Balance Fractions

Topics: division/sharing, fractions
Materials: Pencil & paper or whiteboards, worksheet
Common Core:
4.NF.2 Compare fractions using any method, e.g., benchmarks, same numerator, same denominator
4.NF.3 Solve problems involving addition and subtraction of fractions.
4.NF.4 Solve problems involving multiplication of a fraction by a whole number.
MP1 Make sense of problems and persevere in solving them.

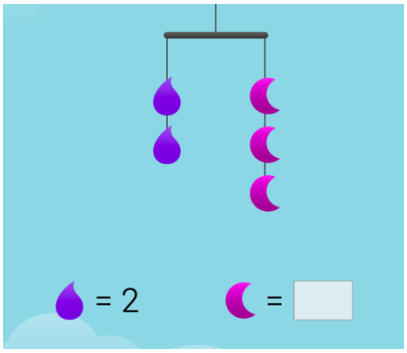
Essential Question. What number makes it balance?

Why we love Balance Fractions

The balance gives a perfect metaphor for the meaning of the equals sign. The problems themselves are simple to pose and interesting to solve.

Launch

Show students a balance fraction problem.

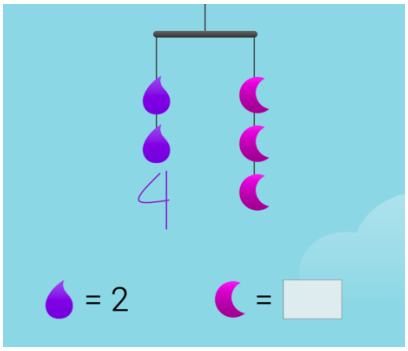


Composing and Decomposing Fractions
The balancing problems in the lesson invite students to draw on a range of fraction ideas, including decomposing fractions (e.g., 2 moons is equal to 3, so each moon equals 1 ½), composing fractions (e.g., each star is equal to ¼ and there are 5 of them, so that side of the balance "weighs" 5/4)

Ask them to discuss what they notice about this picture with a partner. Then check in with the group.

Teacher: What did you notice about this picture?
Student A: There's two sides of the scale, and they balance.
Student B: There's a pink moon shape and a purple water drop shape.
Student C: The purple shape is heavier.
Teacher: How do you know that?
Student C: Well, 2 of the purple shapes are the same as 3 of the pink shapes. So it must be heavier.
Teacher: I want to pause on that, because there are two very important things you just said there. First, like Student A said, there are two sides, and they balance. That means they are equal or the

same. So those 2 purple shapes equal or are the same weight as the 3 pink shapes, as you said. And the other point is that the purple shape must weigh more. Anyone have any rough estimates for what number the pink shape equals?
Student D: What do you mean?
Student E: Oh, does the purple shape equal 2? Because it says it equals 2 at the bottom.
Teacher: Right. So does the pink shape equal more than 2 or less than 2? Let's vote. Thumbs up if it's more than 2, thumbs down if it's less than 2. Thumbs sideways if you aren't sure. [Students vote.] Discuss with your neighbor, and defend your thinking, or let them defend theirs. [Students discuss.]
Student F: I see that the pink shape is actually less than 2. Because if it were more than two, the right side of the balance would be more than 6. But the left side is 4. So that can't be right.



Teacher: So you're saying the pink moon is less than 2. Is it more or less than 1? Discuss with a partner. [Students discuss.] Thoughts?
Student G: It's got to be more than 1. Because if it were less than 1, the right side of the balance would be less than 3. But it's got to equal 4.
Teacher: So we know the moon shape is less than 2 but greater than 1. Any thoughts on what it actually is?
Student H: One and a half?
Teacher: Seems like a good guess. Take a minute and figure out with your partner if the pink moon is one and a half. [Students discuss.]
Student I: It's a little too big. Because if there were 3 one-and-a-halves, then that would be 4 and a half altogether. So it's a little too big.
Teacher: Could someone confirm that calculation for us? How do you know that

$$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 4\frac{1}{2}?$$

Student J: It should be 3 and 3 halves. Because it's 1 + 1 + 1, which is 3. And then there are 3 halves.
Student I: But 3 halves is 1 and a half. So it's 3, plus another 1, plus a half.
Student J: Oh, right. So that's 4 and a half.
Student K: I know what it really is! It's got to be $1\frac{1}{3}$.
Teacher: Why?
Student K: Because $1\frac{1}{3} + 1\frac{1}{3} + 1\frac{1}{3}$ is 3 ones plus 3 thirds. That makes 4!

Explore

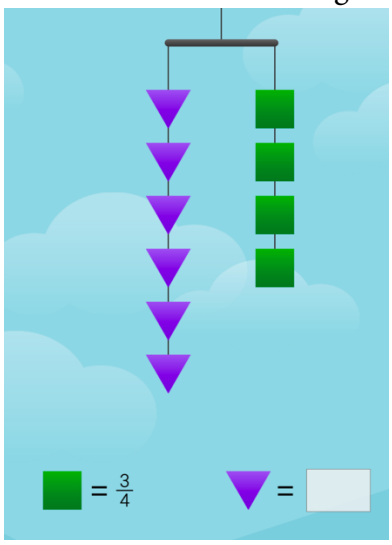
Once students have discussed a balance problem and understand the central premise that "balancing" means "equaling," give them the worksheet of balance problems. They can solve these in pairs to help each other think through their ideas and get unstuck. Students who finish the entire worksheet can make up their own balance problems to pose to their peers.

Questions and Prompts

- What is equal to what in this problem?
- What does the left side of the balance equal altogether?
- What's a number that's too big for the shape to be in this problem?
- What about a number that's too small?
- What would be a better guess for the value of that shape?
- Is your guess too big or too small? Or just right?

Summarize/Discuss

Ask students to consider this problem and share their strategies for solving it.



Student A: I knew the green square was three fourths. So I tried to figure out what 4 of those would be worth together. It went 3 fourths, 6 fourths, 9 fourths, 12 fourths, so it was 12 fourths in all. But 4 fourths makes 1. So the entire right side is 3.

Teacher: First of all, great work figuring out what the right side of the balance is! Can anyone help us figure out what the square is?
Student B: It's less than 1. Because if it were 1, then that would be 6 on the left side, and that's too much.
Student C: That means it must be 1 half! Because 6 is double what it should be. It should be 3.
Student D: I agree with one half. If the triangle is one half, then the left side is 6 halves. And 6 halves is the same as 3 wholes, which is 3. So it balances.

Teacher: (Time permitting) You've gotten the hang of these. So let's do something especially fun: pick two shapes and design a balance problem of your own. You have to check it and make sure you can solve it. But when you know you can, you can share it with someone else and see if they can solve it too.

Tips for the Classroom

1. The balance problems can be challenging. Encourage students to guess at answers. If they can find estimates that are too high or too low, they'll do lots of important math thinking, even if they don't have a direct route to the answer.
2. There is a common misunderstanding of "equals" as meaning "do this operation." This sets students up for confusion in later grades, when equations like 3 + 4 = 6 + 1 cause them confusion. The balance is an excellent visual for equality, and can help them not only with fractions, but with their growing algebraic intuition.
3. Answers to the problems on the following pages:
   hexagon = ¾
   moon = 4/3
   triangle = ½
   square = ½. Diamond = 1
   Circle = 1/3. Square = 2 and 2/3. (or 8/3)
   moon = 15/8

Name_____

Balance Fractions

Find the value of the missing shapes.