increased by 3.

The pattern for an odd number of cards is a little trickier. As before, the odd spots are filled with the first roughly half of the numbers (1 to 4 in the case of 7). If you look at the examples, the first card after the arrow is going to be moved to the end, so it should be the card you want last in that sequence. After that observation, the answer proceeds as in the even case.

Puzzle of the Week

Coin Flipping – 1

A coin may be placed heads up or down in any empty spot. When a coin is placed, all coins touching its sides must be flipped over.

THE CHALLENGE: Fill this grid so all the coins end up heads up or heads down.

EXPLORATION: Put a few coins in other starting positions and see which of these work out. Can you find any patterns in which ones work and which ones don't?

Puzzle of the Week

Coin Flipping – 1 – Notes

THE CHALLENGE: This puzzle can be solved enjoyably and satisfactorily with a lot of trial and error. And that's a fine approach.

To reduce the trial and error, predict the number of flips an existing coin will have. For example, the coins in the upper left and lower right corners have two neighbors, so they will each be flipped twice (ultimately leaving them the same as they started). Similarly, the coin in the center has four neighbors, so it will be flipped four times and return to its original state. So, no matter the choices that are made, the three diagonal coins will all end up being heads down.

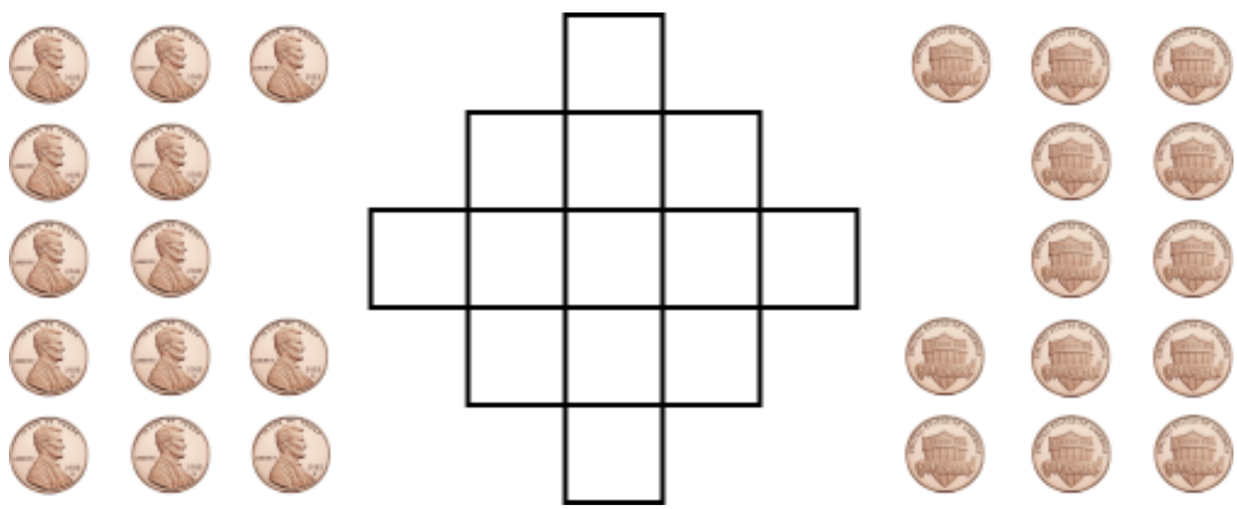
Once you know how to handle the diagonal coins, the rest of this small puzzle is pretty straightforward.

EXPLORATION: For any puzzle with three coins in a single row, column, or diagonal, to be doable the coins must all be heads up or heads down. For other arrangements, it will be determined by whether the initial number of empty slots around each coin is odd or even.

Puzzle of the Week
Coin Flipping – 2

A coin may be placed heads up or down in any empty spot. When a coin is placed, all coins touching its sides must be flipped over.

THE CHALLENGE: Fill this grid so all the coins end up heads up or heads down.



EXPLORATION: See what happens with other patterns of squares. Are there any that are impossible?

Puzzle of the Week

Coin Flipping – 2 – Notes

THE CHALLENGE: There are many simple strategies for doing this puzzle. Perhaps the easiest is to start by putting heads up coins in the four squares that have a single neighbor. Then, in the 3 by 3 empty box that remains, put heads down coins along a diagonal of the 3 by 3 box. Now you can finish solving this exactly as you did the "Coin Flipping - 1" puzzle.

A different strategy is to start at the top and fill them in order from top to bottom and left to right. Look at how many coins have yet to be filled in that will neighbor the current square - if that number is even, make that coin heads up, and if it is odd, make that coin heads down.

EXPLORATION: They should all be possible. It is just a matter of keeping track of evens and odds.

Puzzle of the Week

Combining Digits – 1 2 4 8

Here are some ways to get 0 and 1 using 1, 2, 4, and 8.

0 = 8 - 1 * 2 * 4

0 = 8 * 1 - 2 * 4

1 = 8 - 2 * 4 + 1

1 = 8 - 4 - 2 - 1

THE CHALLENGE: How many numbers can you get using each of the numbers 1, 2, 4, and 8 in any order, using addition, subtraction, and multiplication?

EXPLORATION: What happens with other groups of four numbers? What happens if you use the five numbers: 1, 2, 4, 8, and 16?

Puzzle of the Week

Combining Digits – 1 2 4 8 – Notes

THE CHALLENGE: Here are some solutions from 0 to 26. Of course, there are many more. Have fun comparing different people's solutions!

0 = 8 - 1 * 2 * 4
1 = 8 - 4 - 2 - 1
2 = 8 - 4 - 2 * 1
3 = 8 - 4 - 2 + 1
4 = 8 - 4 * (2 - 1)
5 = 8 - 4 + 2 - 1
6 = 8 - 4 + (2 * 1)
7 = 8 - 4 + 2 + 1
8 = 8 * (4 - 2 - 1)
9 = 8 + 4 - 2 - 1
10 = 8 + 4 - (2 * 1)
11 = 8 + 4 - 2 + 1
12 = 8 + 4 * (2 - 1)
13 = 8 + 4 + 2 - 1
14 = 8 + 4 + (2 * 1)
15 = 8 + 4 + 2 + 1
16 = 8 * (4 - (2 * 1))
17 = 8 * (4 - 2) + 1
18 = (8 + 1) * (4 - 2)
19 = 8 * 2 + 4 - 1
20 = 8 * 2 + 4 * 1
21 = 8 * 2 + 4 + 1
22 = 8 * (4 - 1) - 2
23 = (8 - 2) * 4 - 1
24 = (8 - 2) * 4 * 1
25 = (8 - 2) * 4 + 1
26 = 8 * (4 - 1) + 2

Puzzle of the Week

Combining Digits – Easy as 1 2 3 4

Here are some ways to get 0 and 1 using 1, 2, 3, and 4.

0 = 1 + 4 - 2 - 3

0 = (3 - 1 - 2) * 4

1 = (2 - 1) * (4 - 3)

1 = 4 - 3 * (2 - 1)

THE CHALLENGE: How many numbers can you get using each of the numbers 1, 2, 3, and 4 in any order, using addition, subtraction, and multiplication?

EXPLORATION: How many more numbers can you make if you are also allowed to make two-digit numbers with the digits? For example, 26 = 24 + 3 - 1.

Puzzle of the Week

Combining Digits – 1 2 3 4 – Notes

THE CHALLENGE: Here are some solutions, with one missing, from 0 to 21. Of course, there are many more. Have fun comparing different people's solutions!

0 = 1 + 4 - 2 - 3
1 = 4 - 3 * (2 - 1)
2 = (2 - 1) + (4 - 3)
3 = 4 – (3 – 2 * 1)
4 = 4 * (3 – 2*1)
5 = 4 + (3 – 2*1)
6 = 4 + 3 – 2 + 1
7 = 4 + 3 * (2 – 1)
8 = 4 * (3 – (2 – 1))
9 = 3 * (4 – (2 – 1))
10 = 1 + 2 + 3 + 4
11 = 3 * 4 – (2 - 1)
12 = 3 * 4 * (2 – 1)
13 = 3 * 4 + (2 – 1)
14 = 2 * (3 + 4) * 1
15 = 2 * (3 + 4) + 1
16 = 2 * (1 + 3 + 4)
17 =
18 = 4 * (3 + 1) + 2
19 = 4 * (2 + 3) – 1
20 = 4 * (2 + 3) * 1
21 = 4 * (2 + 3) + 1

EXPLORATION: Here are solutions up to 37 making use of two-digit numbers.

18 = 23 - 4 - 1
19 = 23 - 4 * 1
20 = 24 - 3 - 1
21 = 24 - 3 * 1
22 = 2 * 13 - 4
23 = 4 * 3 * 2 - 1
24 = 4 * 3 * 2 * 1
25 = 2 * 14 - 3
26 = 24 + 3 - 1.
27 = 23 + 1 * 4
28 = 23 + 4 + 1
29 = 32 - 4 + 1
30 = (4 + 1) * 3 * 2
31 = 34 - 2 - 1
32 = 34 - 2 * 1
33 = 34 - 2 + 1
34 = 34 * (2 - 1)
35 = 34 + 2 - 1
36 = 34 + 2 * 1
37 = 34 + 2 + 1

Puzzle of the Week

Combining Digits – Four 4's

Here are some ways to get 0 and 1 using four 4's.

0 = 4 - 4 + 4 -4

0 = 44 - 44

1 = 4 / 4 * 4 / 4

1 = 4 / 4 + 4 - 4

1 = 44 / 44

THE CHALLENGE: How many numbers can you get using four 4's using addition, subtraction, multiplication, division and creating double-digit numbers?

Puzzle of the Week

Combining Digits – Four 4's – Notes

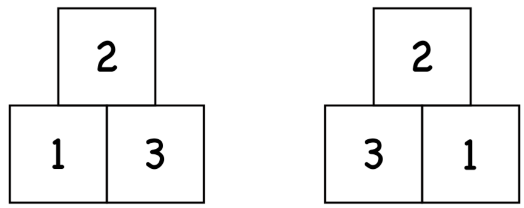
THE CHALLENGE: Here are some solutions, with a few missing, from 0 to 16. Of course, there are many more. Have fun comparing different people's solutions!

0 = 4 - 4 + 4 -4
1 = 4 / 4 + 4 - 4
2 = 4 / 4 + 4 / 4
3 = (4 + 4 + 4) / 4
4 = 4 + ((4 – 4) * 4)
5 = (4 * 4 + 4) / 4
6 = 4 + (4 + 4) / 4
7 = 4 + 4 – (4 / 4)
8 = (4 + 4) + 4 – 4
9 = 4 + 4 + (4 / 4)
10 = (44 – 4) / 4
11 =
12 = 4 * (4 – 4 / 4)
13 =
14 =
15 = 4 * 4 – (4 / 4)
16 = 4 * 4 + 4 – 4

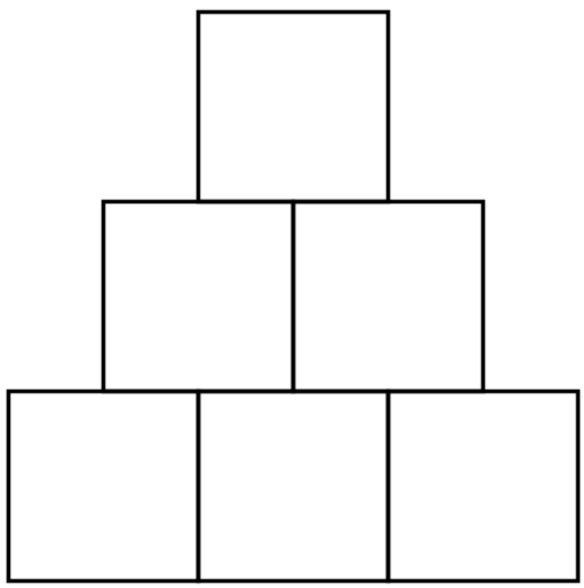
Puzzle of the Week

Difference Pyramids – 1

These pyramids are called *Difference Pyramids*. The number on top is the difference of the two numbers below.



THE CHALLENGE: Place the numbers from 1 to 6 to make a Difference Pyramid.



1 2 3 4 5 6

EXPLORATION: Find different ways this can be done. Are some of these essentially the same?

Puzzle of the Week

Difference Pyramids – 1 – Notes

THE CHALLENGE & EXPLORATION: A good start is to realize that because 6 cannot be the difference of two cards, it must go on the bottom row.

Next, the only way 5 can be the difference is if it is above the 6 and the 1. So, either 5 goes directly above the 6 (with the 1 next to the 6), or 5 is in the bottom row.

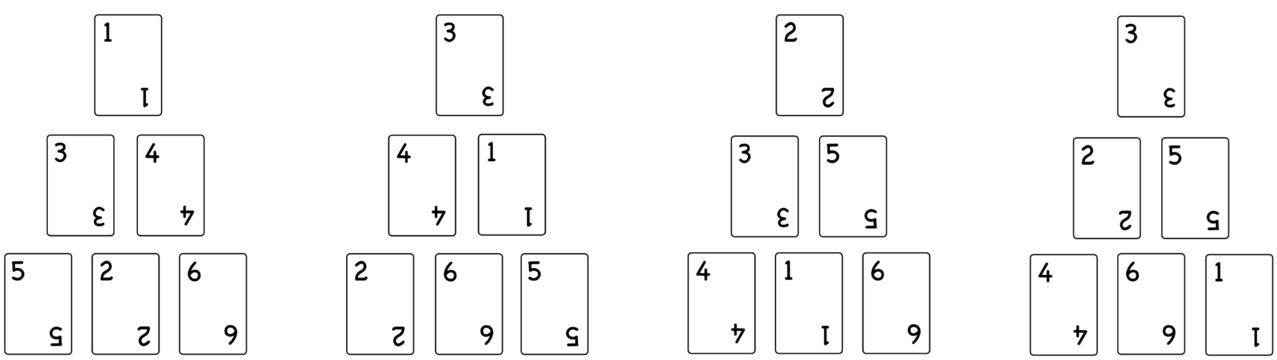
At this point it is useful to consider what makes solutions different. Because the mirror image of any solution is also a solution, it makes sense to ignore those. Ignoring mirror images will reduce the number of solutions to consider by half.

For example, we can assume that not only is the 6 in the bottom row, but it is either in the middle or the right side of the bottom row - if it were on the left side, we could take the mirror image of the whole puzzle and put it on the right side.

Using this thinking, the bottom row can have five possible layouts (up to using mirror images): 5 X 6, X 5 6, X 6 5, X 1 6, X 6 1.

At this point, it is a matter of working through the various possibilities. The only 5 X 6 that works is 5 2 6. It turns out X 5 6 never works. The only X 6 5 is 2 6 5. The only X 1 6 is 4 1 6, and the only X 6 1 is 4 6 1.

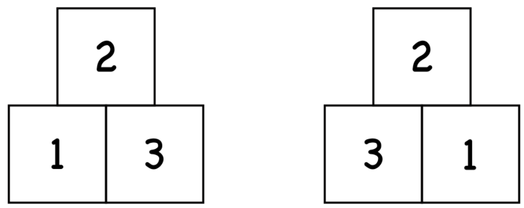
So, ignoring mirror images, there are four solutions:



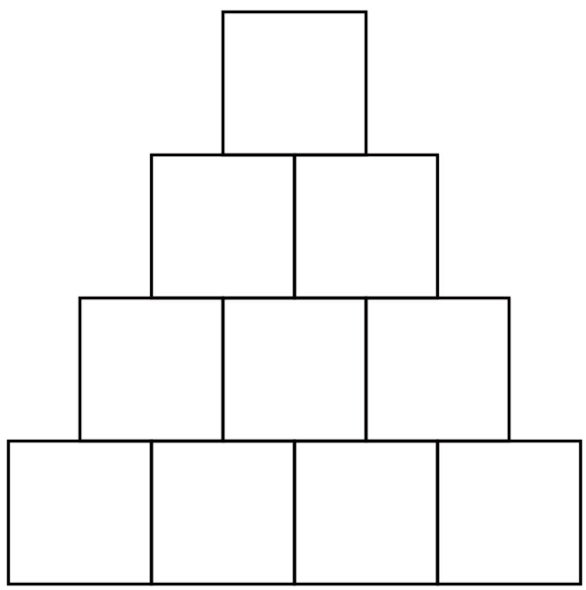
Puzzle of the Week

Difference Pyramids – 2

These pyramids are called *Difference Pyramids*. The number on top is the difference of the two numbers below.



THE CHALLENGE: Place the numbers from 1 to 10 to make a Difference Pyramid.



1 2 3 4 5 6 7 8 9 10

EXPLORATION: Play with even larger pyramids.

Puzzle of the Week

Difference Pyramids – 2 – Notes

THE CHALLENGE: Because 10 cannot be the difference of two cards, it must go on the bottom row. Similarly, either 9 is in the bottom row or it is in the next-to-the-bottom row above the 1 and the 10. The 8 and 7 cards are also good cards to focus on to get rid of possibilities.

This means the bottom row looks like one of the following (ignoring mirror images):

X Y 9 10,   X 9 Y 10,   9 X Y 10,   X Y 10 9,   X 9 10 Y,   9 X 10 Y,   X Y 1 10,   X 1 10 Y, X Y 10 1

That is a lot of possibilities to consider!

Fortunately, if you consider where 8 and 7 can go, the possibilities are reduced to the following list (assuming we haven't missed any). It is easy to finish each of these once you have the bottom row.

8 3 10 9,   9 3 10 8,   6 1 10 8,   8 1 10 6

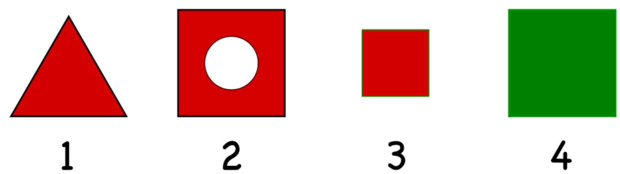
Pyramids of size 15, 21, or higher are left to the truly dedicated. Good luck and enjoy!

Puzzle of the Week

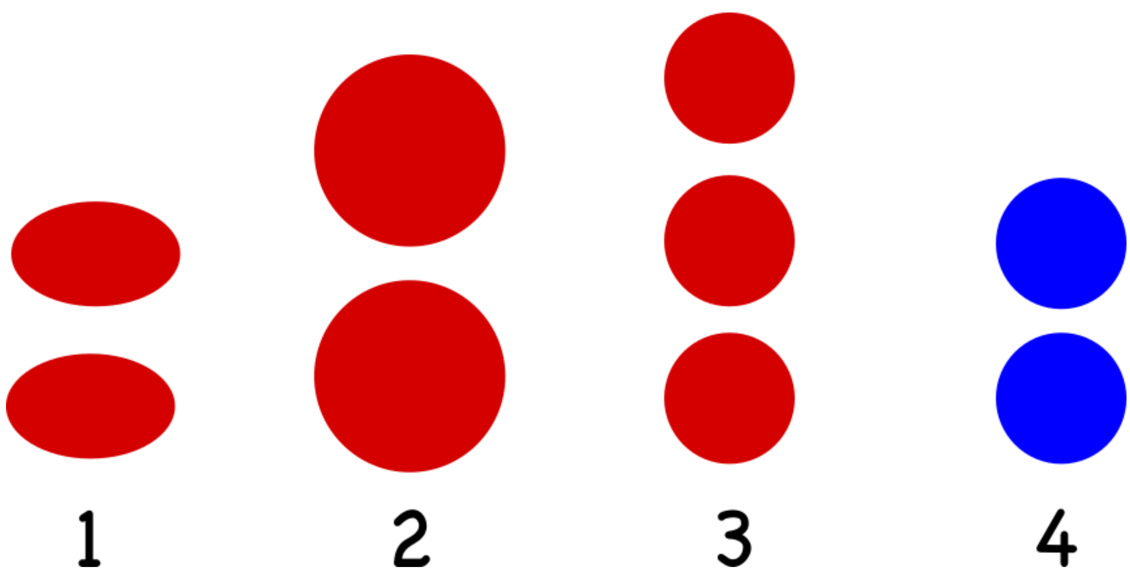
Each of These is Not Like the Others - 1

The objects in this group share some properties, but the objects also differ in some distinctive ways.

1. This is a triangle, the other three are squares.
2. This has a whole in it, the other three are solid.
3. This is a small shape, the other three are larger.
4. This is green, the other three are red.



THE CHALLENGE: For each of these next four groups of objects, describe a property that the remaining three groups have that it does not.



EXPLORATION: Find other groups of four things where each group of three of them has a property the fourth does not. Can you think of a group of five things like this?

Puzzle of the Week

Fill in the Blanks – 1

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use the numbers from 1 to 8 at most once to fill in these blanks.

$$\square + \square = \square + \square = \square - \square$$

1 2 3 4 5 6 7 8

EXPLORATION: Explore other number ranges. What happens if you use 1 to 7, 1 to 9, or 1 to 10? How do things change if you use 0 to 7?

Puzzle of the Week

Fill in the Blanks – 1 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good stuff, and there is no reason to avoid it.

To be more systematic, the key observation is that the subtraction drives the solution.

For a difference of 5, the sums must be 1 + 4 and 2 + 3, and that uses up all the numbers from 1 to 4.

For a difference of 6, if the subtraction is 7 - 1 or 8 - 2, there aren't two ways of getting a sum of 6 without using a 1 (if it's 7 - 1) or a 2 (if it's 8 - 2).

So, the difference must be 7, and the last subtraction must be 8 - 1. Without using a 1, the sum of 7 can be achieved as 2 + 5 or 3 + 4, and that's our single solution.

EXPLORATION: We saw above that 1 to 7 cannot work.

Using the range 1 to 9 opens up more possibilities involving the 9.

- 9 - 1 = 8 gives 2 + 6 = 3 + 5 = 8.
- 9 - 2 = 7 gives 1 + 6 = 2 + 5 = 7
- 9 - 3 = 6 gives 1 + 5 = 2 + 4 = 6

Using the range 1 to 10 now allows us to use the 10.

- 10 - 1 = 9 gives 2 + 7 = 3 + 6 = 4 + 5 = 9
- 10 - 2 = 8 gives 1 + 7 = 3 + 5 = 8
- 10 - 3 = 7 gives 1 + 6 = 2 + 5 = 7
- 10 - 4 = 6 gives 1 + 5 = 2 + 4 = 6
- 10 - 5 = 5 gives 1 + 4 = 2 + 3 = 5

Putting 0 in the range produces quite a few surprises. There are solutions for the ranges as small as 0 to 5!

- 1 + 4 = 2 + 3 = 5 - 0
- 1 + 5 = 2 + 4 = 6 - 0
- 0 + 5 = 2 + 3 = 6 - 1
- 0 + 4 = 1 + 3 = 6 - 2
- 0 + 3 = 1 + 2 = 7 - 4

- 1 + 6 = 2 + 5 = 3 + 4 = 7 - 0
- 0 + 6 = 2 + 4 = 7 - 1
- 0 + 5 = 1 + 4 = 7 - 2

Puzzle of the Week

Fill in the Blanks – 2

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 1 to 9 at most once to fill in these blanks.

$$\square = \square + \square = \square + \square + \square$$

1 2 3 4 5 6 7 8 9

EXPLORATION: Explore other number ranges. What happens if you use 1 to 8 or 1 to 10?

Puzzle of the Week

Fill in the Blanks – 2 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good stuff, and there is no reason to avoid it.

To be more systematic, you want to look for a driver or focus that helps reduce the search. For this puzzle, that driver is the overall sum - we need to keep it small. The smallest sum for the three numbers is 1 + 2 + 3 = 6, but that leaves the other two numbers to add up to at least 4 + 5 = 9. To balance those two things, we can add them both up and divide by two - the smallest the single number on the left can be is (1 + 2 + 3 + 4 + 5) / 2 = 7 ½. So the sum will either be 8 or 9, which we can try out individually.

If it's 8, we have 1 solution:

- 8 = 1 + 7 does not work
- 8 = 2 + 6 = 1 + 3 + 4 works!
- 8 = 3 + 5 = does not work

For 9, we have 3 solutions:

- 9 = 1 + 8 = 2 + 3 + 4 works!
- 9 = 2 +7 = 1 + 3 + 5 works!
- 9 = 3 + 6 does not work
- 9 = 4 + 5 = 1 + 2 + 6 works!

EXPLORATION: We saw above that 1 to 8 gives one solution. The range from 1 to 10 will give us many more new solutions.

- 10 = 1 + 9 = 2 + 3 + 5 works!
- 10 = 2 + 8 = 1 + 2 + 7 = 1 + 3 + 6 = 1 + 4 + 5 - 3 ways!
- 10 = 3 + 7 = 1 + 4 + 5 = 2 + 3 + 5 - 2 ways!
- 10 = 4 + 6 = 1 + 2 + 7 = 2 + 3 + 5 - 2 ways!

Puzzle of the Week

Fill in the Blanks – 3

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 1 to 8 at most once to fill in these blanks.

$$\square = \square + \square = \square + \square = \square + \square$$

1 2 3 4 5 6 7 8

EXPLORATION: Explore other number ranges. What happens if you use 1 to 9, 0 to 7, or 0 to 8?

Puzzle of the Week

Fill in the Blanks – 3 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good stuff, and there is no reason to avoid it.

To be more systematic, use that you have three pairs of numbers that have the same sum. To make that triple sum as small as possible, we could attempt to use the numbers 1 through 6 for them. The sum of the numbers 1 through 6 is 21, and if you break that into three equal parts, that would be a sum of 7 for each individual sum.

Let's look at the two possibilities - the sum is either 7 or 8. For each number, there are only three possible ways to produce that as a sum, and we quickly find the two solutions.

7 = 1 + 6 = 2 + 5 = 3 + 4

8 = 1 + 7 = 2 + 6 = 3 + 5

EXPLORATION: Let's explore the three suggested ranges to look at.

The range 1 to 9: The only new possibilities introduced by using 1 to 9 is having 9 as the sum. That creates

9 = 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5

We can select any three of those four ways to add up to 9.

The range 0 to 7: You can think of 0 to 7 as subtracting 1 from each member of the range 1 to 8. Subtracting 1 from both members of a sum will reduce the sum by 2. The sum can now be one of 5, 6, or 7.

5 = 1 + 4 = 2 + 3 - there aren't enough ways

6 = 1 + 5 = 2 + 4 - there aren't enough ways

7 = 1 + 6 = 2 + 5 = 3 + 4 - the same as before.

It turns out that the 0 doesn't help. To use 0 in a sum would force the digit to be used both in the sum and as the total, which isn't allowed.

The range 0 to 8: As we just found out for the range 0 to 7, the 0 isn't going to help. The only solution we find here is the same one we got before: 8 = 1 + 7 = 2 + 6 = 3 + 5.

Puzzle of the Week

Fill in the Blanks – 4

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 0 to 9 at exactly once to fill in these blanks.

$$\square + \square = \square + \square = \square + \square = \square + \square = \square + \square$$

0 1 2 3 4 5 6 7 8 9

EXPLORATION: Can you solve similar puzzles that break up a number range into common sums? How about four pairs using the numbers 0 to 7 or 1 to 8? How about 3 triplets from 0 to 8 or 1 to 9? How about 2 groups of 5 for the numbers from 0 to 9? Do you see any patterns for when it works and when it doesn't?

Puzzle of the Week

Fill in the Blanks – 4 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good stuff, and there is no reason to avoid it.

If you want to be more systematic, the first question is: What is the common sum for these pairs of numbers? The five pairs have the same sum, and when we add them all up we get the same thing as adding the numbers up from 0 to 9. The sum from 0 to 9 is 45, so when we divide that by 5 we get 9 - the sum for each pair must be 9. Once that is established, the rest is simple:

0 + 9 = 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5.

EXPLORATION: The first step is to see whether the sum of the range of numbers can be broken into that many equal pieces. Also, note that it makes no difference whether we start at 0 or 1, so we'll just look at starting at 0.

0 to 7 using 4 pairs: The sum of the numbers from 0 to 7 is 28. Dividing 28 into 4 pairs gives a sum of 7 for each pair. This is simple enough: 7 = 0 + 7 = 1 + 6 = 2 + 5 = 3 + 4.

0 to 2n - 1 using n pairs: After looking at 0 to 7 and 0 to 9, the pattern is clear: write n as the n possible sums.

0 to 8 using 3 triplets: The sum from 0 to 8 is 36. Dividing 36 into 3 triplets gives a sum of 12 for each triplet. The triplets will be largely driven by the three largest numbers (6, 7, 8), no two of which can be in a triplet together. This produces triplets (8, 0, 4), (7, 2, 3), and (6, 1, 5). This could also be done as (8, 1, 3), (7, 0, 5), and (6, 2, 4).

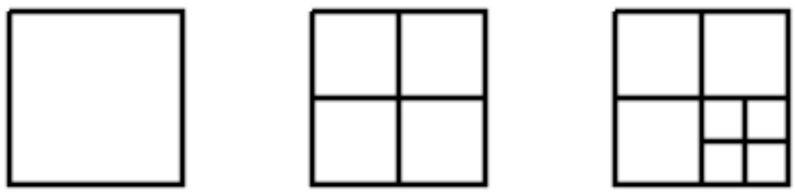
0 to 9 using 2 groups of 5: The sum from 0 to 9 is 45. 45 cannot be divided evenly into two equal groups!

The very interested child may want to go exploring further to see more examples of when this works and when it doesn't. What fun!

Puzzle of the Week

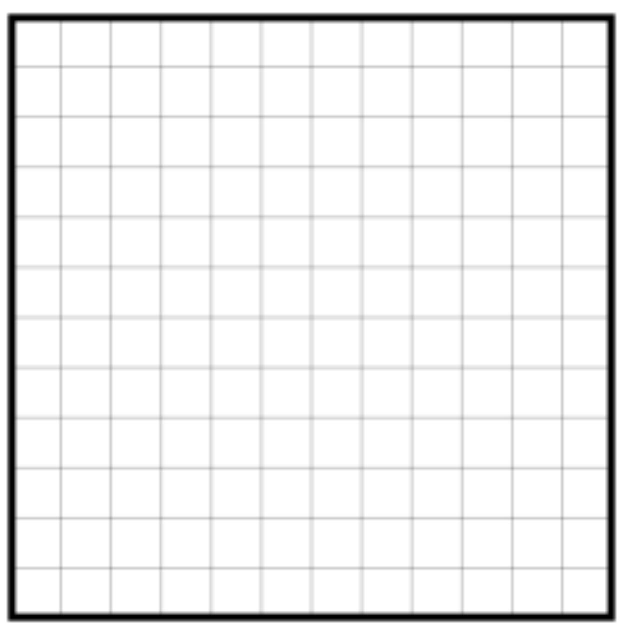
Filling Squares with Squares

Here is how to fill one large square with 1, 4, or 7 squares.



1   4   7

THE CHALLENGE: Find other square counts for filling a large square. Can you do it for 2, 3, 5, 6, 8, 9, or 10 squares?



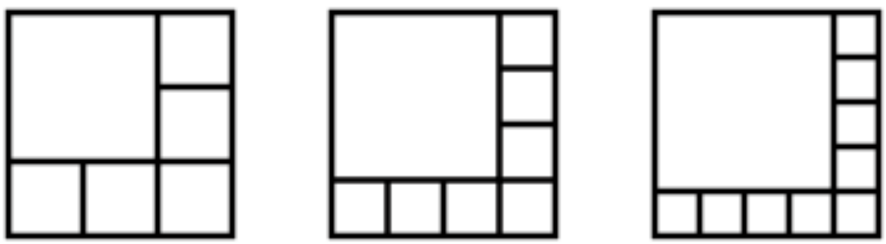
EXPLORATION: When possible, find more than one way to get some of these numbers.

Puzzle of the Week
Filling Squares with Squares – Notes

THE CHALLENGE: Here is a systematic way of building up various counts.

Squares: A good place to start is with square numbers. It is easy to fill a square with 1, 4, and 9 squares by filling it, respectively, in a 1 by 1, 2 by 2, and 3 by 3 pattern.

Even Numbers: After some experimentation, you can create patterns for 6, 8, 10, or any other larger even number as follows. Start with a large square and then put smaller squares around two sides of it as in these drawings.



Replacing One Square: The next big step is to see that any one square in a solution can be replaced by any other existing solution. For example, this was done in producing the pattern for 7 in the introduction – the square in the bottom right corner for “4” was replaced with four smaller squares to produce “7.”.

Whenever one square is replaced by four squares, that will increase the total square count by 3. Start with the list of solutions using square numbers and even numbers: 1, 4, 6, 8, 9, 10, 12, 14, and 16, and then add 3 to each entry on that list to get 4, 7, 9, 11, 12, 13, 15, and 17. Combining these two lists gives all the possibilities up through 17: 1, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17. Using these ideas, every number above 17 is easy enough, and so we reach the conclusion that:

Answer: Every number is possible except 2, 3, and 5.

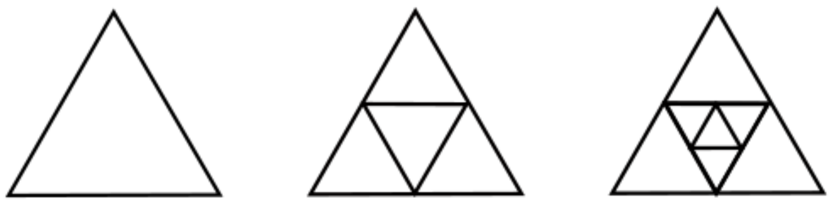
EXPLORATION: Some of these numbers can be produced in more than one way. For example, 9 can be done as a 3 by 3 pattern or as 6 plus three more.

There are many other patterns that can be created that are not made in the same way as these examples. For example, you can start with a 4 by 4 square and group 2 by 2 collections of squares into single 2 by 2 squares - each time you do that you will reduce the total count by 3.

Puzzle of the Week

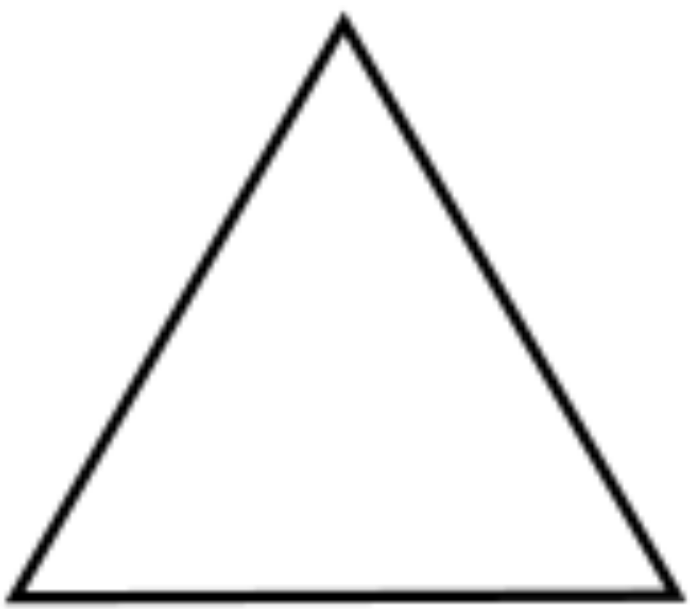
Filling Triangles with Triangles

Here's how to fill one large triangle with 1, 4, or 7 triangles..



1 4 7

THE CHALLENGE: Find other triangle counts for filling a large triangle. Can you do it for 2, 3, 5, 6, 8, 9, or 10 triangles?



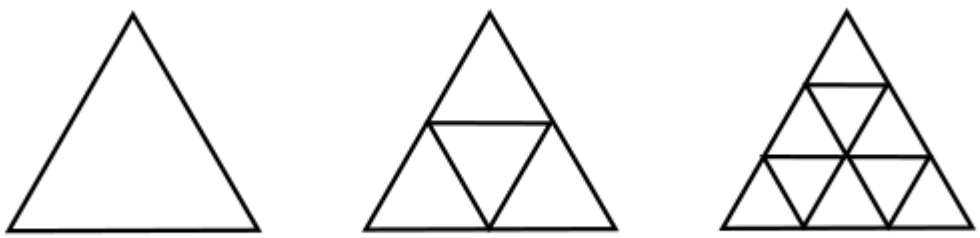
EXPLORATION: When possible, find more than one way to get some of these numbers.

Puzzle of the Week

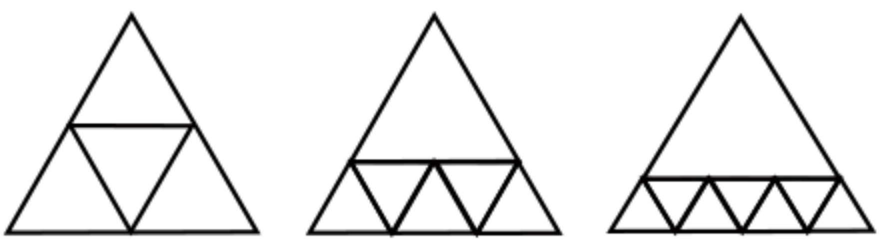
Filling Triangles w/ Triangles – Notes

THE CHALLENGE: Here is a systematic way of building up various counts.

Squares: A good place to start is with square numbers. It is easy to fill a triangle with 1, 4, and 9 triangles by filling it with the regular pattern used in the following illustration.



Even Numbers: After some experimentation, you can create patterns for 6, 8, 10, or any other larger even number as follows. Start with a large triangle and then put smaller triangles along one side.



Replacing One Triangle: The next big step is to see that any one triangle in a solution can be replaced by any other existing solution. For example, this was done in producing the pattern for 7 in the introduction – the triangle in the center for "4" was replaced with four smaller triangles to produce "7.".

Whenever one triangle is replaced by four triangles, that will increase the total triangle count by 3. Start with the list of solutions using square numbers and even numbers: 1, 4, 6, 8, 9, 10, 12, 14, and 16, and then add 3 to each entry on that list to get 4, 7, 9, 11, 12, 13, 15, and 17. Combining these two lists gives all the possibilities up through 17: 1, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17. Using these ideas, every number above 17 is easy enough, and so we reach the conclusion that:

Answer: Every number is possible except 2, 3, and 5.

EXPLORATION: Some of these numbers can be produced in more than one way. For example, 9 can be done as a 3 by 3 pattern or as 6 plus three more. Of course, there are other interesting ways to fill out a triangle to discover.

Puzzle of the Week

Letter Substitutions – 1

Rules:

1. A letter represents a digit from 0 to 9, and has the same value throughout a single puzzle.
2. No number can start with the digit 0.
3. Within a puzzle, different letters must have different values.

$$
\begin{array}{rr} & 8 \\ + & A \\ \hline B & 2 \end{array}
\quad \Rightarrow \quad
\begin{array}{rr} & 8 \\ + & 4 \\ \hline 1 & 2 \end{array}
$$

THE CHALLENGE: Find the value of C, D, E, F and G in these puzzles.

$$
\begin{array}{rr} & C \\ + & 8 \\ \hline & D \end{array}
\qquad
\begin{array}{rr} & E \\ + & E \\ \hline & 8 \end{array}
\qquad
\begin{array}{rr} & F \\ + & F \\ \hline G & 4 \end{array}
$$

EXPLORATION: Make some letter substitution puzzles for your friends to solve.

Puzzle of the Week
Letter Substitutions – 1 – Notes

THE CHALLENGE: Because these involve simple calculations with small numbers, they are straightforward to solve.

In the C + 8 = D, the sum must be less than 10. C cannot be 0 because that would break the rule of not having numbers start with 0. Therefore C is 1 and D is 9, which gives the answer: 1 + 8 = 9.

E must be half of 8, so E is 4. The answer is: 4 + 4 = 8.

This problem involves an important insight about adding: if you add two single-digit numbers, including possibly a carry, the result cannot be larger than 19, so the carry is always either 0 or 1. For this problem, the carry must be 1, so G is 1. F is half of 14, so F is 7. The answer is: 7 + 7 = 14.

EXPLORATION: Here are a few, slightly more challenging, letter substitution puzzles to play with.

H + 4 = KK: K must be 1, so the problem becomes H + 4 = 11, which forces H = 7. The answer is: 7 + 4 = 11.

M + M + 8 = MN: As a carry, M could be 1 or 2. However, if M is 2 then 2 + 2 + 8 must be at least 20, which it isn't. Therefore, M is 1 and the answer becomes 1 + 1 + 8 = 10.

P + P + 4 = T4: To preserve the ones digit, 4, it must be true that the sum of P + P ends in a 0. Therefore, P + P is 0 or 10. If P + P = 0, then P = 0 and that is not allowed. So, the answer is 5 + 5 + 4 = 14.

V + V + 7 = W: The only way that V + V + 7 can be less than 10 is if V is 1. The answer is 1 + 1 + 7 = 9.

Puzzle of the Week

Letter Substitutions – 2

Rules:

1. A letter represents a digit from 0 to 9, and has the same value throughout a single puzzle.
2. No number can start with the digit 0.
3. Within a puzzle, different letters must have different values.

$$
\begin{array}{rr} & 8 \\ + & A \\ \hline B & 2 \end{array}
\quad \Rightarrow \quad
\begin{array}{rr} & 8 \\ + & 4 \\ \hline 1 & 2 \end{array}
$$

THE CHALLENGE: Find the value of C, D, E, F, G, and H in these puzzles.

$$
\begin{array}{rr} & C \\ + & 2 \\ \hline D & E \end{array}
\qquad\qquad
\begin{array}{rr} & F \\ + & G \\ \hline F & H \end{array}
$$

EXPLORATION: Make some letter substitution puzzles for your friends to solve.

Puzzle of the Week

Letter Substitutions – 2 – Notes

THE CHALLENGE: In problems with more letters, it is often helpful to rewrite the problem replacing each letter as you discover its value.

These problems involve an important insight about adding: if you add two single-digit numbers, including possibly a carry, the result cannot be larger than 19, so the carry is always either 0 or 1.

In the first problem, as a leading digit D cannot be 0, so it must be 1. C + 2 must be at least 10, so C is 8 or 9. If C is 9, then DE would be 11 – this would cause D and E to have the same value, which is not allowed. Therefore, C is 8, and the answer is: 8 + 2 = 10.

The second starts off the same way. F must be 1. The problem becomes 1 + G = 1H. The only way 1 + G can be 10 or higher is for G to be 9. The answer becomes: 1 + 9 = 10.

EXPLORATION: Here are a few more letter substitution puzzles to play with.

J + J + K = K0: As a carry, K must be 1 or 2. If K is 1, then J + J + 1 is an odd number, which cannot end in 0. Therefore, K is 2. Now, J + J + 2 = 20 forces J to be 9. The answer is: 9 + 9 + 2 = 20.

L + L + L = M2: Three times L ends in 2 forces L to be 4 and M to be 1. The answer is: 4 + 4 + 4 = 12.

N + N + N = P4: Three times N ends in 4 forces N to be 8 and P to be 2. The answer is: 8 + 8 + 8 = 24.

R + TT = R7: Adding R forces a carry, so R must be one more than T. R + T is 17 and the two numbers are one apart, so this must be 9 + 8 = 17. The answer is 9 + 88 = 97.

V + ZV = VZ: Notice that V + V ends in Z and has a carry. List out the possibilities so you can see them:

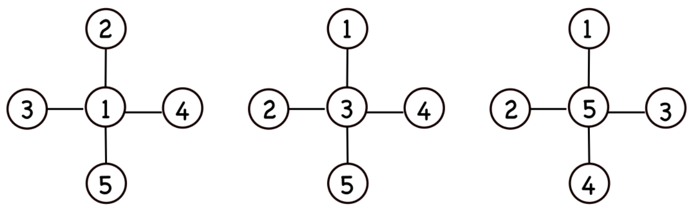
5 + 5 = 10, 6 + 6 = 12, 7 + 7 = 14, 8 + 8 = 16, 9 + 9 = 18.

Because of the carry, V must be one more than Z. This forces V = 9 and Z = 8. The answer is: 9 + 89 = 98.

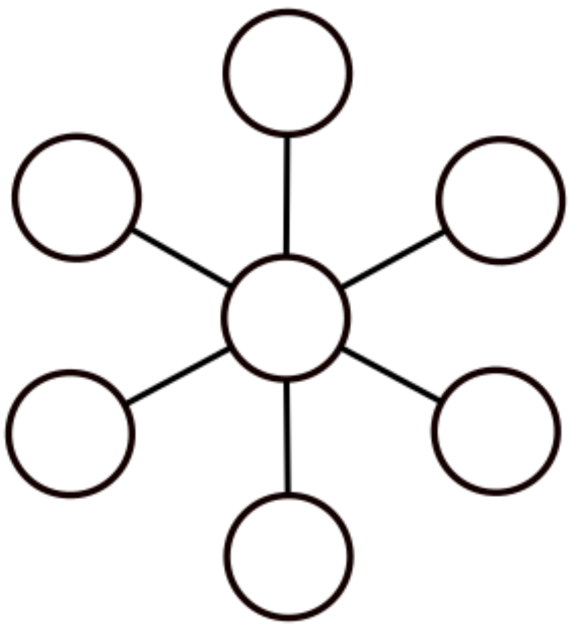
Puzzle of the Week

Magic Flowers – 1

The sums in a *Magic Flower* are the same for all straight lines. These Magic Flowers use numbers from 1 to 5.



THE CHALLENGE: Use the numbers from 1 to 7 to make Magic Flowers.



1 2 3 4 5 6 7

EXPLORATION: Play around with what happens with Magic Flowers with more than three lines.

Puzzle of the Week

Magic Flowers – 1 – Notes

THE CHALLENGE: Use this puzzle after the Magic Pluses puzzle.

As with so many of these puzzles, it is perfectly fine for students to play around with this in an unstructured way until they come across a solution.

Most of these “equal sum” puzzles can be attacked by adding up some of the straight lines. In the case of this puzzle, add up the three directions - this will include all the numbers once, plus the central circle’s number two extra times. The sum of 1 to 7 is 28. So, the sum of the three lines is 28 + 2x1, 28 + 2x2, 28 + 2x3, 28 + 2x4, 28 + 2x5, 28 + 2x6, or 28 + 2x7. Of those, only 30 = 28 + 2x1, 36 = 28 + 2x4, and 42 = 28 + 2x7 are divisible by 3. Dividing them by 3 tells us that the common sums are either 10 = 30 / 3, 12 = 30 / 3, or 14 = 30 / 3.

Let’s look at those three cases.

Common Sum of 10: The central circle will be 1. Making a sum of 10 with a 1 in the center means the other two numbers add up to 9. So, the three directions are: (2 1 7) - (3 1 6) - (4 1 5).

Common Sum of 12: The central circle will be 4. Making a sum of 12 with a 4 in the center means the other two numbers add up to 8. So, the three directions are: (1 4 7) - (2 4 6) - (3 4 5).

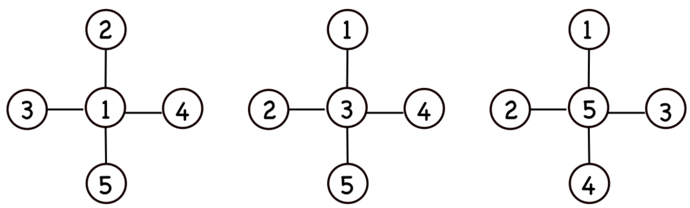
Common Sum of 14: The central circle will be 7. Making a sum of 14 with a 7 in the center means the other two numbers add up to 7. So, the three directions are (1 7 6) - (2 7 5) - (3 7 4).

EXPLORATION: Look at the Notes page for Magic Flowers - 2.

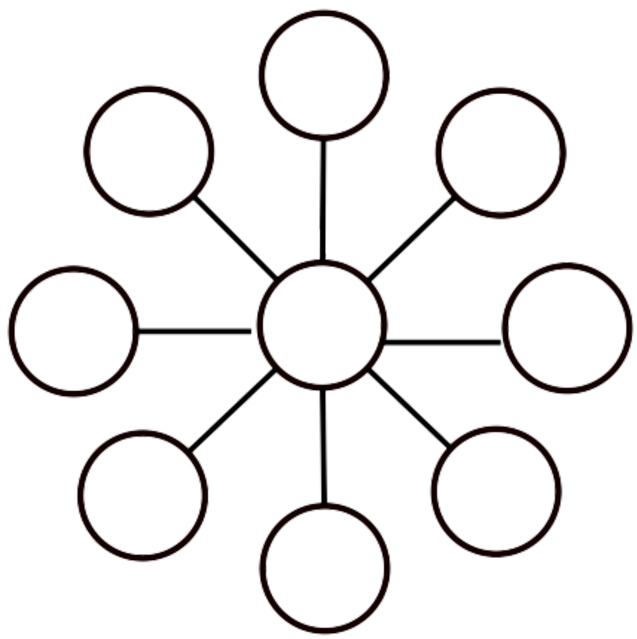
Puzzle of the Week

Magic Flowers – 2

The sums in a *Magic Flower* are the same for all straight lines. These Magic Flowers use numbers from 1 to 5.



THE CHALLENGE: Use the numbers from 1 to 9 to make Magic Flowers.



1 2 3 4 5 6 7 8 9

EXPLORATION: Looking at the solutions for Magic Flowers 1 and 2, what do you expect the solutions to be for Magic Flowers with even more line?

Puzzle of the Week

Magic Flowers – 2 – Notes

THE CHALLENGE: As with so many of these puzzles, it is perfectly fine for students to play around with this in an unstructured way until they come across a solution.

Most of these “equal sum” puzzles can be attacked by adding up some of the straight lines. In the case of this puzzle, add up the four directions - this will include all the numbers once, plus the central circle’s number three extra times. The sum of 1 to 9 is 45. So, the possible sums of the three lines are 45 + 3x1 through 45 + 3x9. Of those, only 48 = 45 + 3x1, 60 = 45 + 3x5, and 72 = 45 + 3x9 are divisible by 4. Dividing them by 4 tells us that the common sums are either 12 = 48 / 4, 15 = 60 / 4, or 18 = 72 / 4.

Let’s look at those three cases.

Common Sum of 12: The central circle will be 1. Making a sum of 12 with a 1 in the center means the other two numbers add up to 11. So, the four directions are: (2 1 9) - (3 1 8) - (4 1 7) - (5 1 6).

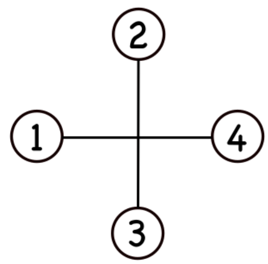
Common Sum of 15: The central circle will be 5. Making a sum of 15 with a 5 in the center means the other two numbers add up to 10. So, the four directions are: (1 5 9) - (2 5 8) - (3 5 7) - (4 5 6).

Common Sum of 18: The central circle will be 9. Making a sum of 18 with a 9 in the center means the other two numbers add up to 9. So, the four directions are (1 9 8) - (2 9 7) - (3 9 6) - (4 9 5).

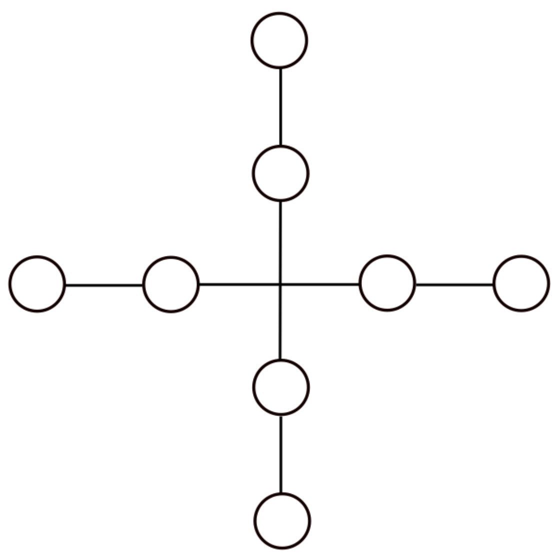
EXPLORATION: For numbers that go from 1 to 2n-1, the central circle has either 1, n, or 2n-1. The sums will be 2n+2 = 1 + 2 + 2n-1, 3n = 1 + n + 2n-1, and 4n-2 = 1 + 2n-2 + 2n-1.

Puzzle of the Week
Magic Pluses

A *Magic Plus* is a plus sign with all the sums the same. This one uses the numbers from 1 to 4.



THE CHALLENGE: Make a Magic Plus with the numbers from 1 to 8.



1 2 3 4 5 6 7 8

EXPLORATION: What happens if you use 1 to 12 with three crossing lines of four circles? Play around with other configurations of circles.

Puzzle of the Week
Magic Pluses – Notes

THE CHALLENGE: Use this as the first “equal sum” puzzle. The Magic Flowers puzzles are slightly harder than this one.

As with so many of these puzzles, it is perfectly fine for students to play around with this in an unstructured way until they come across a solution.

To approach this in a more structured way, start by calculating what the sums must be. The sum of the numbers from 1 to 8 is 36. Breaking this into two equal groups will mean each group has a sum of 18. At this point, there are lots of ways to do this and they all work. If you take any group of numbers that adds up to 18, the remaining numbers will as well.

The solutions are:

- (1 2 7 8) - (3 4 5 6)
- (1 3 6 8) - (2 4 5 7)
- (1 4 5 8) - (2 3 6 7)
- (1 4 6 7) - (2 3 5 8)

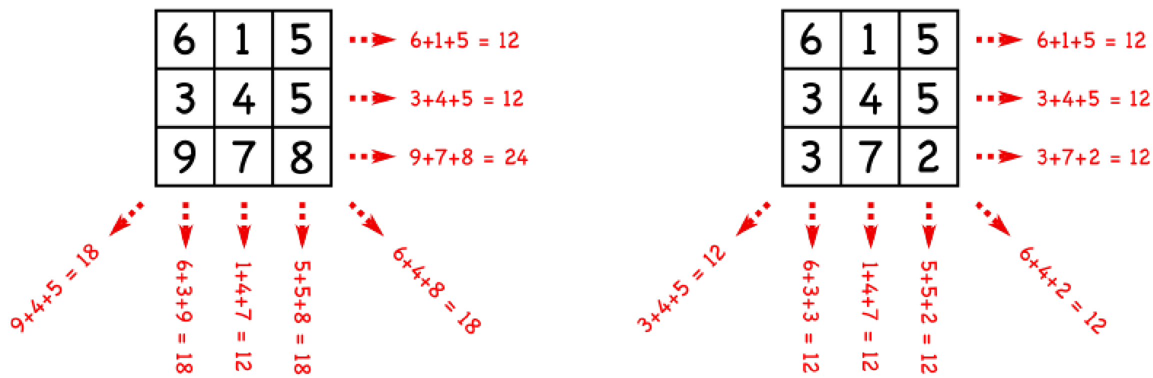
EXPLORATION: The analysis for using 1 to 12 is the same as before. The sum of 1 to 12 is 78. Breaking that into three equal parts gives a sum of 26 in each direction. There are a lot of solutions. Here are a few:

- (1 2 11 12) - (3 4 9 10) - (5 6 7 8)
- (1 2 11 12) - (3 5 8 10) - (4 6 7 9)
- (1 3 10 12) - (2 5 8 11) - (4 6 7 9)

Puzzle of the Week

Magic Squares – 1

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is <u>not</u> a Magic Square. The second one is a Magic Square with a constant sum of 12.



THE CHALLENGE: Use each of the numbers 3, 5, 6, and 9 once to complete this Magic Square.

8	1	
		7
4		2

3 5 6 9

Puzzle of the Week

Magic Squares – 1 – Notes

THE CHALLENGE: This is meant to be an introductory warmup puzzle for Magic Squares, so it doesn't require much careful analysis. If a student wants to just play around with the numbers until they work, that's fine.

Looking at the upper right corner, we know that the common sum is equal to that corner plus 9 more (looking at its row and its column). Considering the diagonal the upper right corner is on, we know that the other two entries on that diagonal add up to 9. So the central square must be 5.

If the central square is 5, then we have a diagonal of (8 5 2), whose sum is 15. Now we've got the common sum.

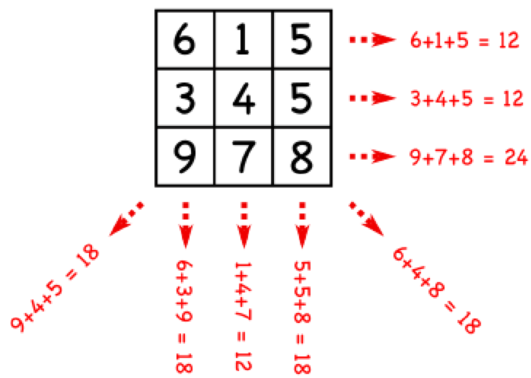
In the bottom row, 15 = 4 + (middle square) + 2 tells us the middle square of the bottom row is 9. We can continue in this way now that we know the common sum.

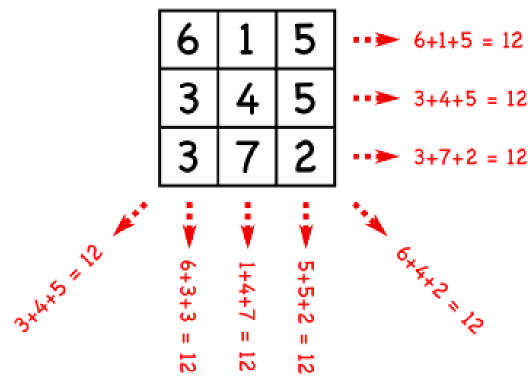
The final solution (by rows) is: (8 1 6) (3 5 7) (4 9 2).

Puzzle of the Week

Magic Squares – 2

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is <u>not</u> a Magic Square. The second one is a Magic Square with a constant sum of 12.





THE CHALLENGE: Use each of the numbers 1, 2, 4, 7, and 8 once to complete this Magic Square.

	9	
	5	3
6		

1 2 4 7 8

Puzzle of the Week

Magic Squares – 2 – Notes

THE CHALLENGE: This is meant to be an intermediate warmup puzzle for Magic Squares, so it doesn't require much careful analysis. If a student wants to just play around with the numbers until they work, that's fine.

The simplest way to start analyzing this puzzle is to find the common sum. Each row adds up to the common sum. Also, the three rows contain the numbers from 1 to 9 and add up to three times the common sum. Therefore, three times the common sum is 45 (the sum of 1 to 9), so the common sum is 15.

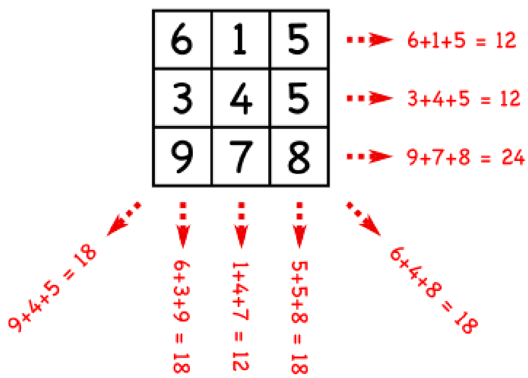
After that, start with the lines that already have two numbers and fill in the missing numbers to make them all add up to 15.

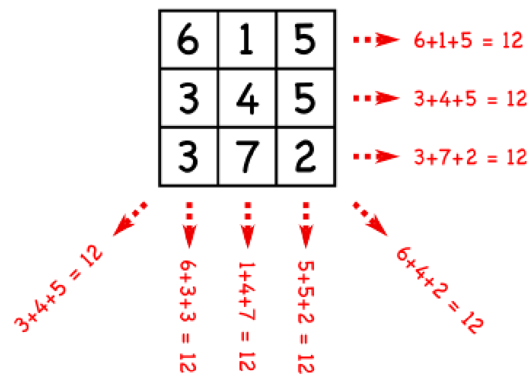
The final solution (by rows) is: (2 9 4) (7 5 3) (6 1 8).

Puzzle of the Week

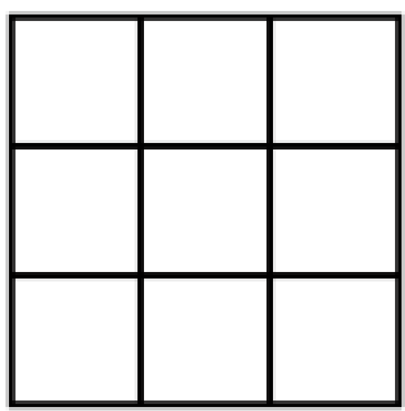
Magic Squares – 3

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is <u>not</u> a Magic Square. The second one is a Magic Square with a constant sum of 12.





THE CHALLENGE: Use each of the numbers from 0 to 8 once to complete this Magic Square.



0 1 2 3 4 5 6 7 8

EXPLORATION: Can you find more than one way to do it? What do the different ways have in common? How would your answer change if you used the numbers from 1 to 9? How about the even numbers from 2 to 18?

Puzzle of the Week
Magic Squares – 3 – Notes

THE CHALLENGE: Let your students play with this. If they pay attention to what they're doing, they'll discover interesting relationships and get a lot out of it. For young students, there is absolutely no need to go into any kind of careful analysis. What follows is a more analytical way to find the solutions.

Command Sum: The simplest way to start analyzing this puzzle is to find the common sum. Each row adds up to the common sum. Also, the three rows contain the numbers from 0 to 8 and add up to three times the common sum. Therefore, three times the common sum is 36 (the sum of 0 to), so the common sum is 12.

Central Square: The next step is to add up the four lines that go through the center square. The common sum is 12 and there are four lines, so their sum must be 4 x 12 = 48. Alternatively, the four lines contain every number once, plus the central square three more times. The sum of the numbers from 0 to 8 is 36. So, 48 equals 36 plus 3 times the central square. So, the central square must be 4.

Adding up to 12: There are surprisingly few ways to add up to 12. They are:

(0 4 8) (1 4 7) (2 4 6) (3 4 5) (0 5 7) (1 3 8) (1 5 6) (2 3 7)

You can figure out a lot for this list. Look at how often a number appears in a triplet:

- 4 times: 4
- 3 times: 1, 3, 5, 7
- 2 times: 0, 2, 6, 8

Next, compare this to how many times a square in the diagram is in one of the lines. You'll see that the center square is in four lines, the corner squares are in three lines, and the middle of the sides are in two lines. This is another way to see that the center square must be 4. Also, the corners must be 1, 3, 5, and 7, and the middle of the sides must be 0, 2, 6, and 8.

Fill up the Square: The hard work is done. Start with 4 in the middle and put 7 in one corner. Note that 0 must go next to the 7 on one side or the other (otherwise the 8 would be forced next to the 7) . You will have no choices after that. One answer, by rows, is: (7 0 5) (2 4 6) (3 8 1). Notice that this is the same as any other answer by rotating the square and possibly flipping it.

EXPLORATION: As noted in the last paragraph, all the solutions are essentially the same - rotate the square until the 7's are in the same corner, and take the mirror image (if needed) along the diagonal to put the 0 in the same position.

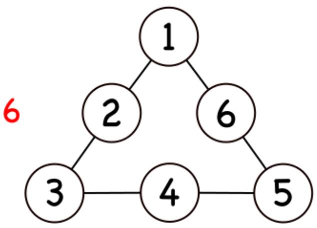
Solving this puzzle for 1 to 9 would mean adding 1 to every entry in the 0 to 8 solution. Solving this puzzle for 2 to 18 would mean doubling all the entries for the 1 to 9 solution.

Puzzle of the Week

Magic Triangles – 1

The sums of the sides of a *Magic Triangle* are all the same. This example is **NOT** a Magic Triangle.

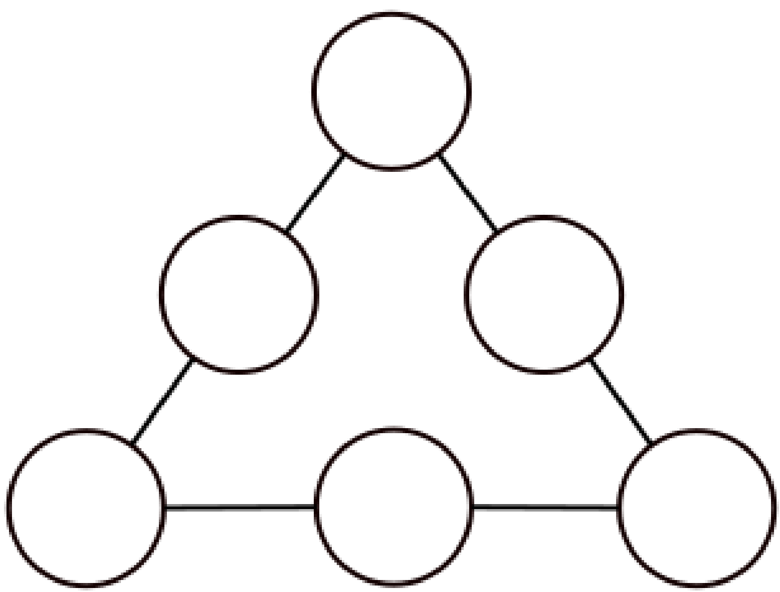
1 + 2 + 3 = 6



1 + 6 + 5 = 12

3 + 4 + 5 = 12

THE CHALLENGE: Use the numbers from 1 to 6 to make a Magic Triangle below.



1 2 3 4 5 6

EXPLORATION: What are the different sums that are possible for Magic Triangles that use 1 to 6?

Puzzle of the Week

Magic Triangles – 1 – Notes

THE CHALLENGE and EXPLORATION: Let your students play with this. If they pay attention to what they're doing, they'll discover interesting relationships and get a lot out of it. For young students, there is absolutely no need to go into any kind of careful analysis.

To be more analytical, add up the three sides. This sum will be the sum of the numbers from 1 to 6 plus the three corners an extra time. The sum of the numbers from 1 to 6 is 21. So, three times the common sum is 21 plus the sum of the three corners. Looked at another way, the common sum will be 7 plus one third of the sum of the corners. The smallest possible sum of three corner numbers is 1 + 2 + 3 = 6, and the largest is 4 + 5 + 6 = 15. So, the common sum might be anything from 7 + (6 / 3) = 9 to 7 + (15 / 3) = 12. Let's look at them 1 at a time.

Common Sum = 9. The corners must be 1, 2, and 3. The number between 1 and 2 must be 6. The number between 1 and 3 must be 5. The number between 2 and 3 must be 4. It works!

Common Sum = 10. The corners add up to 9. The corners could be (1 3 6), (1 3 5), or (2 3 4). (1 2 6) cannot work because there is nothing that can be put between 1 and 2. (1 3 5) can work by putting 6 between 1 and 3, 4 between 1 and 5, and 2 between 3 and 5. (2 3 4) cannot work because there is nothing that can be put between 2 and 4.

Common Sum = 11. The corners add up to 12. The corners could be (1 5 6), (2 4 6), or (3 4 5). (1 5 6) cannot work because there is nothing to put between 5 and 6. (2 4 6) can work because you can put 1 between 4 and 6, 3 between 2 and 6, and 5 between 2 and 4. (3 4 5) cannot work because there is nothing to put between the 3 and 4.

Common Sum = 12. The corners must be 4, 5, and 6. The number between 4 and 5 must be 3. The number between 4 and 6 must be 2. The number between 5 and 6 must be 1. It works!

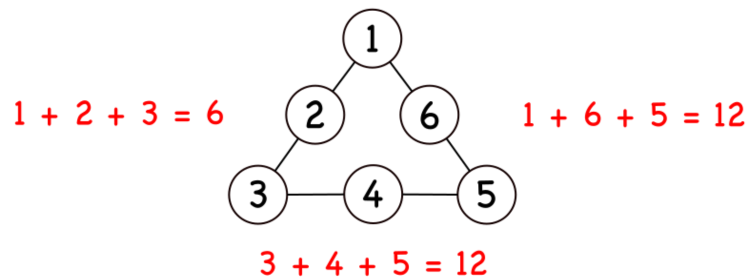
There are four solutions in total.

You can save a lot of work if you realize that you can get all the answers for the Common Sum being 11 and 12 by taking the answers for the Common Sum being 9 and 10 and subtracting those entries from 7. For example, the answer for Common Sum = 9 has sides (1 6 2), (1 5 3), and (2 4 3). If these entries are subtracted from 7, the answer for Common Sum = 12 is found, namely (6 1 5), (6 2 4), and (5 3 4).

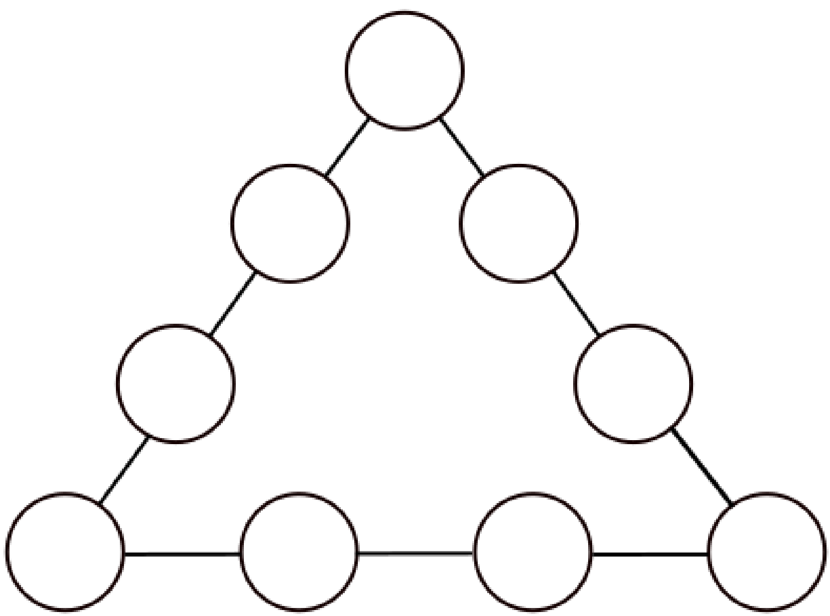
Puzzle of the Week

Magic Triangles – 2

The sums of the sides of a *Magic Triangle* are all the same. This example is **NOT** a Magic Triangle.



THE CHALLENGE: Use the numbers from 1 to 9 to make Magic Triangles.



1 2 3 4 5 6 7 8 9

EXPLORATION: What are the different sums that are possible for Magic Triangles that use 1 to 9?

Puzzle of the Week

Magic Triangles – 2 – Notes

THE CHALLENGE: Let your students play with this. If they pay attention to what they're doing, they'll discover interesting relationships and get a lot out of it. For young students, there is absolutely no need to go into any kind of careful analysis.

To be more analytical, add up the three sides. This sum will be the sum of the numbers from 1 to 9 plus the three corners an extra time. The sum of the numbers from 1 to 9 is 45. So, three times the common sum is 45 plus the sum of the three corners. Looked at another way, the common sum will be 15 plus one third of the sum of the corners. The smallest possible sum of three corner numbers is 1 + 2 + 3 = 6, and the largest is 7 + 8 + 9 = 24. So, the common sum might be anything from 15 + (6 / 3) = 17 to 15 + (24 / 3) = 23. The Common Sum can be 17 to 23.

As noted at the end of the Notes on Magic Triangles – 1, we can cut our work in half by using the answers from 17, 18, 19 and 20 to give us answers for 20, 21, 22, and 23 by subtracting all the entries from 10.

Common Sum = 17. The corners are (1 2 3). The sides of one solution are (1 5 9 2), (1 6 7 3), and (2 4 8 3). Another solution is (1 6 8 2), (1 4 9 3), and (2 5 7 3).

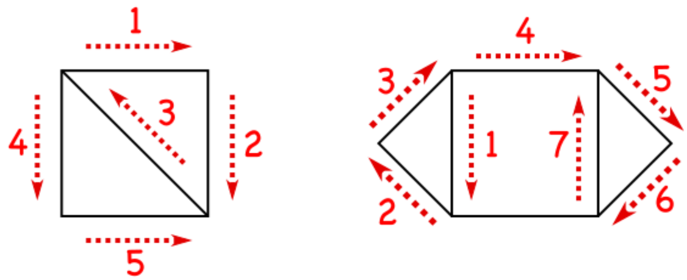
Common Sum = 18. The corners add up to 9. The corners can be (1 2 6), (1 3 5), and (2 3 4), but none of them work out.

Common Sum = 19. The corners add up to 12. The corners can be (1 2 9), (1 3 8), (1 4 7), (1 5 6), (2 3 7), (2 4 6), and (3 4 5). (1 2 9) and (1 3 8) do not work. For (1 4 7) we have the solution (1 6 8 4), (1 2 9 7), and (4 3 5 7). Needless to say, there is a lot to look at if you want to look through every possibility.

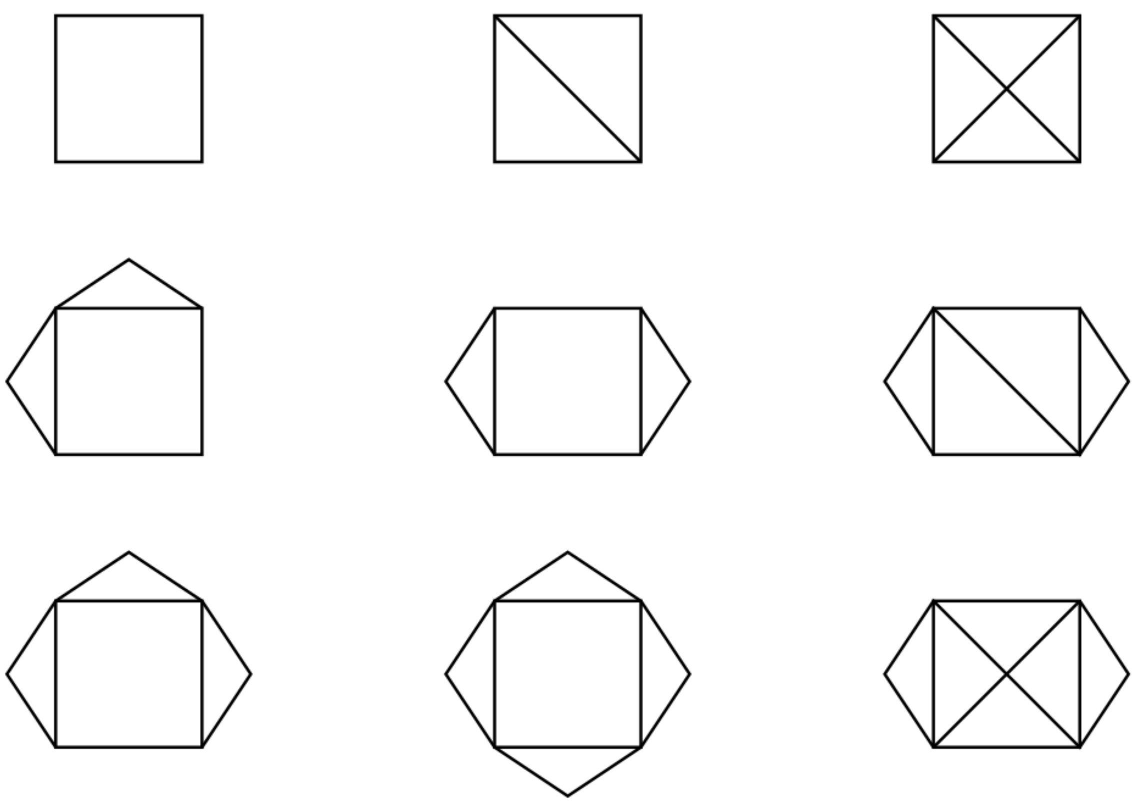
Common Sum = 20. The corners add up to 15. There are even more possibilities here, with no obvious way to shorten the search list.

Puzzle of the Week
Parades

Parades wish to visit each street on their route exactly once. In these two examples, the first one is a successful parade route and the second one is not (one street is left out).



THE CHALLENGE: For each street layout, either find a parade route that visits each street exactly once or decide that it is impossible. For street layouts that have a parade, which ones allow parades to start and end at the same place? Can you find a pattern in your results?



Early Family Math

EarlyFamilyMath.org

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Puzzle of the Week

Parades – Notes

THE CHALLENGE: The key idea is to keep track of where parades start and end, if they exist at all for a street map. Some parades can start and end anywhere, and some must start or end at very specific locations.

After looking at lots of examples, the following observation emerges. It is not important for young children to prove these things.

Result 1: If a corner has an odd number of streets coming to it, the parade must start or end there.

The reason for this is simple. Every time a parade goes into and back out of a corner, that accounts for an even number of streets coming to that corner. Therefore, corners with an odd number of streets must be the start or end of the parade.

Result 2: If there are more than two corners with an odd number of streets coming into them, then this map cannot have a parade.

Result 3: If there are exactly two corners with an odd number of streets, then any parade must begin at one of them and end at the other. In particular, it is impossible to have a parade that begins and ends at the same place.

Result 4: If there are no corners with an odd number of streets, then a parade can start anywhere, and it must start and end at the same place.

Puzzle of the Week

Self-Describing Numbers – 1

The number 1210 is a *Self-Describing Number* because each digit in order describes how many digits of that type there are - there is 1 0, 2 1's, 1 2, and 0 3's. SImilarly, 2020 is a Self-Describing Number because it has 2 0's, 0 1's, 2 2's, and 0 3's.

1	2	1	0
# of 0's	# of 1's	# of 2's	# of 3's

2	0	2	0
# of 0's	# of 1's	# of 2's	# of 3's

THE CHALLENGE: Find a Self-Describing Number that has five digits.

____	____	____	____	____
# of 0's	# of 1's	# of 2's	# of 3's	# of 4's

EXPLORATION: Why are there no Self-Describing Numbers with 1, 2, or 3 digits?

Puzzle of the Week

Self-Describing Numbers – 1 – Notes

THE CHALLENGE: A playful, disorganized approach to this is fine and should be encouraged.

Here are a few analytical ideas for attacking this puzzle.

Result 1: The sum of the digits is the number of digits. The digits count the digits of each type, so the sum of the digits counts all the digits.

Result 2: The sum of the products is the number of digits. Put another way, if you take the sum of multiplying each digit by the size of the digits it is counting, that sum must be the number of digits. This result follows from Result 1 because the sum of the products counts up all the digits involved in the number.

Look at the two given examples. In 1210, the sum of the products is (1 x 0) + (2 x 1) + (1 x 2) + (0 x 3) = 4. In 2020, the sum is (2 x 0) + (0 x 1) + (2 x 2) + (0 x 3) = 4.

Result 3: The rightmost, low-order digit, the ones digit, is 0. From Result 2, this digit must be 0 or 1 (or the product would be too large). Suppose it is 1. To avoid using variables, let's suppose we have a 6-digit number. Then the ones place counts the number of 5's. If there is a 5 someplace, then, by result 1, the only other non-zero digit must be a 1. But then we would have four 0's, which is impossible.

Result 4: The high-order digit is not 0. This must be true for this number to be considered a number.

Okay, let's construct some answers for 5-digit numbers.

There is a basic tension between results 1 and 2. To keep those sums the same, there must be an increase in 0's. For example, for each 2 there must be an extra 0, for each 3 there must be two extra 0's, for each 4 there must be three extra 0's, and so on. That's why most self-describing numbers have a lot of 0's.

If the number of 0's were 1, there would need to be nonzero numbers in every place except the far right place. But having all those nonzero numbers would necessitate having more 0's to make Result 2 work out. So, having only one zero in numbers with more than four digits is impossible. We can call that Result 5 if you like.

If there are 2 0's, the number is either 22100 or 21200. Of those, only 21200 works, and **that is the answer!**

If there are 3 0's, the number would be 32000, which doesn't work.

EXPLORATION: Let's look at the cases individually. Remember the four results when looking at these.

1 digit: Results 3 and 4 make this impossible.

2 digit: The number would have to be 20, which is not self describing.

3 digit: The number would have to be either 300, 210, or 120. None of these is self describing.

Puzzle of the Week
Self-Describing Numbers – 2

The number 1210 is a *Self-Describing Number* because each digit in order describes how many digits of that type there are - there is 1 0, 2 1’s, 1 2, and 0 3’s. SImilarly, 2020 is a Self-Describing Number because it has 2 0’s, 0 1’s, 2 2’s, and 0 3’s.

1	2	1	0
# of 0's	# of 1's	# of 2's	# of 3's

2	0	2	0
# of 0's	# of 1's	# of 2's	# of 3's

THE CHALLENGE: Find a Self-Describing Number that has seven digits.

# of 0's	# of 1's	# of 2's	# of 3's	# of 4's	# of 5's	# of 6's

EXPLORATION: Why are there no Self-Describing Numbers with 6 digits?

Puzzle of the Week

Self-Describing Numbers – 2 – Notes

THE CHALLENGE & EXPLORATION: A playful, disorganized approach to this is fine and should be encouraged.

Here are some useful results from the Notes on Self-Describing Numbers – 1:

Result 1: The sum of the digits is the number of digits.

Result 2: The sum of the products is the number of digits.

Result 3: The rightmost, low-order digit, the ones digit, is 0.

Result 4 & 5: The high-order digit is at least 2 for numbers with at least five digits.

There is a basic tension between results 1 and 2. To keep those sums the same, there must be an increase in 0's. For example, for each 2 there must be an extra 0, for each 3 there must be two extra 0's, for each 4 there must be three extra 0's, and so on. That's why most self-describing numbers have a lot of 0's.

Suppose the high-order digit is 2, so there are only two 0's. For a five-digit number, that means there are three nonzero digits. In Self-Describing Numbers – 1 we found that the only solution was 21200. For a six-digit number, there would be four nonzero digits. Also, since the digits have to add up to six, that would force the number to have two 1's and two 2's. None of the ways of mixing two 1's and two 2's work. The same thing keeps happening for longer numbers that have two 0's. This gives:

Result 6: The high order digit is at least 3 for numbers with at least six digits.

Let's keep playing with the number of 0's. What would be too many? Suppose the number of 0's is within three of the total number of digits. That would leave at most two more nonzero digits. One of those would be a 1 for the large 0 count. The only other possible nonzero entry would have to be for the number of 1's, which is not going to work out. We get yet another result:

Result 7: For numbers with at least six digits, the number of nonzero entries must be more than 3.

If you combine results 6 and 7, you will see that there cannot be any solutions for six-digit numbers. It is surprising that this works for Self-Describing numbers that have 5 or 7 digits, but not for ones that have 6 digits.

For 7-digit numbers, results 6 and 7 together guarantee that, if there is a solution, the number of 0's is 3. The other entries in the number must total 4. A few quick experiments quickly leads to the answer.

The answer is 3211000!

Puzzle of the Week

Self-Describing Numbers – 3

The number 1210 is a *Self-Describing Number* because each digit in order describes how many digits of that type there are - there is 1 0, 2 1’s, 1 2, and 0 3’s. SImilarly, 2020 is a Self-Describing Number because it has 2 0’s, 0 1’s, 2 2’s, and 0 3’s.

1	2	1	0
# of 0's	# of 1's	# of 2's	# of 3's

2	0	2	0
# of 0's	# of 1's	# of 2's	# of 3's

THE CHALLENGE: Find a Self-Describing Number that has ten digits.

# of 0's	# of 1's	# of 2's	# of 3's	# of 4's	# of 5's	# of 6's	# of 7's	# of 8's	# of 9's

EXPLORATION: Do you see a pattern in your answer for ten digits that will help you find other examples with a large number of digits?

Puzzle of the Week

Self-Describing Numbers – 3 – Notes

THE CHALLENGE: A playful, disorganized approach to this is fine and should be encouraged.

Here are some useful results from the Notes on Self-Describing Numbers – 1 and – 2:

Result 1: The sum of the digits is the number of digits.

Result 2: The sum of the products is the number of digits.

Result 3: The rightmost, low-order digit, the ones digit, is 0.

Result 4 & 5: The high-order digit is at least 2 for numbers with at least five digits.

Result 6: The high-order digit is at least 3 for numbers with at least six digits.

Result 7: For numbers with at least six digits, the number of nonzero entries must be more than 3.

It is probably helpful to have the answer for 7 in front of us: 3211000. Looking at this, it is tempting to think that the number of nonzero entries is four for numbers with at least six digits. From Result 7, we know that the number of nonzero entries cannot be less than four. Can it be more than four?

We immediately know two of the nonzero entries - the high order digit that gives the number of 0's (which is a number that is at least 3), and the place that corresponds to that digit - if that place has a value greater than 1, then that causes there to be that many 0's and that many of some other number, and things quickly spiral out of control. So, there is at least one 1. That gives us three nonzero entries. It is now impossible to have a single 1 (how would we fill in the number of 1's), so the number of 1's must be at least two. So we get an additional 1 corresponding to the place that has the number of 1's.

And that's it. Any additional entries would once again cause a snowball effect that would cause the numbers to get too big. At last, we arrive at the answer.

The answer for 10 is 6210001000!

EXPLORATION: The answer for 7 was 3211000 and the answer for 10 was 6210001000. The pattern X210….01000 looks promising. Let's look at how we can apply it.

It looks like, as we go up from 7, we can increase the number of 0's by 1 each time, and put that new 0 between the two 1's. If you check it out, it always works. Well, it works until we get to 13 with 9210000001000. After 13, the first digit becomes too large to be a single digit, but it was fun while it lasted.

Puzzle of the Week

Square Sums – 1

The rows and columns add up to the numbers on the outside of this 2 by 2 square.

5	3	8
2	7	9
7	10	+

		8
		9
7	10	+

THE CHALLENGE: Using the numbers from 1 to 7, solve for the missing numbers in this square.

		3
		12
9	6	+

1 2 3 4 5 6 7

EXPLORATION: Make a Square Sum challenge for someone else.

Puzzle of the Week

Square Sums – 1 – Notes

THE CHALLENGE: Using the smallest and largest numbers helps to narrow down the possibilities.

The only two numbers that add up to 12 are 5 and 7, so they must go along the bottom row.

The only two numbers that add up to 3 are 1 and 2, so they must go along the top row.

The only way to get 6 using 5, 7, 1, and 2 is to add 5 and 1, so they must be in the rightmost column.

At this point, we have the solution: (7 5 12) (2 1 3) (9 6 +).

EXPLORATION: It is easy to make these puzzles. Start by putting four numbers on the inside, find their sums, and then create the puzzle leaving out the four numbers on the inside.

While creating puzzles that way is easy, they aren't always the most fun to solve. To increase the fun, add some restrictions. For example, say that each number is used at most once. Also, restrict the possible numbers by saying they are in a range or that they all have some characteristic, such as all being odd.

Puzzle of the Week

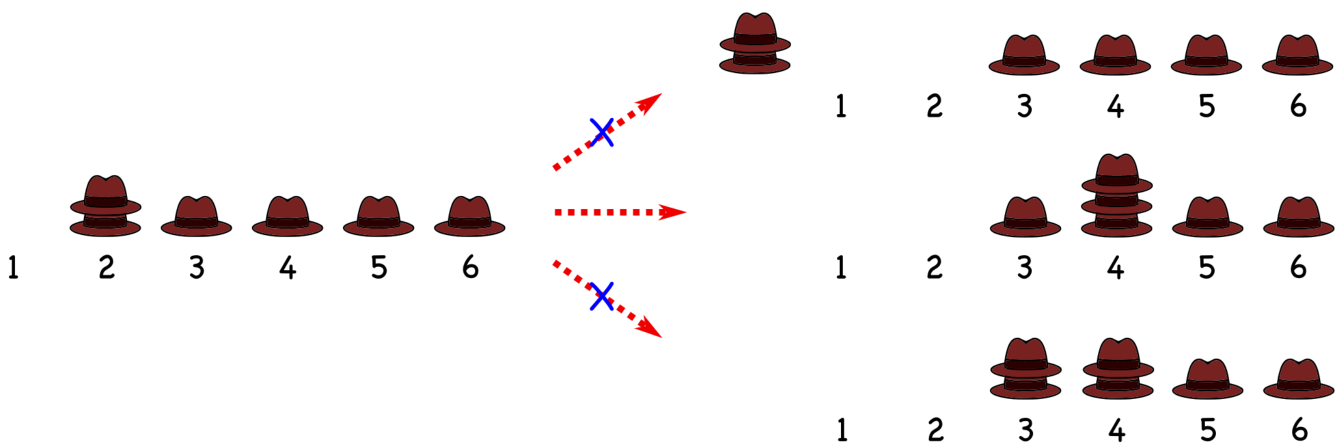
Stacking Hats – 1

Rules for stacking:

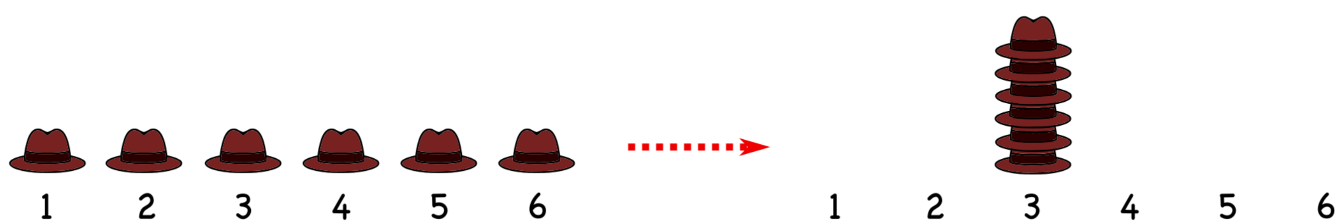
1) When you move a stack, you must move the whole stack onto a place with at least one hat.

2) A stack moves over the number of places for how many hats there are.

3) You can only use the original six spots.



THE CHALLENGE: Use these rules to move the six hats into one stack. Can the final stack of six hats end up in any of the six positions, or do only some of the positions work?



EXPLORATION: What happens if you start with seven hats in seven places? What about other numbers?

Puzzle of the Week

Stacking Hats – 1 – Notes

THE CHALLENGE & EXPLORATION: It is actually easiest to show how to do this with any number of hats ending up in any position. Interestingly, sometimes making a problem more general makes it easier to do.

Doing this puzzle for one, two, or three hats is very easy.

So, assume we have some number of hats that is bigger than three, and also assume we know how to do this puzzle for any number of hats less than this number.

Select the spot where you want all the hats to end up. Leave that hat alone - it does not ever need to move. I'll assume there are hats on both sides of that special hat - if there aren't, then you only need to concern yourself with the one side.

For the hats that form a group of hats on the left side, use your expertise to pile them up in one stack on the far left side. Similarly, for the hats that form a group of hats on the right side, use your expertise to pile them up in one stack on the far right side. Finally, jump the hats on the far left onto the special hat and jump the hats on the right side onto the special hat. We now have all the hats in one pile where we wanted them to be.

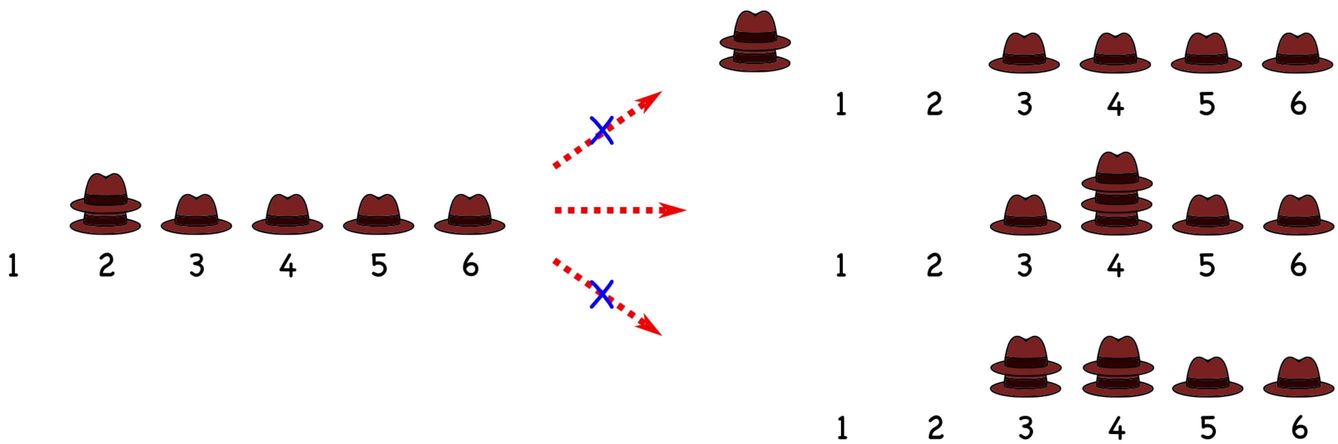
Puzzle of the Week
Stacking Hats – 2

Rules for stacking:

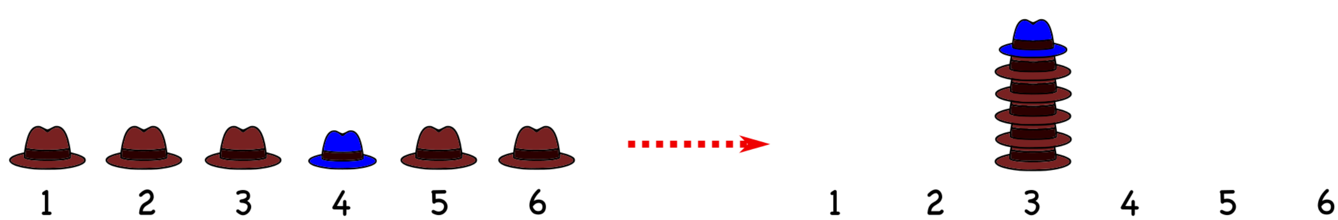
1) When you move a stack, you must move the whole stack onto a place with at least one hat.

2) A stack moves over the number of places for how many hats there are.

3) You can only use the original six spots.



THE CHALLENGE: Use these rules to move the six hats into one stack. The small blue hat needs to end up on top of the stack. Can the final stack of six hats end up in any of the six positions, or do only some of the positions work? Can the blue hat start in any position, or do only some of the starting positions work?



EXPLORATION: What happens if you start with seven hats in seven places? What changes if you allow hats to move to empty positions?

Puzzle of the Week

Stacking Hats – 2 – Notes

THE CHALLENGE & EXPLORATION: Dealing with a blue hat makes the Stacking Hats puzzle much trickier! If you look at the results below, you'll notice some clear patterns. However, I'll just describe what happens with different numbers of hats and leave it at that.

A couple general thoughts first. Because it is not legal to move a stack into an empty position, and the Blue hat must be moved away from its initial position so that it will be on top, it is never possible to have the final stack end up where the Blue hat started. To save work, I will only consider Blue hat positions for the positions on the left side - the other positions can be easily analyzed by taking mirror images of the left side positions.

The shorthand I will use is BL for Blue, BR for Brown, Y for yes, and N for no. For example, (BL BR BR) - (N Y N) means that when there are three hats ordered Blue - Brown - Blue, it is possible to have a stack of all three hats with the Blue hat on top only ending in the middle position.

1 Hat: (BL) - (Y)
2 Hats: (BL BR) - (N Y)
3 Hats: (BL BR BR) - (N Y N); (BR BL BR) - (Y N Y)
4 Hats: (BL BR BR BR) - (N Y N Y); (BR BL BR BR) - (Y N Y Y)
5 Hats: (BL BR BR BR BR) - (N Y N Y Y); (BR BL BR BR BR) - (Y N Y Y Y); (BR BR BL BR BR) - (Y Y N Y Y)
6 Hats: (BL BR BR BR BR BR) - (N Y N Y Y Y); (BR BL BR BR BR BR) - (Y N Y Y Y Y);
(BR BR BL BR BR BR) - (Y Y N Y Y Y)
7 Hats: (BL BR BR BR BR BR BR) - (N Y N Y Y Y Y); (BR BL BR BR BR BR BR) - (Y N Y Y Y Y Y);
(BR BR BL BR BR BR BR) - (Y Y N Y Y Y Y); (BR BR BR BL BR BR BR) - (Y Y Y N Y Y Y)

Allowing stacks of hats to jump into empty spots makes it possible to almost always succeed. I have put in red Y's for the new places that are possible.

1 Hat: (BL) - (Y)
2 Hats: (BL BR) - (N Y)
3 Hats: (BL BR BR) - (**Y** Y **Y**); (BR BL BR) - (Y N Y)
4 Hats: (BL BR BR BR) - (**Y** Y **Y** Y); (BR BL BR BR) - (Y **Y** Y Y)
5 Hats: (BL BR BR BR BR) - (**Y** Y **Y** Y Y); (BR BL BR BR BR) - (Y **Y** Y Y Y); (BR BR BL BR BR) - (Y Y **Y** Y Y)
6 Hats: (BL BR BR BR BR BR) - (**Y** Y **Y** Y Y Y); (BR BL BR BR BR BR) - (Y **Y** Y Y Y Y);
(BR BR BL BR BR BR) - (Y Y **Y** Y Y Y)
7 Hats: (BL BR BR BR BR BR BR) - (**Y** Y **Y** Y Y Y Y); (BR BL BR BR BR BR BR) - (Y **Y** Y Y Y Y Y);
(BR BR BL BR BR BR BR) - (Y Y **Y** Y Y Y Y); (BR BR BR BL BR BR BR) - (Y Y Y **Y** Y Y Y)

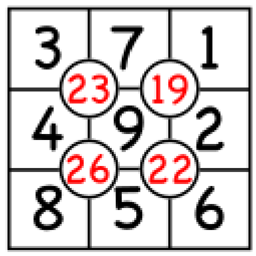
Puzzle of the Week

Sujiko Puzzle – 1

In a Sujiko puzzle, use each of the numbers from 1 to 9 once in the nine squares. The number in each circle must be the sum of the four squares that surround it.

3+7+4+9 = 23

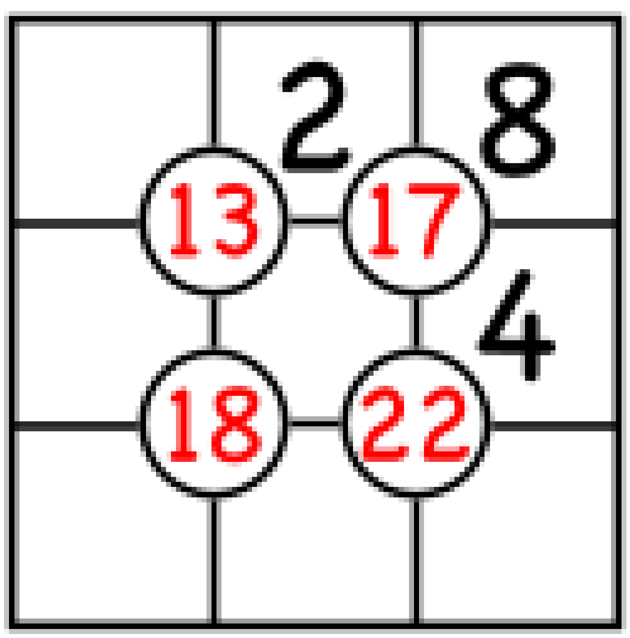
7+1+9+2 = 19



4+9+8+5 = 26

9+2+5+6 = 22

THE CHALLENGE: Fill in this Sujiko puzzle.



1 3 5 6 7 9

Puzzle of the Week

Sujiko Puzzle – 1 – Notes

THE CHALLENGE: This is the first Sujiko Puzzle, so solving it is fairly straightforward.

The upper right 2 by 2 corner has three of the four squares filled in, so that is the place to start. The three squares filled in, (2 8 4), add up to 14. To make the sum of the four squares equal to 17 (in the circle), we need the central square to be 17 - 14 = 3.

The smallest and largest numbers are often good places to start. For the bottom right 2 by 2 corner, the two numbers we have, (3 4), plus the two missing numbers must add up to 22. So, the missing two numbers on the right side of the bottom row add up to 15. We can get 15 as the sum of 6 + 9 or 7 + 8. However, the 8 is not available, so they must be 6 + 9.

Let's look at the bottom left 2 by 2 corner. On the right side of that corner we will either have 3 + 6 or 3 + 9. If it were 3 + 6, we would need two more numbers that add up to 9 and those are not available. So it must be 3 + 9. The remaining two numbers in that bottom left corner must add up to 6, and the only possible way to do that is with 1 + 5.

The only unused number at this point is the 7, so it must go in the upper left corner.

Three of the four numbers in the upper left 2 by 2 corner are (7 2 3), so the remaining number must be 1.

At this point we have the complete solution! Here it is row by row:

(7 2 8)

(1 3 4)

(5 9 6)

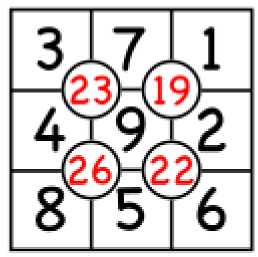
Puzzle of the Week

Sujiko Puzzle – 2

In a Sujiko puzzle, use each of the numbers from 1 to 9 once in the nine squares. The number in each circle must be the sum of the four squares that surround it.

3+7+4+9 = 23

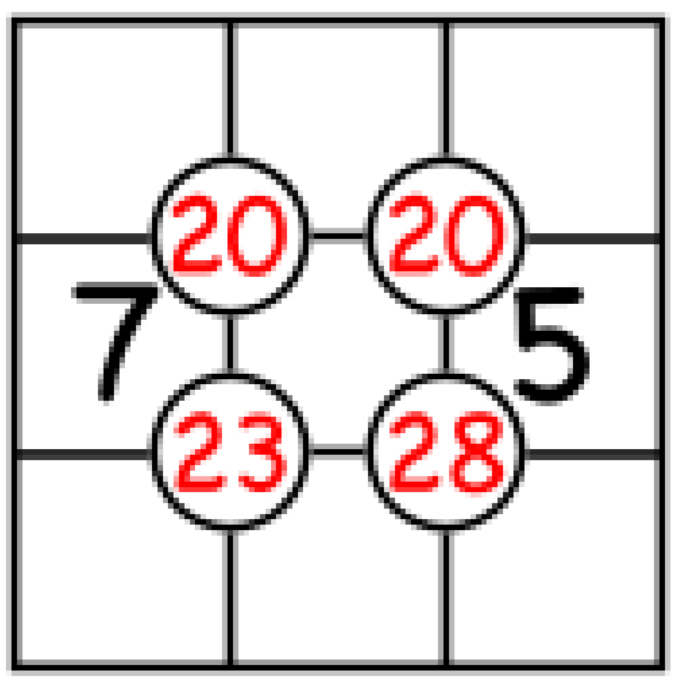
4+9+8+5 = 26



7+1+9+2 = 19

9+2+5+6 = 22

THE CHALLENGE: Fill in this Sujiko puzzle.



1 2 3 4 6 8 9

Puzzle of the Week

Sujiko Puzzle – 2 – Notes

THE CHALLENGE: When solving a puzzle, look for its weak points, its easiest points of entry.

For this puzzle, the 28 in the circle is quite a big number and seems a good place to start. The three missing numbers in the bottom left 2 by 2 corner must add up to 23. The only way to get 23 from the list of numbers we have is to use 6, 8, and 9. So, we know those three numbers must be in that bottom corner (and nowhere else).

Looking at the bottom right 2 by 2 corner, 3 and 4 are the largest remaining numbers, so the central square must be 8 or 9. This means that the two numbers on the right side of the upper row are (3 4) or (2 4).

Suppose the central square is 8. This means the right side of the upper row is (3 4).

> Looking at the bottom left 2 by 2 corner, the two squares shared with the bottom right 2 by 2 corner are (8 6) or (8 9). However, (8 9) makes it impossible to get a sum of 23 in the bottom left 2 by 2 corner. Therefore, it must be (8 6), and so the bottom left corner must be 2.
>
> The only unused value is 1, which must go in the upper left corner. That makes the sum of the upper left corner either 1 + 3 + 7 + 8 = 19 or 1 + 4 + 7 + 8 = 20. The latter of the two is a solution!

Suppose the central square is 9. This means the right side of the upper row is (2 4).

> Looking at the bottom left 2 by 2 corner, the two squares shared with the bottom right 2 by 2 corner are (9 6) or (9 8). However, (9 8) makes it impossible to get a sum of 23 in the bottom left 2 by 2 corner. Therefore, it must be (8 6), and so the bottom left corner must be 1.
>
> The only unused value is 3, which must go in the upper left corner. That makes the sum of the upper left corner either 3 + 2 + 7 + 9 = 21 or 3 + 4 + 7 + 9 = 23, neither of which work!

Therefore, the only solution is given by (showing triplets as rows):

(1 4 3)

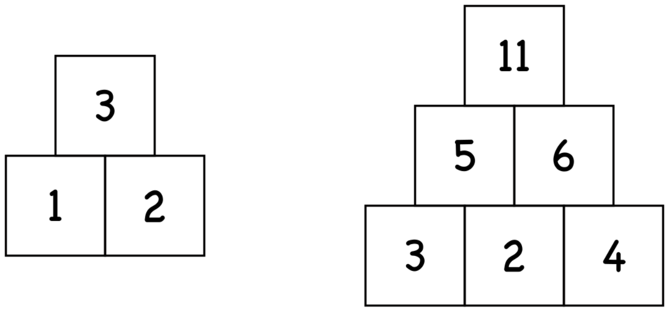
(7 8 5)

(2 6 9)

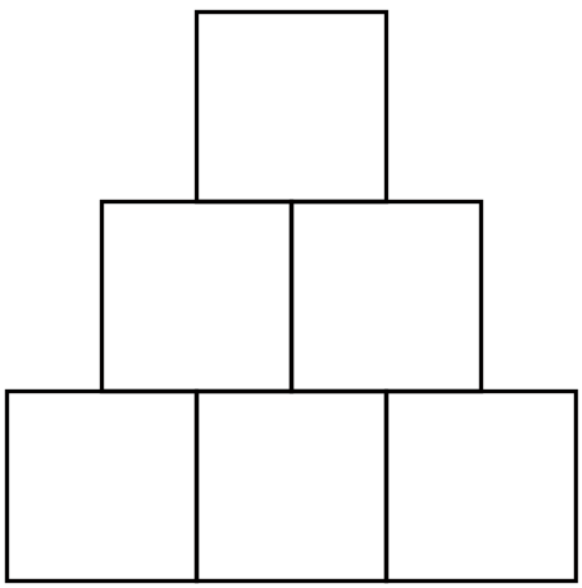
Puzzle of the Week

Sum Pyramids – 1

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 10, not repeating any number, to make a Sum Pyramid with the smallest possible number on top. Can you do better than 11?



1 2 3 4 5 6 7 8 9 10

Puzzle of the Week

Sum Pyramids – 1 – Notes

THE CHALLENGE: The center square in the bottom right will contribute to both squares in the middle row, and so its value will show up twice in the top number. To keep the top number as small as possible, we should put a 1 in the middle of the bottom row. We would like to put 2 and 3 in the two remaining squares in the bottom row, but that is not possible because 1 + 2 = 3 would cause 3 to be used twice.

To make the sum as small as possible, we will use (2 1 4) in the bottom row. This produces the pyramid:

(8)

(3 5)

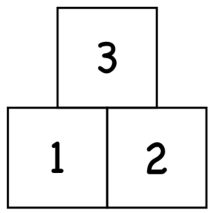
(2 1 4)

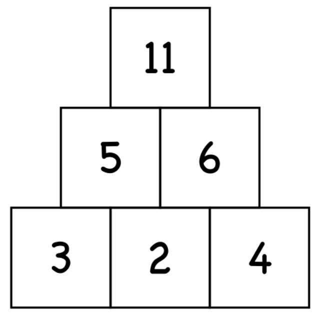
This is a considerable improvement over having 11 on top!

Puzzle of the Week

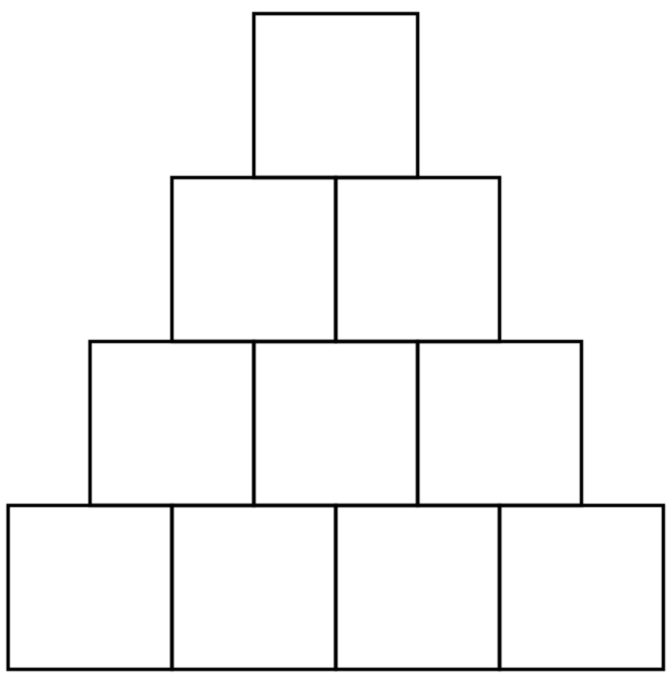
Sum Pyramids – 2

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.





THE CHALLENGE: Place some of the numbers from 1 to 25, not repeating any number, to make a Sum Pyramid with the smallest possible number on top. Can you do better than 25?



Puzzle of the Week

Sum Pyramids – 2 – Notes

THE CHALLENGE: The pyramid is completely determined by the entries in the bottom row.

Each of the middle numbers in the bottom row will contribute a total of three times to the number on the top of the pyramid. The corner numbers in the bottom row will each only contribute once to the number on the top of the pyramid. Therefore, let's use 1 and 2 for those middle numbers on the bottom row.

1 and 2 creates a 3 on the next row, so we can't use it in the bottom row. Exploring how to finish out the bottom row we should consider (4 1 2 5), (5 1 2 4), (4 1 2 6), and (6 1 2 4). All of these produce duplicates. Looking further, we can try (5 1 2 7) or (6 1 2 8). However, (5 1 2 7) produces a duplicate entry, and (6 1 2 8) has a top number of 26, which is certainly not better than 25.

Going back, let's consider using 1 and 3 in the middle of the bottom row. Some exploration produces the following, which is our best answer.

(20)

(11 9)

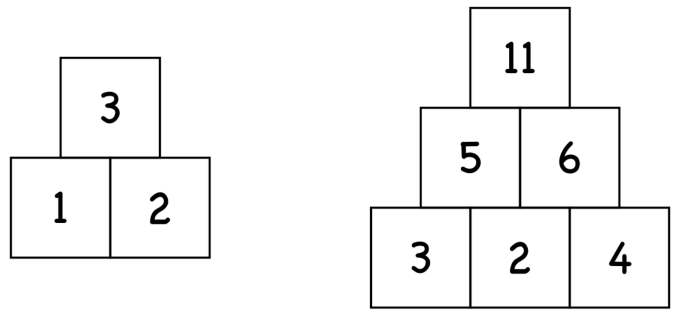
(7 4 5)

(6 1 3 2)

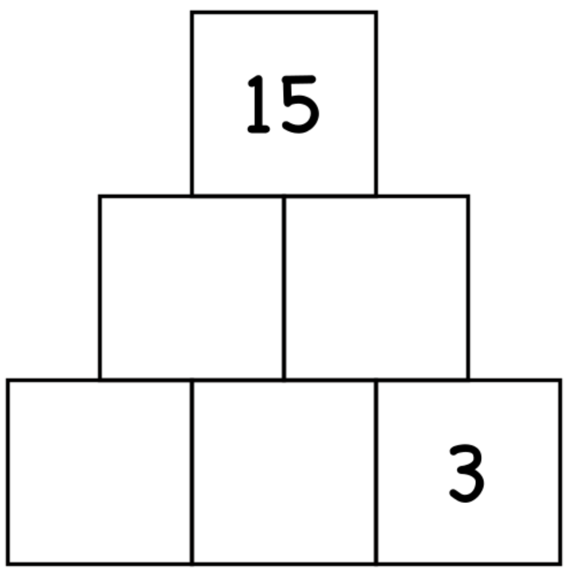
Puzzle of the Week

Sum Pyramids – 3

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 15, not repeating any number, to complete this Sum Pyramid. Can you find more than one solution?



Puzzle of the Week

Sum Pyramids – 3 – Notes

THE CHALLENGE: Following the numbers up, 15 will equal 3 plus twice the middle number in the bottom row plus the leftmost number in the bottom row. So, 12 will be twice the middle number plus the leftmost number.

The possibilities for the bottom row are: (2 5 3), (4 4 3), (6 3 3), (8 2 3), and (10 1 3). This gives us three possible solutions:

(15)

(7 8)

(2 5 3)

or

(15)

(10 5)

(8 2 3)

or

(15)

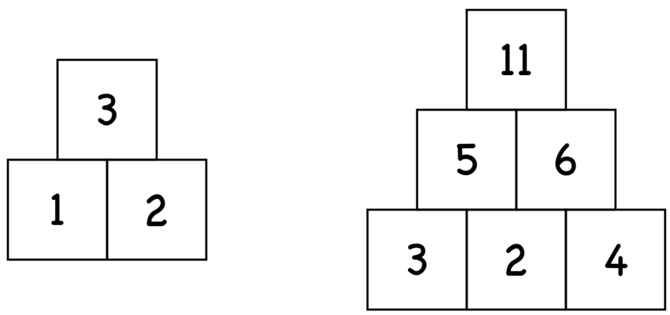
(11 4)

(10 1 3)

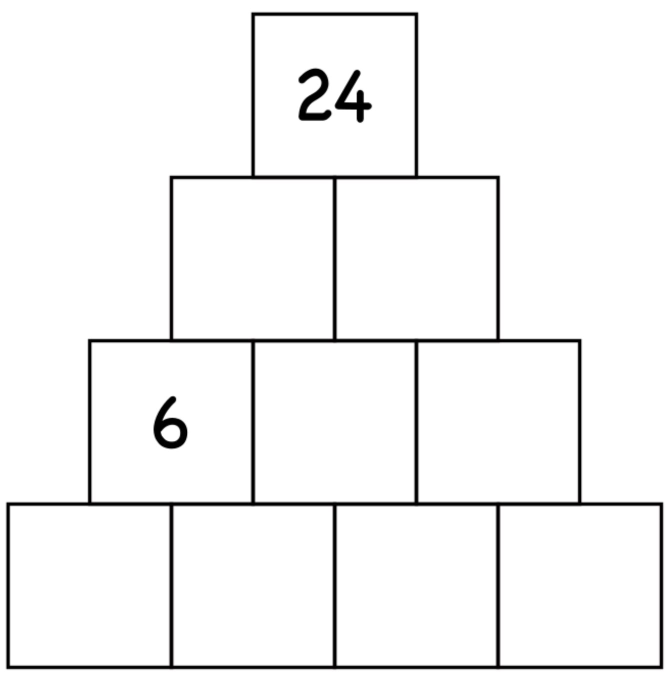
Puzzle of the Week

Sum Pyramids – 4

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 24, not repeating any number, to complete this Sum Pyramid. Can you find more than one solution?



Puzzle of the Week

Sum Pyramids – 4 – Notes

THE CHALLENGE: The puzzle is completely determined by its bottom row.

Tracing numbers up, we have two immediate constraints on the bottom row. The leftmost two numbers on the bottom row must add up to 6, so they are 1 + 5 or 2 + 4. Also, three times the sum of the two middle numbers, plus the sum of the two corner numbers, must be 24.

There are four possible contributions from the left side of the bottom row. In each case, we see the right side will contribute the remainder using 3 times its central number plus 1 times its corner number.

Case 1: (1 5) which contributes 1 + 3 x 5 = 16. We need 8 more from the right side. That cannot be done without duplicating numbers.

Case 2: (5 1) which contributes 5 + 3 x 1 = 8. We need 16 more from the right side. This can work with (3 7) or (2 10). If the bottom is (5 1 3 7), there is a duplication. If the bottom is (5 1 2 10), it works!

Case 3: (2 4) which contributes 2 + 3 x 4 = 14. We need 10 more from the right side. This can work with (3 1) or (1 7). (f the bottom is (2 4 3 1), there is a duplication. If the bottom is (2 4 1 7), it works!

Case 4: (4 2) which contributes 4 + 3 x 2 = 10. We need 14 more from the right side. This can work with (3 5) or (1 11). If the bottom is (4 2 3 5), there is a duplication. If the bottom is (4 2 1 11), it works!

Putting this together, there are the three solutions:

(24)
(9 15)
(6 3 12)
(5 1 2 10)

or

(24)
(11 13)
(6 5 8)
(2 4 1 7)

or

(24)
(9 15)
(6 3 12)
(4 2 1 11)