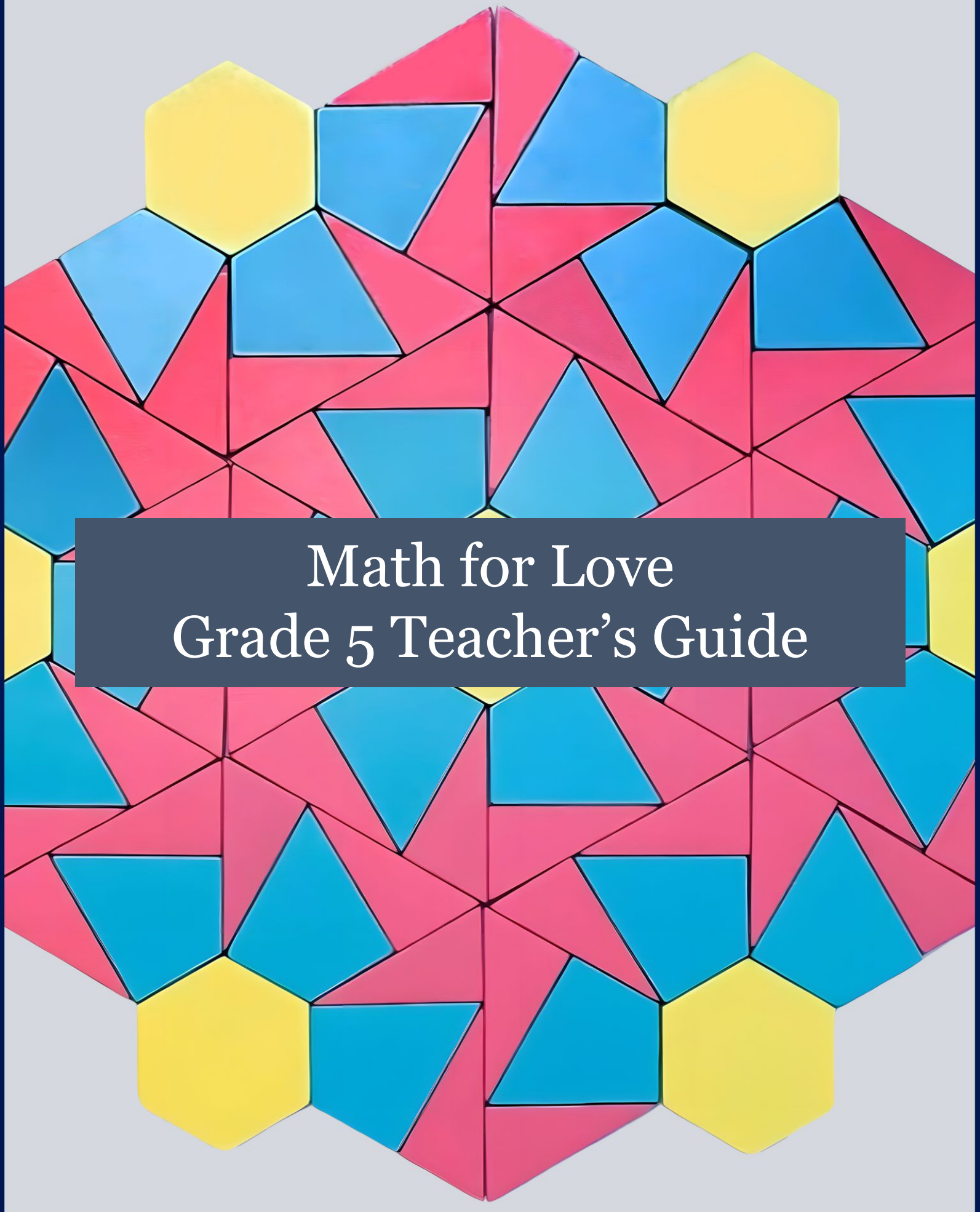


Math for Love
Grade 5 Teacher's Guide

Introduction

Welcome to the Math For Love Supplemental Curriculum! We are thrilled that you will be using this curriculum with your students. Like the lesson plans, we'll make this introduction quick, easy to read, and useful.

We are Math For Love, an organization dedicated to transforming how math is taught and learned. Our passion is connecting students and teachers with opportunities to experience excellent mathematics, deepening everyone's skill and enjoyment in the process.

The Math For Love Supplemental Curriculum is built on our belief that *play* and *rigor* go hand in hand, and that the best of mathematics is accessible to students and teachers who are ready to work hard and have fun. You and your students will learn a lot of math over the next 80 lessons, and by the end we hope you'll see why we think math is one of the best parts of the day.

The Big Picture

We built this curriculum with a few key principles in mind.

Principle 1.
Every student can participate in rigorous mathematical thinking.
Rigorous mathematical thinkers want to understand *why*, not just get the answer. They make connections and seek underlying structure and coherence. They develop powerful tools to solve problems, including fact fluency and procedural efficiency. Rigorous mathematical thinkers ask questions, make conjectures and predictions, test out their ideas relentlessly, and expect to be surprised.

Principle 2.
Play is the engine of learning.
Mathematicians engage in play constantly: exploring, wondering, noticing, and being led by curiosity. Play can transform math class from tedious to joyful, from shallow to deep, from mundane into fascinating. Students at play are more likely to persist, to build tenacity, to remember, and to learn. Play is the secret sauce that helps students come to love and succeed in mathematics.

Principle 3.
Without rigor, mathematical play is formless.
Without play, mathematical rigor is unsustainable.
We need both, together, to get the most out of mathematics.

In this introduction, we'll discuss some specific teacher moves that can help encourage rigorous mathematical play.

But first, some details.

The Details

The Math For Love Supplemental Curriculum is built to provide eighty days of 1-hour (or longer) classes, intended to complement a standard curriculum. It can be used for small groups, enrichment, remediation, after school programs, and summer programs. Every lesson is written to accommodate a wide range of student skill level, making it easy and enjoyable to differentiate and support each individual's learning. Our belief is that beautiful and interesting math problems — when designed to be appropriately accessible — should be offered to everyone, no matter where they are in their math journey.

Materials included with curriculum:

- Teacher Guide
- Student Workbooks
- Manipulative Kit
- Math Games

The Lessons

Each lesson follows a standard format with four sections:

- Opener
- Main Activity
- Closer
- Choice Time

We sometimes provide a sketch of how a lesson might unfold, with prompts and questions to help you respond organically to what your students bring to the conversation. Any sample dialog is never meant to be a script, and precisely how the lesson goes will depend on you and your students.

We include guidelines for how long we expect each part of the day will take; however, times will vary depending on student engagement and your decisions.

When preparing for a lesson, review all sections of the lesson in advance. This will help you make decisions on how to group students, how to arrange materials, and what images to project. Even a little bit of preparation will help you be ready to emphasize what's important in the lesson and respond naturally to your students' ideas.

Choice Time Days

Occasionally a full lesson - after the Opener - is devoted to Choice Time. These Choice Time Days are intended to give students a chance to dig deeper into any lessons, or relax with some extra time to play the games they already know. As with normal Choice Time, you can use the suggestions we provide, or substitute in other options.

See the sample lesson templates on the next pages for more details about the lesson plans.

SAMPLE DAY	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

This is where we highlight the main standards we're focusing on for the day, particularly in the main activity. We usually highlight one practice standard and one content standard.

Materials: This section will mention everything you need for the day. The main activity will also include a materials list.

Opener	**We'll say what the opener is here**	10 – 15 minutes
Main Activity	**We'll say what the main activity is here**	20 – 40 minutes
Closer	**We'll summarize what's happening in the Closer here**	5 – 10 minutes
Choice Time	• We'll provide a short list of good options for Choice Time here. • You're always welcome to choose different options!	5 – 25 minutes

Standards Connections

These are additional standards that are connected (or could connect) to today's lesson.

Opener

The lessons cycle through a short collection of our favorite opening routines. The first time you see a particular Opener, there will typically be more detail included. Later, these writeups will become shorter and more succinct. Don't be surprised to see the Opener instructions look almost identical on different days - once you're confident with a given opener, it should take very little time to prep for using it with class.

Here are the main Openers we use in this grade.

➔ Unit Chats

Project an image with multiple options for what to count.
Prompt: "How Many?"

➔ Number Talks

Share an expression to evaluate.
Prompt: "How many ways can you find the answer?"

➔ Fraction Talks

Project an image that includes several colors.
Prompt: "What fraction of the image is each color?"

➔ Counterexamples

Make a false claim or conjecture.
Invite students to find an example that proves you wrong.

➔ Broken Calculator

Project a calculator with some broken keys, and a target number.
Prompt: how many ways can you hit the target number using the broken calculator?

➔ Teacher-led Games

For example, Bullseyes and Close Calls, Don't Break the Bank, Penny Nickel Dime.

Prompts and Questions

- Look here for useful things to say to students to help them get started or push deeper in their thinking.

Tips for the Classroom

1. Look here for some specific ideas for increasing student interaction, adjusting challenge, and more.

2. If there's an image to project for an opener, it's typically on the next page.

Main Activity

Materials and Prep

Here's where we describe what students will need for the main activity (doesn't include choice time materials). You'll need to read the lesson to make some decisions about how to arrange the materials for the day. In general, keep this simple – offer containers of manipulatives rather than exact amounts.

Motivating Question (OR How to Play)

To begin working or playing on their own, students should either have a question that frames the day's exploration— along with the knowledge and skill to begin thinking about it — or know the rules of the game they're about to play. We essentialize that question (or summarize those rules) here.

Launch

This is how to introduce the motivating question and get students excited and curious to think about it, or to teach the game in a way students will understand and find irresistible. In the case of games, demonstrating with a student volunteer is almost always the most powerful way to communicate how the game is played.

In general, the Launch should be as thorough as necessary *and* as short as possible. The goal should always be to have the students spending as much time as possible doing the thinking during math class. Whenever you are speaking to the whole class, pose questions and look for opportunities to ask for student ideas, questions, and contributions.

Work

As soon as they're ready, students go to work on their own or in pairs or small groups. This section will have some ideas of what to look for, the lesson flow, extensions, good hints, and (occasionally) solutions.

While students work, circulate in the room, offering help, prompts, hints, asking questions, making connections between ideas, and getting a sense of your students' strengths and where they could benefit from greater support.

Tips for the Classroom

1. Look here for additional ideas on how to implement this activity.
2. We'll often include extensions or simplifications to help with differentiation.
3. Student workbook pages will typically be included on the page right after the Tips for the Classroom.

Launch Key Points

- We try to include some key points for how to help the launch succeed in getting students excited to work.
- Points about the essential knowledge or skills might be here too.

Prompts and Questions

- This section gives ideas for what you might say to students during the "Work" section of the lesson, when they're working on their own or in small groups.
- Sometimes a prompt, hint, or nudge to talk to someone else is all students need.

Closer

Gather the students together for a whole-class discussion when the Main Activity is done. This is where students reflect, consolidate their learning, and potentially try an extension or variation of the Main Activity. To make sure the engaged thinking continues during this part of the day, rather than just summing up what everyone should have learned, take the opportunity to pose questions, invite student comments, and use partner sharing to give everyone a chance to participate.

Prompts and Questions

- These prompts are for the Closer.
- They might be useful things to say to the class as a whole.
- They also might be helpful replies to anticipated student contributions to a closing discussion.

Choice Time

Choice Time is when students get a chance to revisit games, puzzles, and other material they want to spend more time with. Getting to choose their activity helps with buy-in and self-regulation, and is a chance for students to reflect on what they want to think about more.

Choice Time works like this:

1. Present students with a short list of suggested activities.
2. Students choose the game, worksheet, challenge problem, block set, or other activity they'd like to pursue and commit to sticking with it for at least 5 - 10 minutes.
3. If time permits, students can try more than one activity.

The suggestions for Choice Time are only suggestions. If there is another activity from the curriculum that you think would be a better fit here, or if a student has a strong preference for something not on the suggested list, feel free to make a swap.

You may need to print some materials in advance to prepare for Choice Time. Since the final Choice options are up to you, we don't give a list of materials you'll need for them.

Here are some options that can be freely offered any Choice Time:

- Challenge Problems
- Free Block Play
- Multiplication by Heart (once students know how to play it)
- Prime Climb (once students know how to play it)
- Work on problems from an earlier lesson

Teacher Moves

Here are some useful ways to support your students during these lessons.

- **Model enthusiasm and curiosity.** Ask questions. Statements like "I wonder if…" and "I notice that…" go a long way. If students see you enjoying the work, they'll be much more likely to enjoy it too.

- **Keep instructions and launches as brief as possible** (but as long as necessary) and look for places to invite student questions or ideas. As much and as often as possible, we want students to be spending classroom time doing mathematics and thinking mathematically.

- When launching games, **play a demo game with a volunteer** to help students learn the rules. When students play games against each other during work time, try these ways of grouping students:
    - Students play one against one and switch opponents often.
    - Students play in groups of three. Two play while one watches as a referee. When the game is over, the referee position rotates.
    - Students play two against two, and have to agree on moves with their teammate.
    - Students play collaboratively with a partner, and try to get the highest score they can, rather than beat an opponent.

- **Resist solving students' problems for them.** While working on hard problems, it's natural to feel stuck, or unsure of what to do next. Sometimes a key insight requires a lot of exploration first. Give students the time they need.

- **On the other hand, support students when they need it.** There's no use in leaving students feeling dispirited or unsuccessful, and the goal is for students to be productive, even if stuck. We provide ideas for questions, prompts, and hints to keep students motivated and engaged. Even when students are playing or exploring, understand your job as looking for opportunities to help students develop greater efficiency, organization, and power in their methods.

- **Have a plan for how to respond to wrong ideas and answers**. One of the strongest ways to handle these moments is to turn them back to the students by treating the idea seriously and asking for counterexamples or supporting arguments. A very good phrase to keep in your back pocket is: "Convince me."

- **Be willing to be the slowest person in the room**. This means asking for elaboration and clarification if you think there is even one student in the room who doesn't understand an argument yet.

- **Care and respect**. Show students you care about them, respect their thoughts, and that it matters to you that they learn, and enjoy, mathematics.

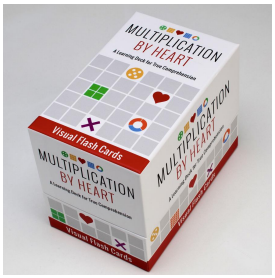
Materials

We provide just about everything you need to use this curriculum with a classroom of 25 (or more) students. The only extras you'll need are scratch paper, pencils, and crayons or colored pencils. You may occasionally need to make some additional photocopies for Choice Time, though students can often turn to earlier pages in their Student Workbook and find what they need. In addition to this Teacher's Guide and the student workbooks, manipulatives and games include:

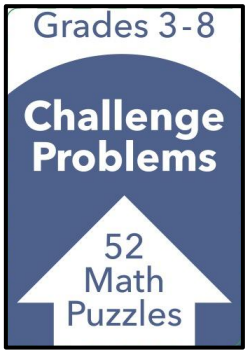
21st Century Pattern Blocks. These blocks include 8 shapes, with enormous possibilities for exploring multiplication, division, fractions, ratios, geometry, and more. These are also great for students to explore with during Choice Time.

Number Rods. Another excellent tool for understanding arithmetic operations, fractions, measurement, and more. Rods go from 1 cm to 10 cm long, in the colors named to the left.

Prime Climb. One of the world's most popular mathematical board games. Includes a unique visual for prime factorizations of numbers that acts as a guide for multiplication and division. Always a good Choice Time option once students learn how to play. Video instructions available at mathforlove.com/prime.



Multiplication by Heart. A visual flash card deck with three subdecks. Great in the classroom for small group fact practice and for simple fluency games. Ideas included in the lessons, and at mathforlove.com/multiply.



Challenge Problems Deck. These extra puzzles and problems are great options for Choice Time. These generally get harder the higher the number.

Also included: **Square Tiles**, **6-sided and 10-sided Dice**, and **Polyominoes** (which include dominoes, triominoes, and tetrominoes).

Other Stuff

- Email errata@mathforlove.com if you notice an error that should be fixed.
- **Additional Material:** We'll gather corrections and additional material at mathforlove.com/curriculum/grade5.
  Password: M4LCurriculum
- Problem with access? Email info@mathforlove.com.

Thanks and Acknowledgements

These lesson plans were built from the lessons we developed over our years working with teachers and students of all ages. However, putting together this more ambitious curriculum required a team, and we were lucky to have an amazing one.

Our curriculum writers were Karen Gallagher, Mark Goldstein, Tara Hofmann, Becky Holden, and Chase Orton. Our editors were Hana Murray and Jen Moffett. We had help with images from Bella Christianne and Hana Murray. Hana Murray also created the cover using photos of 21st Century Pattern Blocks.

This team of writers and editors worked with incredible focus and skill to build the teacher's edition you're holding now — big thanks to all of them for their dedication and contributions to this project.

Some images for this book were made, with permission, using Mathigon Polypad (polypad.amplify.com) - thanks to the good folks there for building such a fantastic tool. Mathigon also partnered with us to create digital versions of the Multiplication by Heart and Addition by Heart cards included in the curriculum kits. If you'd like to use the digital versions, they are free to use at fluency.amplify.com.

Finally, thanks to all the teachers, coaches, students, and staff who have used versions of our materials over the years, and welcomed us into their classrooms.

Daniel Finkel | Founder | Math for Love

Katherine Cook | Creative Director | Math for Love

Table of Contents

Table of Contents

Table of Contents

Table of Contents

DAY 1	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP6 Attend to precision.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Materials: 21st Century Pattern Blocks, scratch paper and pencil.

Opener	**Bullseyes and Close Calls**	10 – 15 minutes
Main Activity	**Forty Faces**	20 – 40 minutes
Closer	**Strategies for Calculating Values of Faces**	5 – 10 minutes
Choice Time	• Block Free Play • Bullseyes and Close Calls • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP6 | MP7 | 4.NBT.5

Bullseyes and Close Calls

Secretly choose a number with no repeated digits and write it down where no one can see it. Play with a 2-digit number for the first game, and a 3-digit number for games after that.

Students attempt to guess the number. After each guess, give feedback using a combination of "Close Call" and "Bullseye", or possibly "Nothing". See chart to the right.

Note: you might need to say "2 Close Calls" or "1 Bullseye and 2 Close Calls" or some other combination. DON'T say "first digit Close Call, second digit Bullseye." Your responses apply to the entire numbers, not individual digits.

As soon as the rules are mostly clear, begin to play the game. Any confusion about the rules will get worked out during play itself.

Outcome of Guess	Feedback
Correct digit in the wrong place	"Close Call"
Correct digit in the correct place	"Bullseye"
No correct digits	"Nothing"

Example Game

Your secret number is 487.

Guess	Feedback
139	Nothing
820	Close Call
468	1 Close Call, 1 Bullseye
568	Close Call
482	2 Bullseyes

Tips for the Classroom

1. Note that students DON'T get a Bullseye or Close Call for each digit. The clue applies to the entire 2- or 3-digit number.

2. Write the guesses and the responses somewhere that everyone can see it.

3. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses. For example, after guessing 139 and finding that none of those digits are in the number, cross off the 1, 3, and 9 from the list of possible digits.

4. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

Prompts and Questions

- What numbers can I cross off after that guess?
- Is there anything you know after that guess? Any number that is or isn't in our mystery number?
- Why are you so sure the number doesn't have a 5?

Forty Faces

Materials and Prep

21st Century Pattern Blocks, scratch paper and pencil, Number Rods (optional).

Motivating Question

How can you make a face with an area equal to exactly forty (or fifty) green triangles?

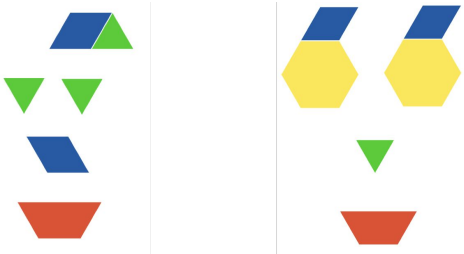
Launch

Hold up a pattern block hexagon and a green triangle, and ask students if they know how many green triangles it would take to build the hexagon. Repeat for the blue diamond, the red trapezoid, and the purple concave hexagon (stop there–leave the other 21st Century Pattern Blocks for later). Students who have worked with Pattern Blocks before will be able to answer. You can confirm by building.

Project the images below (see next page), or build them where everyone can see. Ask for guesses about how many green triangles' worth of area each face is.

After discussing the guesses, show how the faces are worth 10 or 20 triangles. For the second face, for example, there are 2 hexagons, 2 rhombuses, one trapezoid, and one triangle. In terms of triangle area, the total "value" would be 12 (in hexagons)+ 3 (in trapezoids) + 4 (in rhombuses) + 1 (in triangles) = 12 + 3 + 4 + 1 = 20 triangles worth of area. Demonstrate writing this equation down.

Once students understand how to count the "value" of the face, challenge them to create their own faces from pattern blocks that have value (i.e., area) 40 and 50.



Launch Key Points

- Discuss how each pattern block can be built from triangles–pose a quick series of questions to students to find the values of the hexagon, blue diamond, and red trapezoid.
- Ask students to guess the value of the faces before discussing them.
- Make sure students understand how to model a face with an equation.

Forty Faces

Work

Circulate the room as students construct their faces. Encourage them to check their neighbors' work to make sure the faces everyone builds are actually worth 40 and 50.

Tips for the Classroom

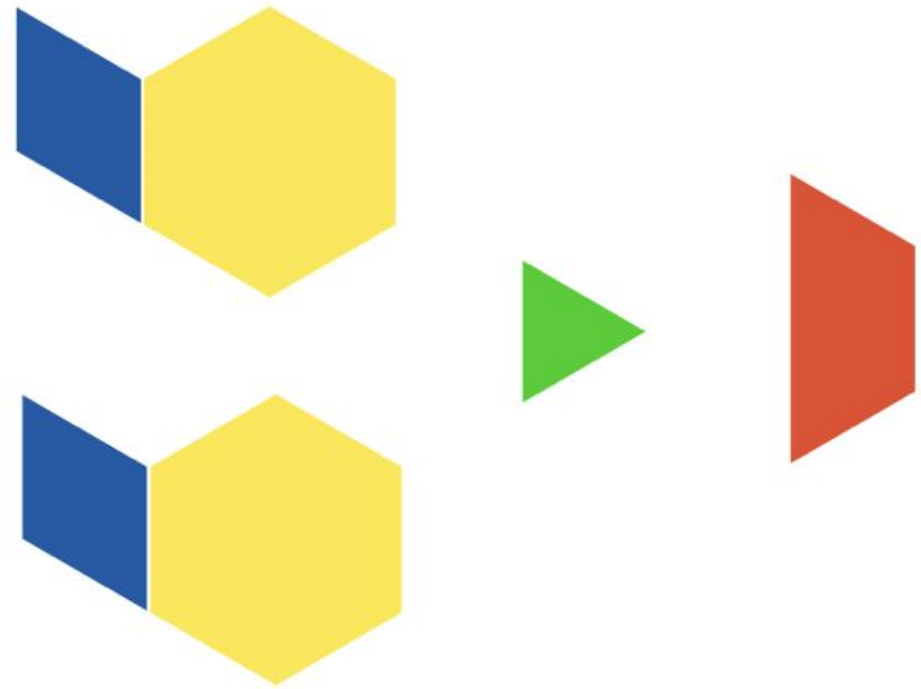
1. Let students challenge themselves when they're ready. Can they make a 100 face?

2. Encourage students to use pencil and paper to track the arithmetic. It gets difficult to find the answer without making a mistake once the faces get larger.

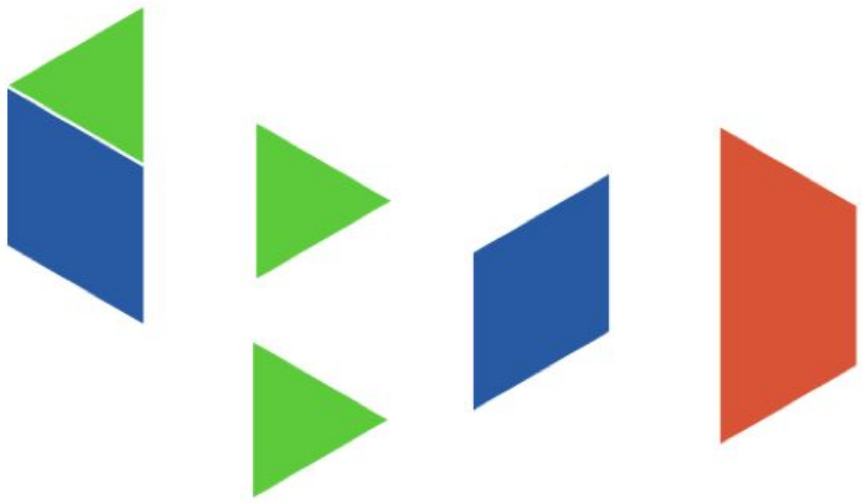
3. The new shapes of 21st Century Pattern Blocks create an additional challenge: what are those blocks worth? Figuring out the answer requires some subtle arguments. This will come up in the Closer, and in future lessons.

Prompts and Questions

- How much more area do you need to add to get to 40?
- Show me how you found the area.
- Let's count how much the hexagons are worth.
- The trapezoids came to 18 area? Let's write that down.
- Do you think the two of you could make a face with an area of 75?



20 Face



10 Face

Closer

Ask students to tour the room and find a face that looks like it has area 40 but actually doesn't. Can they find any errors? (You can plant a face with area 39 somewhere, though chances are there will be errors in the room.)

Once students get a chance to appreciate each other's creativity and check their work, gather students and show a face that has area close to 40. Ask students to find its value, and be ready to defend their thinking about it. Ask students to share their approaches and discuss some of the different ideas with the class. Possible strategies include skip-counting, multiplying, or finding the value of specific blocks. Once everyone agrees on the area of the figure, take student suggestions for how it could be adjusted to come to forty exactly.

A question that should come up is what the value of the blocks that can't be built from green triangles are actually worth. These include the pink triangle, the gray dart (concave quadrilateral), and the teal kite. Ask students if they have any ideas what the value of these blocks might be. Don't solve this problem for them - it'll be the topic of an upcoming lesson. If they can solve it on their own, though, that's wonderful.

Prompts and Questions

- Can you explain why this face has an area that is almost forty triangles?
- Could you use skip-counting or multiplication to find the value of specific blocks?
- How might we change this face so that the area is exactly forty?

Choice Time

Today's Choice Time Options

- Block free play
- Bullseyes and Close Calls
- Challenge Problems

Prepare students for Choice Time by explaining that they will choose from some pre-selected options. Tell students that once they choose an activity, they should spend at least 10 minutes on it before trying something else.

Show the Challenge Problem Deck and explain that it will always be among the Choice Time options. Students can simply take a challenge problem card and try to solve it, on their own or with a partner. Be sure to provide scratch paper and pencils to support student work.

Block free play is also an option. This is an opportunity for students to build with Pattern Blocks or other manipulatives.

Overview

Focus Standards

MP1	Make sense of problems and persevere in solving them.
3.NBT.2	Fluently add and subtract within 1,000 using strategies based on place value.

Materials: Six sided dice, Pig Scoring Sheet and pencil.

Opener	**Unit Chats**	10 – 15 minutes
Main Activity	**Pig & Big Pig**	20 – 40 minutes
Closer	**Pig Strategy Conversation**	5 – 10 minutes
Choice Time	• 21st Century Pattern Block Free Build • Pig & Big Pig • Challenge Problems	5 – 25 minutes

Standards Connections

MP5 | MP6 | MP7 | 3.OA.7 | 3.NBT.2

Unit Chats

This simple-to-launch exploration emphasizes not just how many, but also the unit involved. With a variety of images to use, unit chats are productive, fun, differentiated, and delightful.

Project a Unit Chat image (on the following page). It will include a variety of objects to count.

Prompt the students to look at the image and answer the question **how many**. The question is ambiguous on purpose. Let them know that there are many correct ways to answer the question, depending on what they choose to count.

Students can take a minute or two to look at the image and chat with a partner about what they see. Talk to groups and hear how people are approaching the image, and be prepared with prompts for those who are having trouble getting started.

Once each group has some ideas to share, transition to a whole-class discussion. Groups can share what they counted, and how they know their count is correct. Ask questions, clarify their ideas as necessary, and make notes on the image, or write equations or other notes on another surface.

Do 1 - 2 unit chats, as time permits.

Some possible student answers to today's Unit Chat:

- I see two colors.
- I see two different types of dice.
- 8 (total number of dice, total number of dots on the white dice.)
- 12 (total number of dots on the black dice.)
- 20 (total number of dots on all the dice.)

Prompts and Questions

- How did you see that?
- How did you count that?
- Does anyone else think they can explain what Therese is saying?
- Did anyone count something different?
- Did anyone count the same thing but with a different strategy?

Tips for the Classroom

1. You can emphasize how students counted, or shift the conversation to what they counted, depending on what will be the most engaging and enlightening.

2. Each unit chat image usually gives you 5 - 10 minutes of conversation. Don't try to get every possible observation out of an image - if the conversation starts to flag, just move on to the second image.

Image 1

Image 2

Pig & Big Pig

Materials and Prep
Six-sided dice, Pig Scoring Sheet, and pencil.

How to Play
Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a 2, 3, 4, 5, or 6, the player adds that many points to their score for the turn. After a roll, a player may choose to end their turn and "bank" their points. If a player rolls a 1, they lose all their unbanked points and their turn is over. The first player with 50 points or more in their bank wins.

See Closer for rules for Big Pig.

Launch
Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students advise you on whether they think you should keep rolling by giving a thumbs up/down.

Play until it feels like most students are clear on the rules - usually 3 - 4 turns for you and your volunteer.

Work
Students play Pig in pairs. Consider having both students in a pair keep score for both players, to catch any arithmetic errors.

Tips for the Classroom
1. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!

2. Beginner Game: The first player to score 50 or more points wins. Advanced Game: The first player to score 100 or more points wins.

3. Whoever isn't rolling is in charge of writing. That way, every student has something to do every turn.

Launch Key Points

- Take risks in the demo game! Ask students when they think you should stop rolling; they can indicate their opinion quietly with a thumbs up or down.
- Demonstrate how to use the score sheet, where to keep track of rolls, and when to bank.
- It can be useful to roll recklessly until you get a 1 for one of your turns, to show losing all the points for that turn.

Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?
- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?

Closer

Gather the whole class together for a discussion of Pig. Ask students if they think they are using any strategies as they play. For example, a student might roll 3 times, then bank, or roll till they have 10 points, then bank.

You may find that some students play a riskier game than others. If time permits, consider selecting a student who plays a risky game (roll 10 times before banking) and one who plays a conservative game (bank after one roll) and have them play against each other in a demonstration game. You can take a vote ahead of time about which strategy is more likely to win.

There is an opportunity here to discuss the difference between a good strategy and a single loss–luck plays a big role here!

If time remains, share the extended game of Big Pig with students as a choice time option for today, and for the future.

Big Pig. Big Pig is the same as Pig, except that players roll 2 dice each turn. If they roll a 1 on *either* die they lose all their unbanked points. If they roll doubles, they get twice as many points as normal (e.g., double 5s are worth 20 points instead of 10.)

If they roll a pair of 1s, they gain 25 points and can keep rolling!

Prompts and Questions

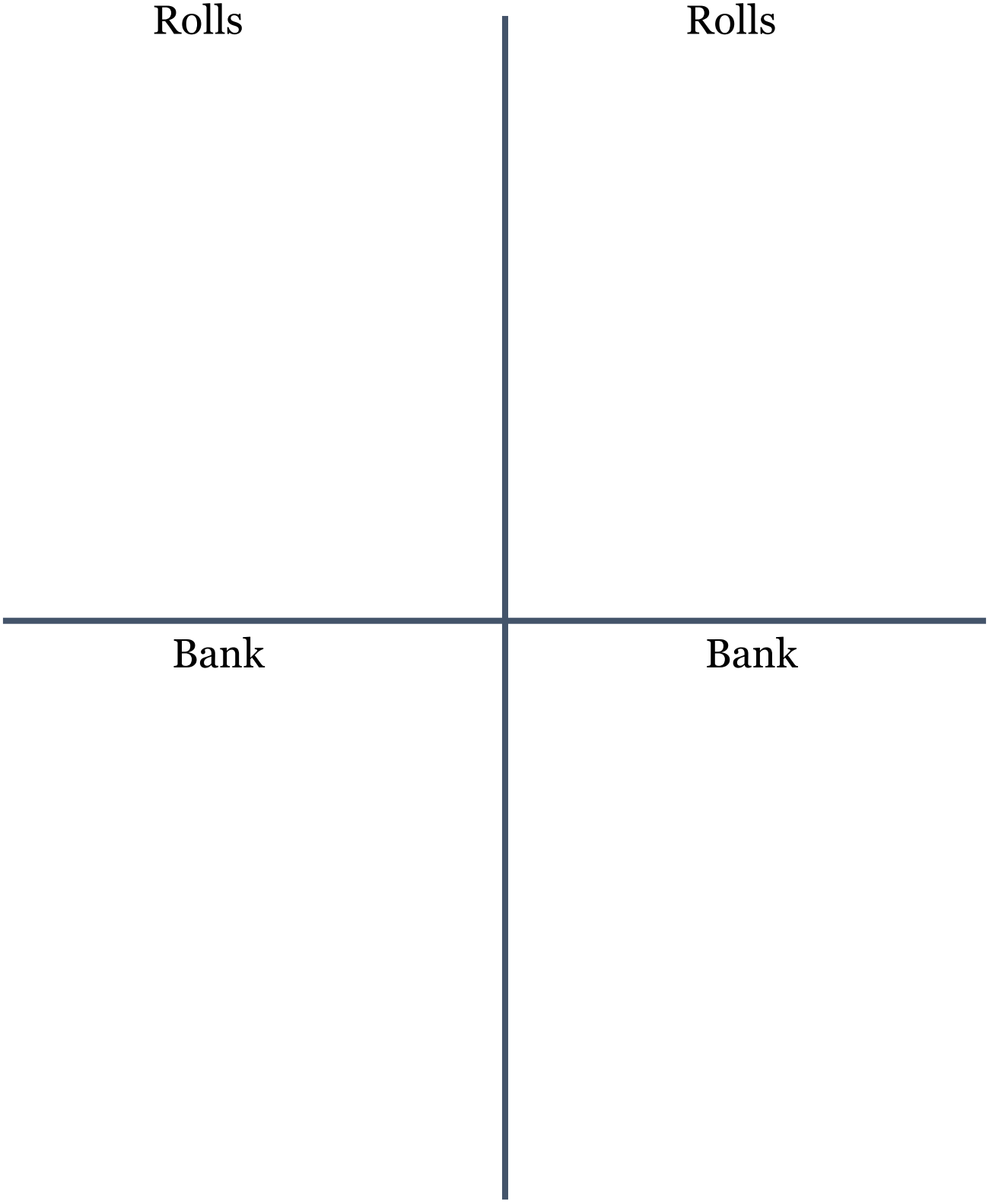
- What's your favorite winning strategy so far?
- Do you prefer to be more risky or more cautious? Which seems like it works better in the long run?
- Do you have a rule for when you stop rolling and bank your points?

Choice Time

- 21st Century Pattern Block Free Build
- Pig & Big Pig
- Challenge Problems

Day 2

Pig



DAY 3	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

5.OA.A Write and interpret numerical expressions.

Materials: 21st Century Pattern Blocks, scratch paper, and pencil.

Opener	**Counterexamples**	10 – 15 minutes
Main Activity	**Pattern Block Proofs**	20 – 40 minutes
Closer	**Areas of 21st Century Pattern Blocks**	5 – 10 minutes
Choice Time	• Blockout • Pig & Big Pig • Challenge Problems	5 – 25 minutes

Standards Connections

MP3 | MP4 | MP5 | MP7 | MP8 | 6.G.1 | 6.EE.6 | 5.NBT. | 4.OA.2 | 4.NBT.5

Counterexamples

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All _____s are _____" or "No _____s are _____."

For the first day you play Counterexamples, get the students used to the game with the statement **"All birds can fly."**

Tell students their job is to come up with an example that proves your statement false. In this case, they are likely to suggest penguins as a counterexample. If they do, modify your statement: **"All birds can fly except penguins."** Counterexamples here could include baby birds, injured birds, ostriches, etc.

Next, use the following motivating examples and an "I notice" or "I wonder" statement. This allows students to see you model the process of making a conjecture.

Step 1. Share motivating examples
Write these down where everyone can see them.

- 4 + 5 = 9
- 10 + 11 = 21
- 12 + 13 + 14 = 39
- 1 + 2 + 3 + 4 + 5 = 15

Share the following observation: "I notice that every time I add consecutive numbers, the sum is an odd number." (*Consecutive* means "next to each other.")

Step 2. Pose a conjecture
State this as an 'obvious' conclusion from the motivating examples. You might say, "I bet it's true that every time I add consecutive numbers, the sum is odd. I'm going to *conjecture* that this is true." Then write:

Conjecture. Any sum of consecutive numbers is odd.

Step 3. Invite students to try to find counterexamples - that is, examples that prove your conjecture false.
They may need some time. Every time a counterexample is offered, ask the class to consider it fully and see whether it satisfies the goal. Sometimes counterexamples can be deceiving!

Possible counterexamples students might share include:

Prompts and Questions

- I think I see a pattern here. I'm going to make a conjecture...
- You think my conjecture is wrong. But how can you *show* me it's wrong?

Counterexamples (continued)

3 + 4 + 5 = 12
4 + 5 + 6 + 7 = 22

These are counterexamples because they represent consecutive numbers with a sum that is not odd. This contradicts the original conjecture, so the conjecture must be false!

Step 4. At this point, extend the game by offering a revised conjecture that accounts for the counterexamples the students came up with.
Some examples of possible refined conjectures include the following:

Revised Conjectures
Any sum of two consecutive numbers is odd. (This is true.)
Any sum of an even number of consecutive numbers is even. (This is false.)

How you frame the refined conjecture will depend on what counterexamples the students found.

Once you have a refined conjecture, students can continue searching for new counterexamples. As long as students are engaged, feel free to continue cycling between refining the conjecture and seeking counterexamples, though in general one or two iterations will be sufficient.

Pattern Block Proofs

Materials and Prep
21st Century Pattern Blocks, scratch paper, and pencil.

Motivating Question
How can you determine the relative areas of all the 21st Century Pattern Blocks?

Launch
Refer students to all 8 of the 21st Century Pattern Blocks. Remind them of which areas they can find (in terms of the green triangle) and which are still unknown (the gray dart, the teal kite, and the pink right triangle). Today we'll try to find them all!

You can start by taking student guesses on what the missing values are. Students may have intuitions, but math will allow everyone to create arguments and know for sure.

Build large hexagons (see image on the right) in two different ways, so all students can see them. Suppose we didn't already know that the concave purple hexagon had an area of 4 triangles. Ask students how these two large hexagons could help them prove it.

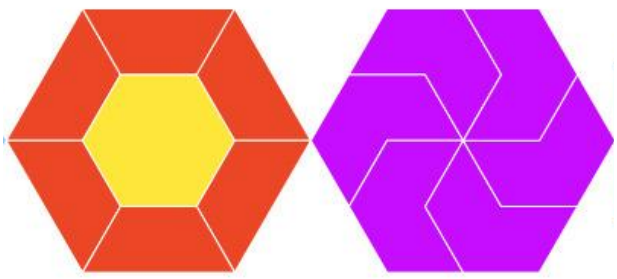
Students can discuss in trios, then as a class. Students will, ideally, be able to articulate an argument: we can find the area of the shape on the left in terms of green triangles (24) and the right in terms of the "unknown" shape (6 concave hexagons).

So 6 purple = 24 green, which means 1 purple = 4 green.

We already knew that, since you can build the purple shape from 4 green triangles. But this is still a powerful approach, since it could work for shapes we CAN'T make out of triangles. Thus, the challenge: **Find the areas of the gray dart, the pink right triangle, and the teal kite even though none can be built from equilateral triangles.**

Launch Key Points

- If students can build a shape twice, once with blocks they know, once with blocks they don't, they'll be able to make these kinds of arguments.
- It's tempting to write full equations (6p = 24g) and use algebra, but applying the rules of algebra abstractly will confuse students more than enlighten at this early stage. Keep working visually, and emphasize the equations as connected to the images.



Work

Students work in pairs or small groups to find the areas of the three unknown shapes. Circulate to give hints, help students write equations to match their drawings, suggest next steps in their arguments, or just listen.

A good first hint: the same hexagon shape we used in the beginning can be used to relate all the shapes to each other. (See image below.)

If students can successfully find all the missing areas, encourage them to find smaller shapes to create simpler relationships between blocks.

Tips for the Classroom

1. Tracking the unit (the area of 1 green triangle) is important. You can shorten it to simply "g".

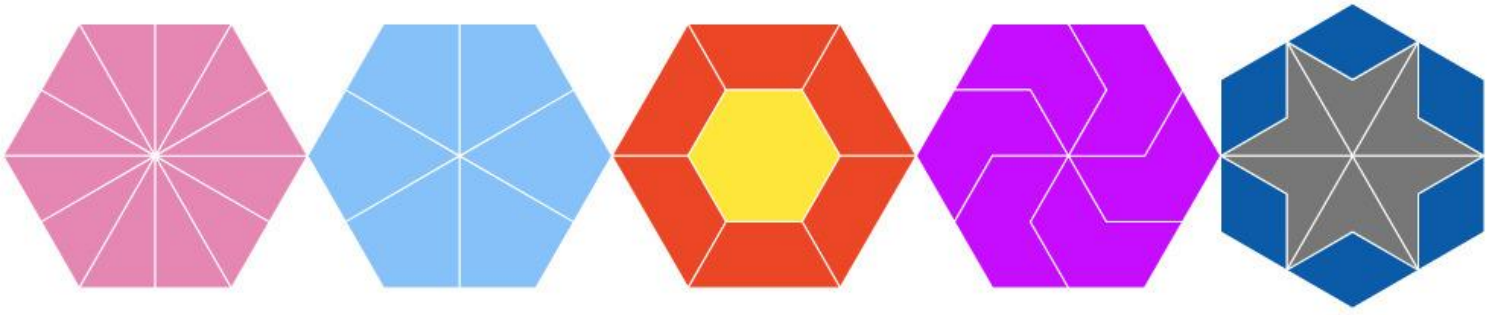
2. This activity isn't just about getting the answers. It's about connecting geometry with equations to solve for unknowns. Don't let students satisfy you with right answers only. Clear arguments are the goal.

3. If you need them, however, the answers are:
   Gray Dart = 2g
   Teal Kite = 4g
   Pink Triangle = 2g

4. Make sure each member of each group is engaged. This usually means that everyone should be manipulating blocks themselves. If you see kids who aren't touching blocks at all, intervene to help them engage.

Prompts and Questions

- Can you make a shape out of blocks you know, and also out of blocks you don't know?
- What if we try the same hexagon shape again. Could you make it out of kites?
- What do you know based on what you've built?
- Can you find another shape that allows you to make a simpler argument?



Closer

Show the image below on a projector or document camera, and ask students to find an argument to relate the orange square and the light gray rhombus.

Students can work in groups of 1 – 3 for about 5 minutes. Then students present ideas to the class.

The argument to expect from this image is that the shapes are the same area, and both include 12 triangles. So what's left when you remove 12 triangles from each must also be the same. That means 6 squares has the same area as 12 rhombuses. So the area of each square is double the area of a rhombus.

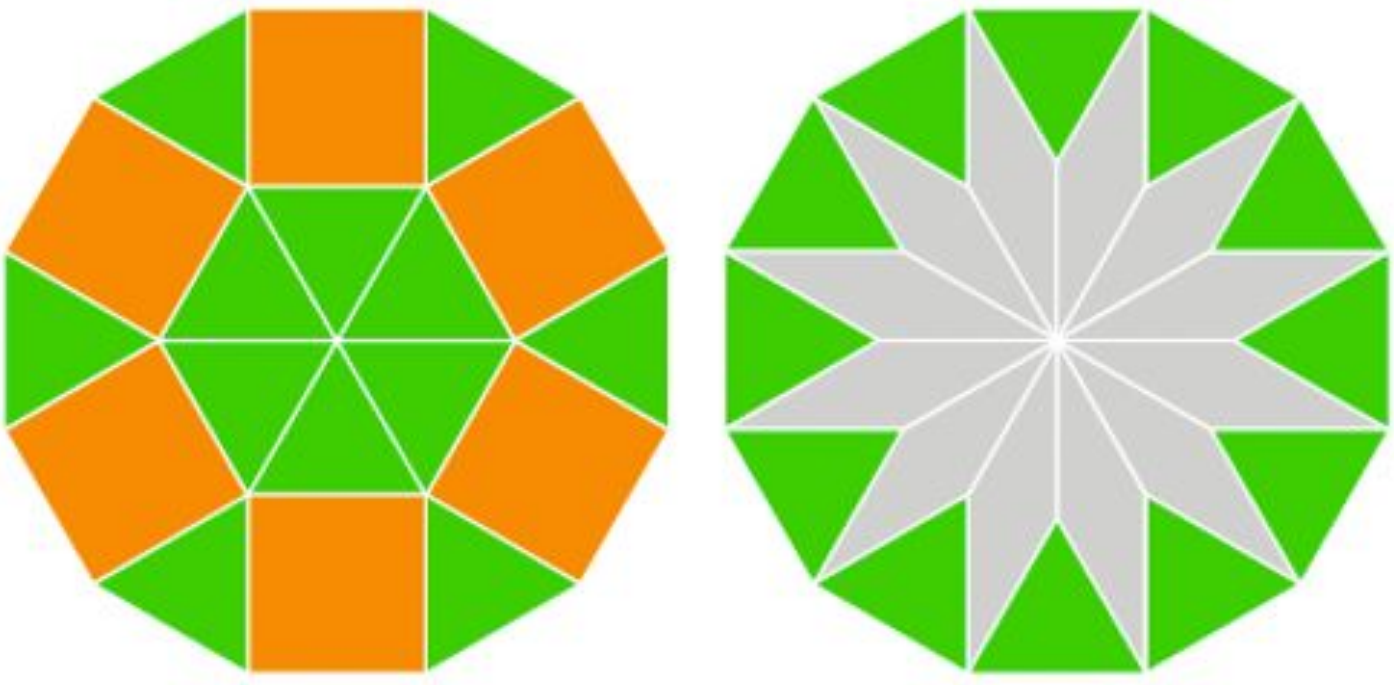
If you wrote this out algebraically, you might write:
6 squares + 12 triangles = 12 rhombuses + 12 triangles
So 6 squares = 12 rhombuses.
That means 1 square = 2 rhombuses.

Prompts and Questions

- What can you figure out from this image?
- Do the two shapes have the same area?
- How many triangles are in each?
- How many squares/rhombuses?
- If you took the triangles away, would what's left still be equal?

Choice Time

- Blockout
- Pig & Big Pig
- Challenge Problems



Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

5.NBT.B Perform operations with multi digit whole numbers.

Materials: 21st Century Pattern Blocks, Sunflowers templates, scratch paper and pencil.

Opener	**Unit Chats**	10 – 15 minutes
Main Activity	**Pattern Block Sunflowers**	20 – 40 minutes
Closer	**Do the patterns you see make sense?**	5 – 10 minutes
Choice Time	• Pig and Big Pig • Blockout • 21st Century Pattern Block Free Build • Challenge Problems	5 – 25 minutes

Standards Connections

MP2 | MP6 | MP7 | MP8 | 4.OA.2 | 4.NBT.4

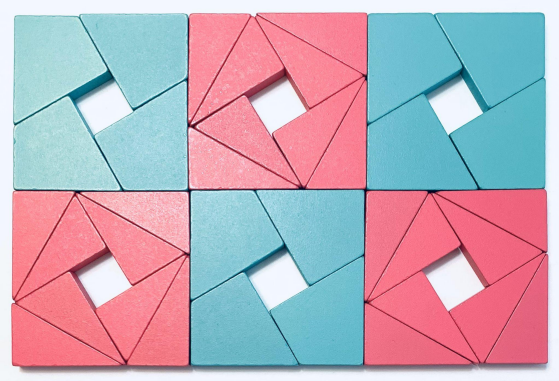
Unit Chats

Project the first Unit Chat image.
Ask students **How many?**

Give them a minute to think privately. Then invite students to share their answers with their partner.

After they've had enough time, discuss 3 - 4 student answers with the class.

Repeat with the second Unit Chat image, if time allows.



Prompts and Questions

- How did you see that?
- How did you count that?
- Does anyone else think they can explain what Therese is saying?
- Did anyone count something different?
- Did anyone count the same thing but with a different strategy?

Image 1

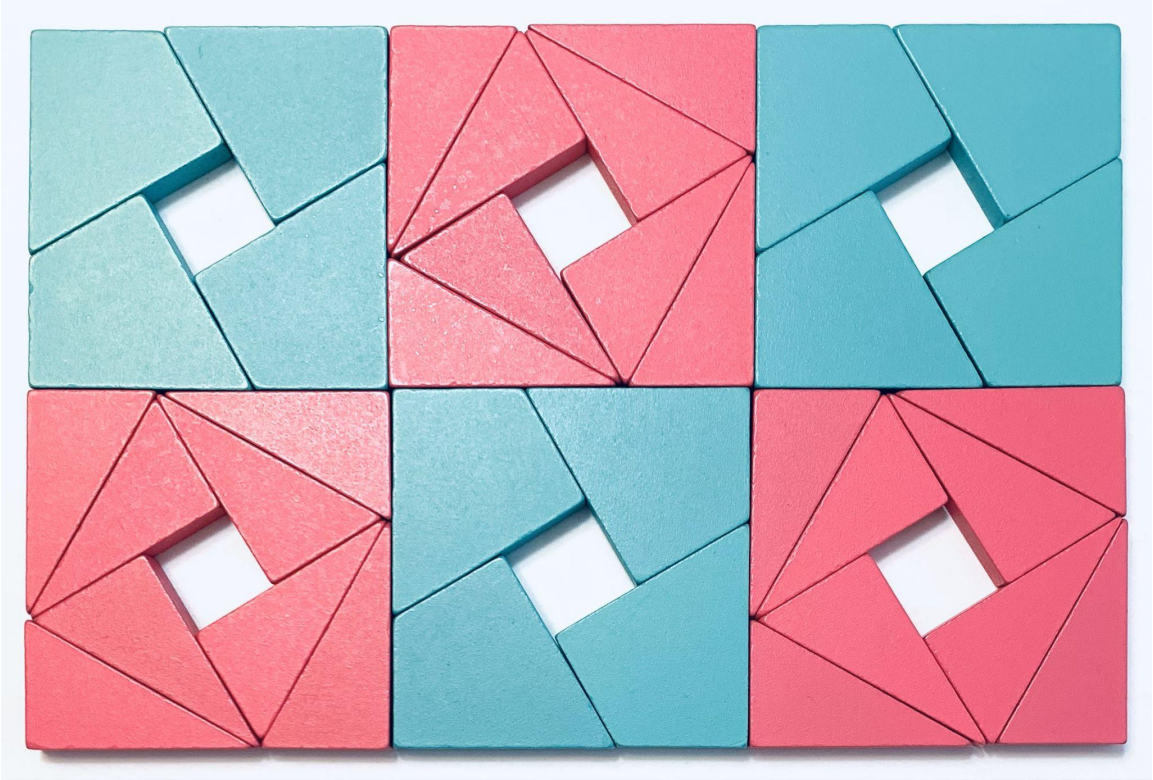
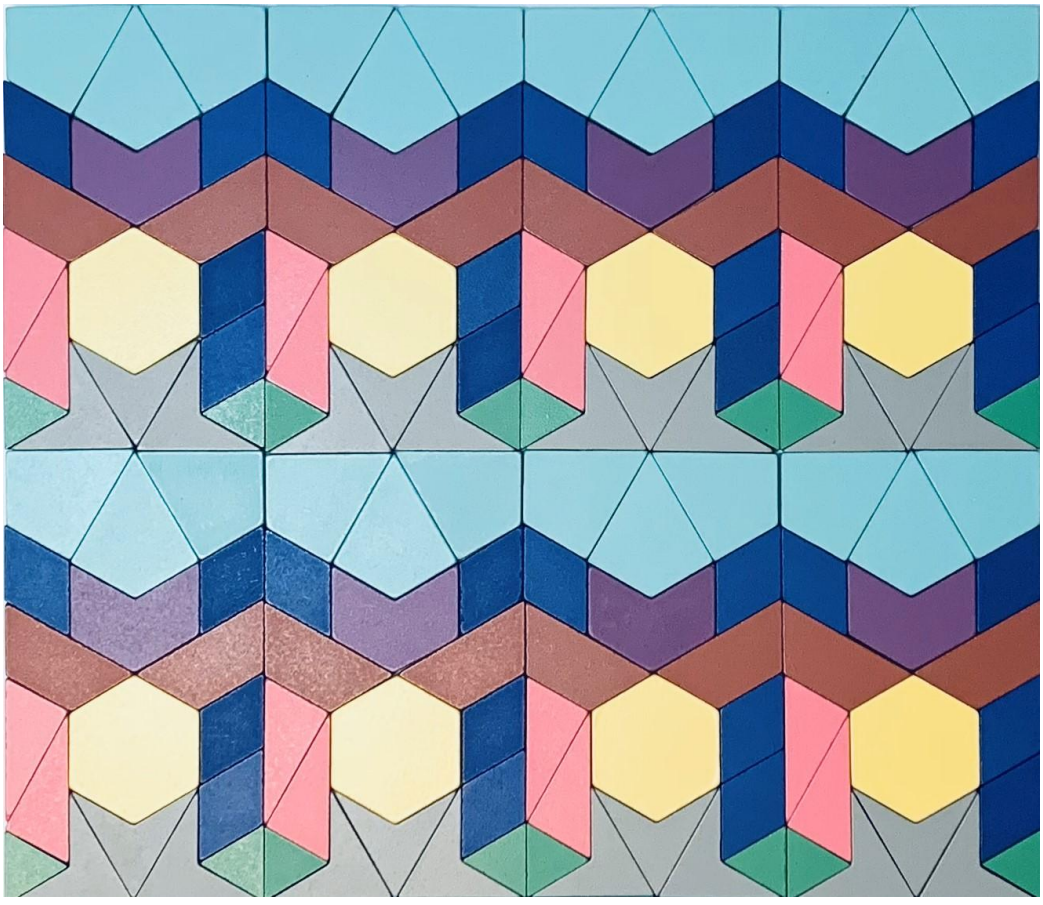


Image 2



Pattern Block Sunflowers

Materials and Prep
21st Century Pattern Blocks, sunflowers templates, and pencil.

Motivating Question
If you know how many blocks it takes to make one sunflower, can you figure out how many it will take to make more of them?

Launch
Build or display the "sunflower" made of 21st Century Pattern Blocks, and ask students how many of each type of block it uses. (1 hexagon, 2 trapezoids, 3 rhombuses, 6 triangles.) These are the ingredients it takes to build this sunflower. Ask students how many of each block they'd need in total to build 2 sunflowers. Give them a bit of time with the blocks and a partner to work it out. Once students understand the idea, display the chart and have students guide you on filling in the 2nd row.

Next, tell students that their goal is to know how many of each block it takes to make anywhere from 1 - 15 sunflowers.

Work
Circulate and help students as they complete the chart. Encourage students who need more concrete examples to build three sunflowers and count the blocks, and then record them clearly in the table. Nudge students toward/away from building sunflowers when they need more concrete/abstract work. For students ready for more of a challenge, ask them to continue the table for 6, 7, etc. sunflowers. Or they can jump ahead: how many of each block would it take to make 12 sunflowers? How do they know?

If students can complete the entire chart, challenge them repeat the exercise with a new sunflower of their own design. Students should make their own chart for this, depending on which blocks they use.

Tips for the Classroom
1. A key element in the flow of the class will have to do with nudging students toward or away from actually building sunflowers depending on their readiness. Encourage students to make predictions about what's coming next in the table. See if they can explain why their prediction makes sense to them, or if it's just a hunch.

2. Don't worry if you don't have enough 21st Century Pattern Blocks for everyone to build 5 sunflowers! Students should be motivated by the very lack of blocks to come up with other ways of handling these problems, aside from counting one by one, say.

Launch Key Points
- Students may have different approaches to count the blocks necessary for two flowers.
- Students do not need to build each flower (though they may choose to).
- Demonstrate how to use the table to keep track of quantities.

Prompts and Questions
- How do you know that's the number of rhombuses you need for three sunflowers?
- What patterns do you see in the table that might help you?
- Are you sure that pattern will still be true when you add another sunflower?
- (Challenge) How many <u>total</u> blocks would it take to build all 15 sunflowers?

Closer

Gather students and ask them to look at the completed table. Ask what patterns they notice. Discuss patterns, reasoning, and predictions.

Once they've shared what they see, challenge them to figure out how many of each block it would take to make 25 pattern block sunflowers. They can work in pairs to solve that problem. Once students have had enough time, discuss their approaches. Students might notice, for example, that you can add the 10 and 15 rows together to get the 25 row.

Choice Time

- Pig & Big Pig
- Blockout
- 21st Century Pattern Block Free Build
- Challenge Problems

Prompts and Questions

- Can you explain why any of these patterns are there?
- Do the patterns make sense or are they a mystery?
- Can you use any of these patterns to predict how many of each block it would take to make a whole garden of sunflowers? How?
- How much do you trust your own predictions?

Day 4

Pattern Block Sunflowers

Number of Flowers	Hexagons	Trapezoids	Rhombuses	Triangles	Total
1	1	2	3	6	12
2					
3					
4					
5					
10					
15					

Day 4

Make your own!
Pattern Block Sunflowers

Number of Flowers	Triangles	Quadrilaterals	Hexagons	Total
1				
2				
3				
4				
5				
10				
15				

Overview

Focus Standards

MP2 Reason abstractly and quantitatively.

5.NBT Perform operations with multi digit whole numbers.

Materials: Six sided dice, Number rods, scratch paper and pencil.

Opener	**Penny, Nickel, Dime**	10 – 15 minutes
Main Activity	**Number Rod Multiplication & Division**	20 – 40 minutes
Closer	**Discussion of a Number Rod Multiplication and/or Division Problem**	5 – 10 minutes
Choice Time	• Penny, Nickel, Dime • Bullseyes & Close Calls • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP3 | MP7 | 3.OA.2 | 3.OA.3 | 3.OA.4 | 3.OA.6 | 3.OA.7 | 4.OA.2

Penny Nickel Dime

How to Play
Roll the die. Whatever number it lands on, each player can choose to take that many pennies, that many nickels, or that many dimes. Each student records their numbers on their own recording sheet as the game continues. Repeat for seven rolls in all. The winner is the person who comes the closest to $1 without going over.

Play a game with your students. Go slowly enough that everyone can make their decisions and keep up, but quickly enough that students can't work out exactly what their current total is, and have to make estimates. Once a game is complete, have students share their score. Who busted? How close were you to getting to $1 without busting?

Invite the player with the best score to explain where they put their coin choices to win - and for the class to check that they didn't make any arithmetic errors!

Play 1 - 3 games, as time allows. Then have students share strategies.

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Penny, Nickel, Dime
[Sample Game]

Roll	Dimes	Nickels	Pennies
1	4		
2		2	
3		3	
4			6
5			1
6	1		
7			5
Totals	5 dimes	5 nickels	11 pennies

Tips for the Classroom

1. Start play quickly, and assume that by the end of the first game students will have absorbed the rules.

2. While kids may bust their first few games, they'll inevitably start estimating and choosing good strategies for themselves.

3. To keep the game novel, ask students what they'd do differently if there were only 5 rolls per game, or six. Or eight! Try those variations and see what happens.

4. Downlevel the game by removing nickels as an option. Uplevel the game by adding quarters and change the target number to $2.50 or higher.

Prompts and Questions

- Do you have a strategy to keep the total close to $1?
- How are you deciding if your roll goes into dimes, nickels or pennies?
- What might you try the next game that will help you win?

Number Rod Multiplication & Division

Materials and Prep
Number rods, scratch paper and pencil.

Motivating Questions
If you know the length of one Number Rod, can you find the others? How can you put Number Rods together to hit target values?

Launch
Project images or physically build the number rods where everyone can see. Tell students you'll be posing a series of problems for them to solve today.

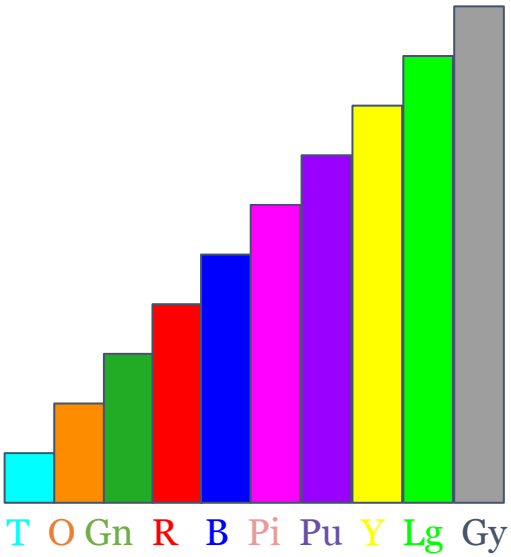
The problems to pose are below. Make sure every student has access to number rods and pencils. As you discuss the problems, emphasize that good arguments are what you are looking for here.

Note: You can pose these problems with almost no words by placing the Number Rods on a white board, and writing the numbers underneath or beside them.

Work
Problem 1. If the turquoise (1 cm) Number Rod equals 1, what are the other rods worth?

If students haven't thought through this kind of problem before, this is a good warm up problem. Students will likely build a staircase from the rods, and see that orange = 2, dark green = 3, and so on, up to gray = 10.

Challenge them to determine what gray + light green + yellow is (gray + light green + yellow = 10 + 9 + 8 = 27). Once students have found what all the rods are worth, you can ask them to prove how they know that the light green rod is 9. There are many ways to prove it using what you know about the smaller. For example, the light green rod is 4 oranges (2 cm each) plus 1 turquoise (1 cm). That's 9. It's also a blue (5 cm) plus a red (4 cm) which is 5 + 4 = 9. It's also one turquoise (1 cm) less than an gray (10 cm) rod, which gives 10 - 1 = 9. And so on.



Launch Key Points

- Give students enough time to think through their arguments and talk to their peers before moving forward.
- The method of checking or proving a rods length is the critical idea students will need. For example, if the orange is 2 and pink is three oranges, then pink is three 2s, or 6.
- Allow time for students to explain their thinking during the discussions.

Work (continued)

Problem 2. If turquoise (1 cm) equals 9, what are the other rods worth?
In this case, every rod will be equal to a multiple of 9. Note that some students may mistakenly mistakenly believe that orange = 10, dark green = 11, etc. This can be proved wrong by noting that turquoise + turquoise = orange, which would mean $9 \times 2 = 18$. Clearly a mistake! Once students have shown their solutions to this problem, you may want to pose several questions at once, so students can work through to harder problems when they're ready.

Problem 3. If orange (2 cm) equals 24, what are the other rods?
In this case, every rod will be equal to a multiple of 12.

Problem 4. If purple (7 cm) equals 84, what are the other rods?

Problem 5. If blue (5 cm) equals 55, what are the other rods?

Problem 6. If orange (2 cm) plus yellow (8 cm) equals 150, what are the other rods?

Have students draw a picture of the patterns they discover.

If more problems are needed, let students make up their own challenges for themselves and each other.

Tips for the Classroom

1. Make sure students can build their own version of the problem and solve physically.

2. Adjust the difficulty of the problems as necessary.

3. Students can always guess and check. It's having a way to check that is critical.

Prompts and Questions

- What if the orange (2 cm) rod equaled 10? Is that too big or too small?
- How do you know that the yellow (8 cm) rod has that value?
- How can you prove that? Can you show it with the rods, or in a drawing?
- How does that translate into using numbers?

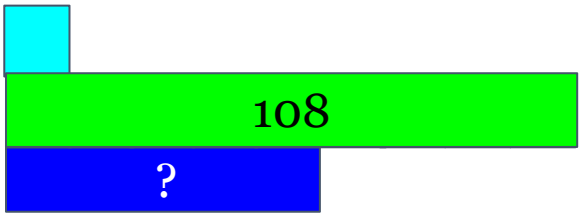
Closer

Take the last problem all students have attempted and spend a few minutes letting students share their answers with each other. You can have them share their methods with a partner, and then take one or two volunteers to share their method with everyone.

One of the things to stress is justification of answers. Make sure that students can defend answers by any combination of rods, pictures, words, and numbers in a convincing way. Having students with different looking pictures come to an agreement that they represent equivalent values can be powerful.

Refer to the image to the right. When determining rod length, you could also make the full connection to fractions: the blue is 5/9 of the light green, because 1/9 of the light green is the turquoise rod (108/9 = 12) and 5 ninths makes 5/9, which would be 5 x 12 = 60, since a ninth of 108 is 12, so 5 (ninths of 108) is 5 twelves.

After you discuss a problem as a group, ask students what they noticed about their solutions. Hopefully someone will have noticed that all the answers are multiples of whatever the value of the turquoise rod was!



Prompts and Questions

- What would happen if.....?
- How do you know your equation matches what you built?

Choice Time

- Penny, Nickel, Dime
- Bullseyes & Close Calls
- Challenge Problems

Overview

Focus Standards

MP2 Reason abstractly and quantitatively.

5.NBT.B Perform operations with multi digit whole numbers.

Materials: Two 6-sided dice, two 10-sided dice, scratch paper and pencil.

Opener	**Broken Calculator**	10 – 15 minutes
Main Activity	**Odd Pig Out**	20 – 40 minutes
Closer	**What are the chances of rolling an odd product?**	5 – 10 minutes
Choice Time	• Odd Pig Out • Blockout • Pig & Big Pig • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP5 | MP6 | MP7 | 3.NBT.2 | 3.OA.7 | 4.OA.2

Broken Calculator

This creative exercise in arithmetic is surprisingly dynamic, with a simple constraint that provides interest and rigor.

Tell the students that you have a calculator with some broken buttons. The challenge for students is to **make a target number on the calculator in as many ways as they can**, despite not being able to use the broken keys.

Students work in pairs or trios to write down a list of solutions. Circulate and help students to talk to each other, extend their thinking, or get unstuck. If students are starting to slow down after 5 minutes or so, pause to highlight some interesting approach from a student, or let students share solutions they found notable.

If students seem to have exhausted their interest in the original question, add challenges.

Example Challenges (optional)

- Solve the problem using the division key.
- Solve the problem starting with a 3-digit number.

Tips for the Classroom

1. Avoid writing faulty "equations" that treat the equals sign as the "compute" button on a calculator, i.e.,
   5 × 6 = 30 - 9 = 21 is false and strange.

   Better is to write a single equation using parentheses if necessary, or rewrite what you've done so far on a new line.
   (5 × 6) - 9 = 21 or 5 × 6 = 30 are clear
   30 - 9 = 21

2. Have a representative from each student group come up at some point while they're working to write one or two of their favorite solutions on the board. This helps other students to get inspired, and also makes the transition to share solutions quicker.

Prompts and Questions

- Good idea, but you used the "1."
- Can you use that same approach to get another solution?
- How many different ways have you come up with so far?
- Do you think it's possible to solve using division?

Target: 21

Odd Pig Out

Materials and Prep
10-sided and 6-sided dice, Odd Pig Out worksheets, pencil.

How to Play
Players take turns rolling two 10-sided dice as many times as they like. After each roll, they multiply the numbers shown on the dice.

- If the product is even, they add that number to their current points for the turn.
- If the product is odd, players lose all their points *from that turn* and their turn is over.

A player may choose to end their turn at any time and "bank" their points. The first player with 500 points wins.

Launch
Odd Pig Out is a natural extension of Big Pig, but for multiplication. Introduce the game, choose a volunteer, and play a demonstration game using the 10-sided dice. If students already know Big Pig, this game should be relatively quick and intuitive to learn. You'll only need to play a few rounds before releasing the class to play their own games.

Work
Students play Odd Pig Out. Try grouping students in pairs to play against each other, one on one, to start. Once they've played a few games, combine the pairs to play in teams of two on two. Team play will force them to talk to their partner about whether they should roll more or stop.

Tips for the classroom
1. Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.

2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. Help them be good winners and losers.

3. If students need more support, they can play with two 6-sided dice instead the 10-sided dice. A worksheet and table for this modification is included below. For a game with 6-sided dice, the winner is the first to claim 300 points.

4. Students can play to 1000 for a longer game.

Launch Key Points

- Students should have some backup method of confirming their products if they need it. Either drawing arrays, "groups of," or using a multiplication table are all possibilities

- The launch can go quickly, since students should know Pig already.

Prompts and Questions

- How many points do you have for this turn, so far?

- Who is ahead?

- Are you sure that's the product for those two numbers? Are you using your multiplication table?

- What strategy are you using?

Closer

Ask students whether they're more likely to roll odd products or even products. Some questions that might help students develop an answer: How many odd numbers are there on the multiplication table (up to 6 by 6)? How many even numbers? How are they distributed? Do students see any patterns?

Depending on how the conversation goes, students may start to articulate a conjecture about the products of even and odd numbers.

Even times even = even
Even times odd = even
Odd times even = even
Odd times odd = odd

Can students defend this pattern? Do they believe that it's true? (It is, but it's always worth approaching rules skeptically.)

The above observation can be extended to an argument that the odds of rolling an even product are three times the odds of rolling an odd product.

Alternatively, students might circle the even or odd numbers on a multiplication table and find the same thing!

Prompts and Questions

- Do you prefer to be more risky or more cautious? Which seems like it works better in the long run?
- What's your favorite winning strategy so far?
- Do you have a rule for when you stop rolling and bank your points?
- What are the odds of rolling an odd product? An even product? How do you know?

Choice Time

- Odd Pig Out
- Blockout
- Pig & Big Pig
- Challenge Problems

DAY 7 | Opener | Main Activity | Closer | Choice Time

Overview

Focus Standards

MP7 Look for and make use of structure.

4.OA.4 Gain familiarity with factors and multiples.

Materials: Six sided dice, Prime Climb coloring chart, scratch paper, pencil, crayons or colored pencils.

Opener	**Don't Break the Bank**	10 – 15 minutes
Main Activity	**Prime Climb Color Chart**	20 – 40 minutes
Closer	**Discussion of Patterns**	5 – 10 minutes
Choice Time	• Don't Break the Bank • Blockout • Odd Pig Out • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP7 | 3.OA.7 | 3.OA.8 | 3.OA.9 | 4.OA.3

DAY 7 | Opener | Main Activity | Closer | Choice Time

Don't Break the Bank

Everyone makes a diagram on their paper. Everyone in the class plays their own game, using the same numbers. You'll need one six-sided die to play.

Roll the die. Whatever number it lands on, everyone enters it in one of the nine spots on their board. After nine turns, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum **without going over 999**. Note that EVERYONE uses the same collection of numbers - they just place them differently on their own board.

4		1
2	2	1
3	6	6

For example, consider the game to the side. After eight turns, there's just one more roll left to go. If a 1 is rolled, this player would have a near perfect score of 998. Any other roll would result in busting - that is, going over 999.

Once the game is complete, find out who busted, and who got the best scores. It's usually fun to play 2 - 3 games.

Tips for the Classroom

1. Make sure students are actually placing their numbers after each roll.
2. Play slowly enough to allow students to think, but quickly enough that they can't do too many calculations. They should be estimating rather than calculating precisely.
3. Let students roll the die.
4. In playing early games, you might choose to bust on purpose, so students know it's okay to do.

Prompts and Questions

- What's a good strategy for this game?
- Where would you put this 5?
- Have you already "broken the bank?" How can you tell?

Prime Climb Color Chart

Materials and Prep
Prime Climb coloring charts, crayons or colored pencils.

Motivating Question
What patterns can you find in the color chart?

Launch
Show students the chart with the coloring to 20, and ask them what they notice, and what they wonder.

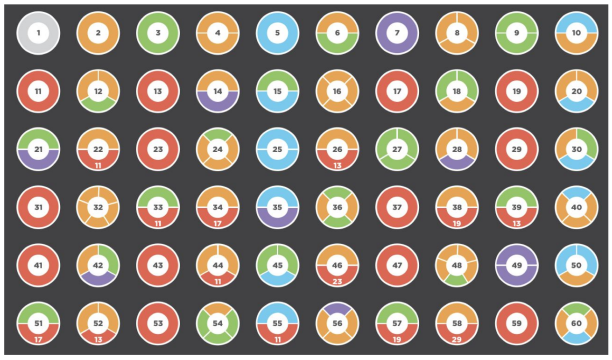
Give students some time in pairs to discuss or write down their ideas. Then discuss as a class. In particular, what are student conjectures to explain the way the numbers are colored? Think of your role during the class discussion as a scribe, asking questions and helping to clarify student ideas. If the wondering/noticing drifts from the mathematics, gently redirect them back.

Possible student observations might include:

- Every even number has orange in it.
- If there is green in the numbers you can reach it if you skip count by 3.
- The circles with blue are numbers end in 5 or 0.
- The circles with red are prime numbers.

Wondering might include:

- Why does 4 have two orange parts?
- Why does 8 have three orange parts?
- If prime numbers are red, why isn't 7 red?



Launch Key Points

- Make sure to leave their questions unresolved. They will be useful questions to come back to in the Closer.
- Similarly, don't be the answer key when it comes to their conjectures. Allow students to determine if a conjecture is always true, sometimes true, or untrue. This is also something that can be discussed in the Closer.

Work

Once students have discussed some ideas for how the coloring works, challenge them to color in the numbers 21 - 30 so that it extends the pattern.

Let students work in pairs. If they are stuck after five minutes or so, you can gather the class together and discuss how to color 21. If the patterns will still work, 21 should be colored green, because it can be reached if you skip count by 3s. It should also be purple, since you can reach it if you skip count by 7s. Another rationale for the coloring: 3 is green, 7 is purple, and $3 \times 7 = 21$, so 21 should be green and purple.

Once students have the hang of how the coloring works, let them work on their own again. They can color in as much of the chart as they can, but getting to 30 is a good initial goal. In general, multiplying and dividing or skip counting is the key to understanding how the coloring works.

Tips for Classroom

1. Don't expect students to finish the entire chart in one lesson. They can come back to it in the future. This is a perfect Choice Time activity.

2. As much as possible, have students explain their reasoning, question the reasoning of others, and determine the truthfulness of their conjectures.

3. If students are having trouble finding any patterns at all, you could switch to the chart that goes to 60, and let them find the colorings for 61 - 70.

4. For composite numbers divisible by primes larger than 10, the number will have a red segment in it's coloration which can be distinguished by writing in the prime it represents on the segment. See the colored-in chart to 60.

Prompts and Questions

- Look at just one color at a time. What's happening with orange? What's happening with blue?

- What numbers have blue? (5, 10, 15, 20.) What do you think comes next in that pattern? So probably 25 and 30 will have a blue part colored in, right?

- Which numbers have two or more orange parts colored in (4, 8, 12, 16, 20)? What pattern do you see in those numbers?

Closer

Pick a few numbers that everyone has at least thought about: 28, 29, 30, for example, or 22, 23, 24 if students haven't gotten that far. Let students defend their choices for coloring.

- Why should 23 be red?
- Why does 24 have three orange segments and one green segment?
- Based on what we know, what colors will 72 have?

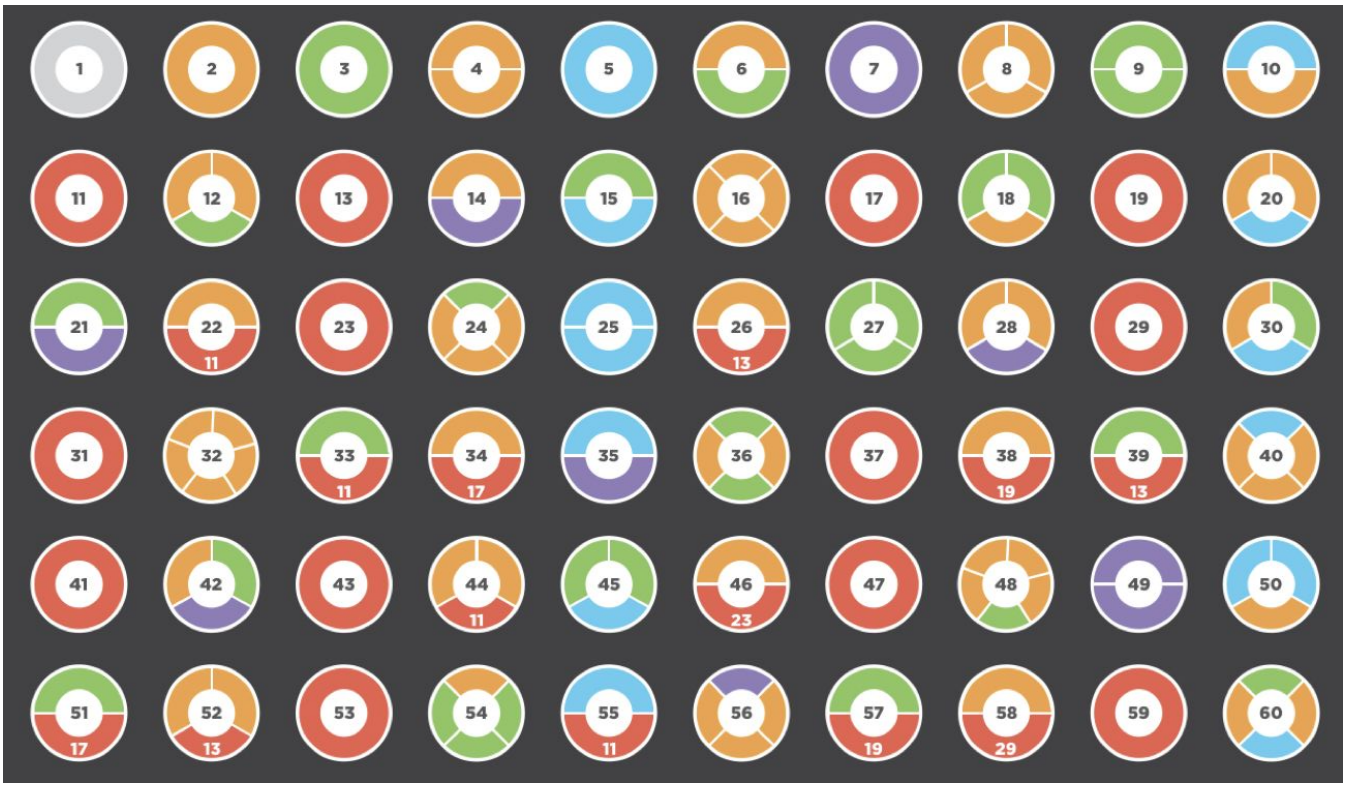
You can use the chart colored up to 60 as a reference, but it's best if students can argue why a given coloration works, and convince other students based on multiplication/skip counting arguments.

Prompts and Questions

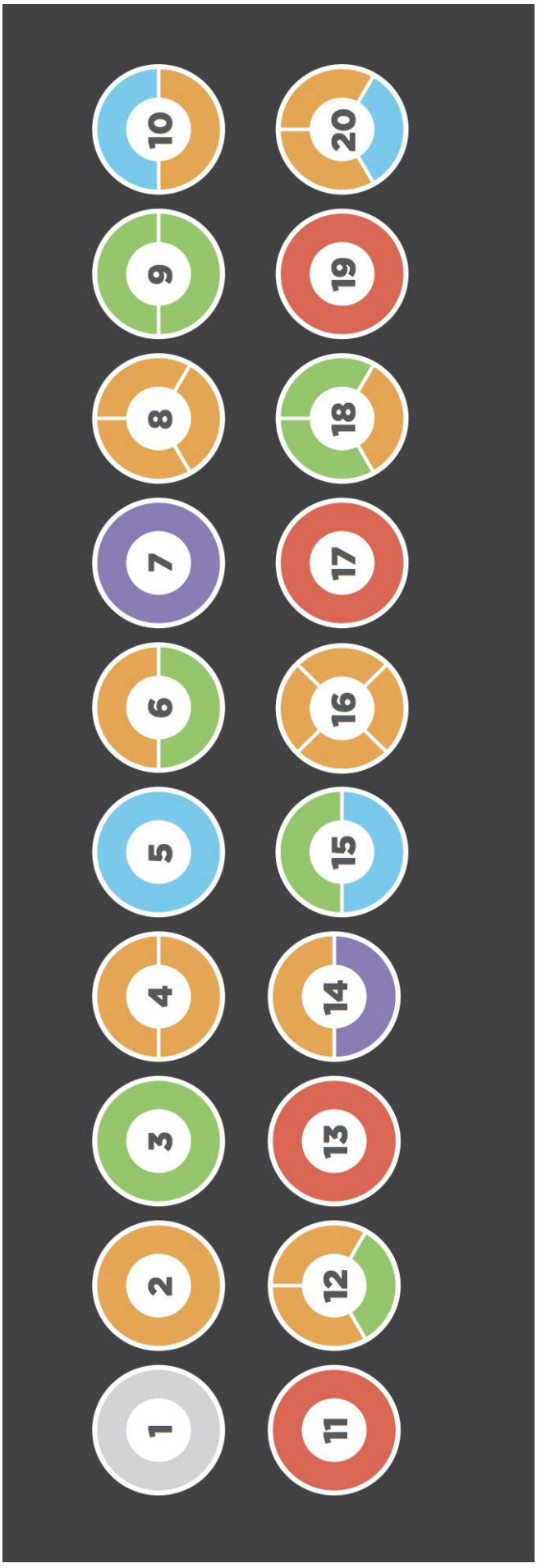
- What patterns do you see in the Prime Climb Color Chart?
- If I wrote out some multiplication problems like $2 \times 5 = 10$, $3 \times 5 = 15$, $4 \times 5 = 20$, what do you notice about the colors of all the numbers in the equations?

Choice Time

- Don't Break the Bank
- Blockout
- Odd Pig Out
- Challenge Problems

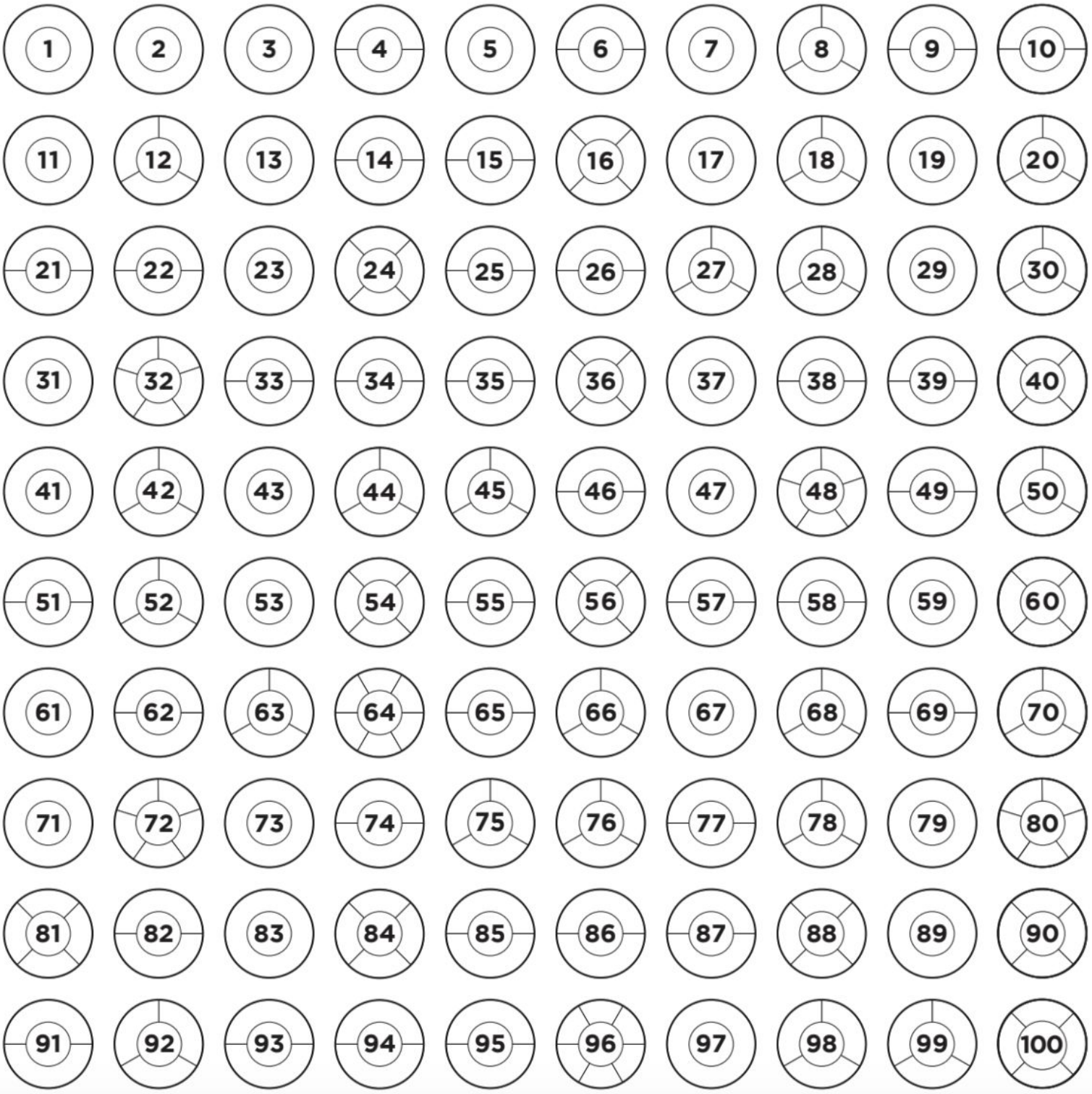


Day 7



Day 7

Prime Climb Coloring Chart



DAY 8	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP7 Look for and make use of structure.

5.NBT.B Perform operations with multi digit whole numbers.

Materials: Prime Climb board game, scratch paper, and pencil.

Opener	**Dot Talks**	10 – 15 minutes
Main Activity	**Prime Climb**	20 – 40 minutes
Closer	**Prime Climb Strategy**	5 – 10 minutes
Choice Time	• Prime Climb • Pig • Odd Pig Out • Challenge Problems	5 – 25 minutes

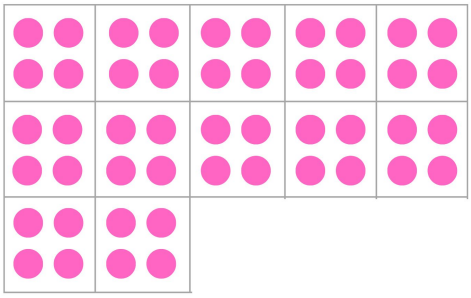
Standards Connections

MP1 | 3.OA.7 | 3.OA.8 | 3.OA.9 | 4.OA.4 | 4.OA.3

Dot Talks

Project the dot image on the board where all students can see it. Then ask students to figure out:

1. **How many dots there are**
2. **As many ways to count them as they can.**



Students can think about the questions on their own first, and then share with a partner. Once students have had time to think the question through, lead a class discussion where students share their answers and approaches. Your job is to restate, clarify, and record student ideas, looking for opportunities to make connections.

The key elements to these talks are a de-emphasis on speed and an added emphasis on process and communication. Expect some disagreements over the answers, and try to use those disagreements as a motivation for students to articulate their ideas to their classmates.

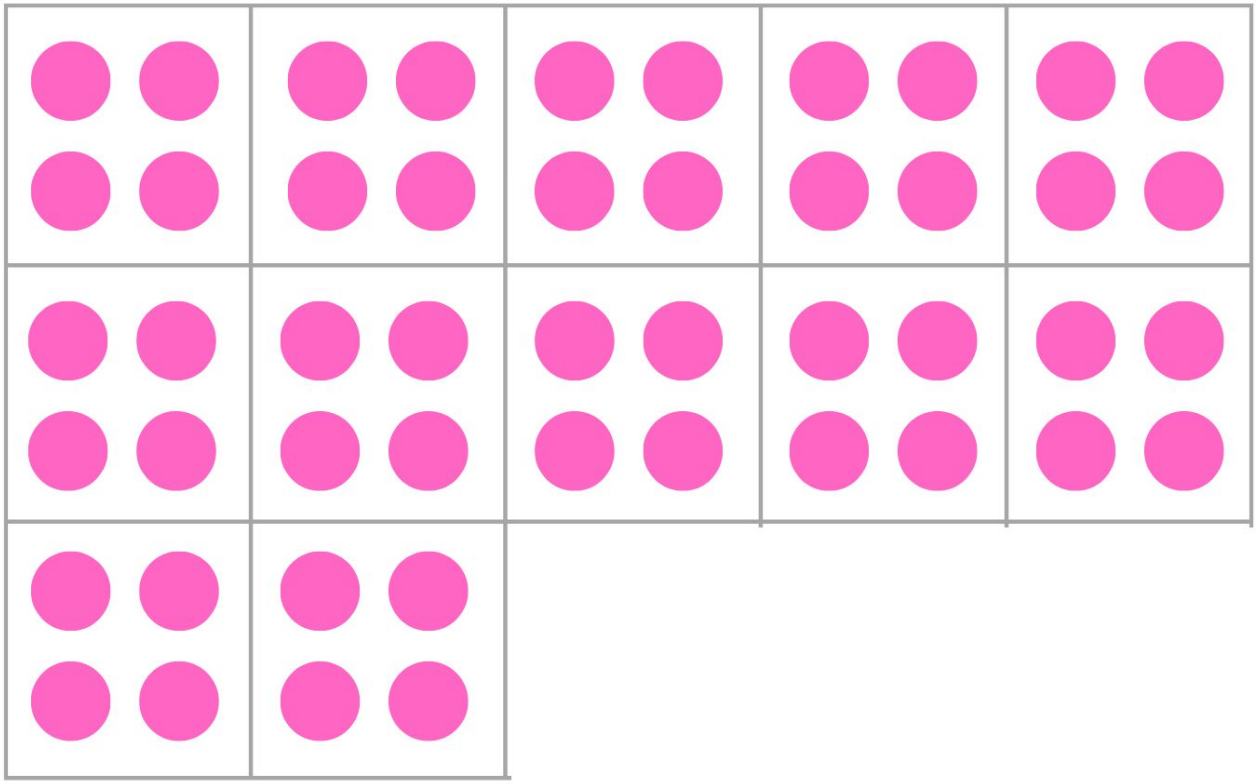
Tips for the Classroom

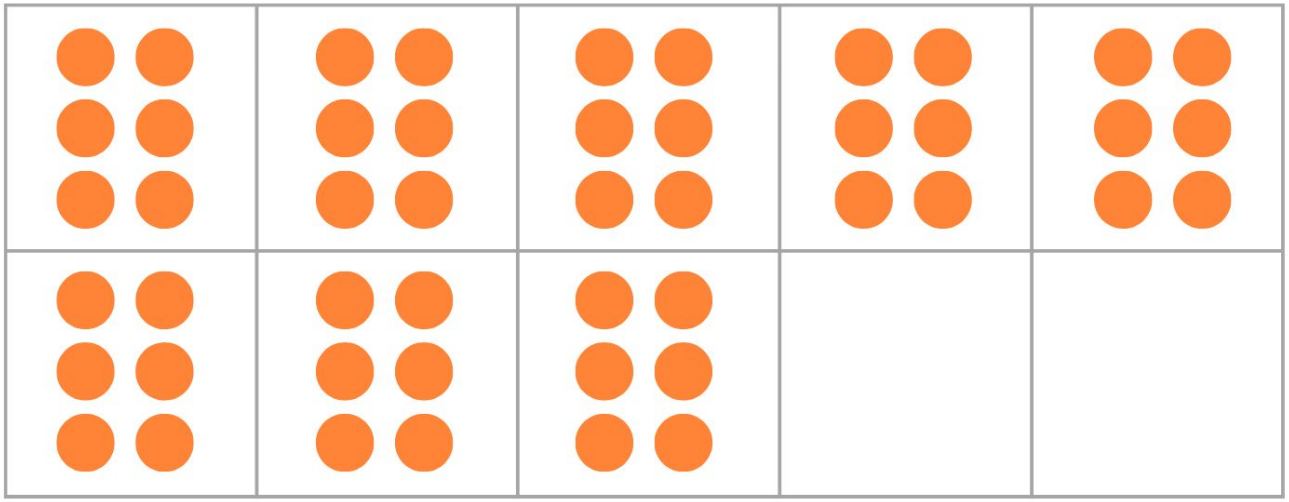
1. Give students constructive language to use in the discussion, like, "I respectfully disagree, because..." and "I agree with _____, because..."

2. Make sure to address incorrect answers or approaches with clear arguments. However, don't worry if you don't reach total consensus on every problem. Sometimes a student will need more time to process. You can move on when it feels like it is time.

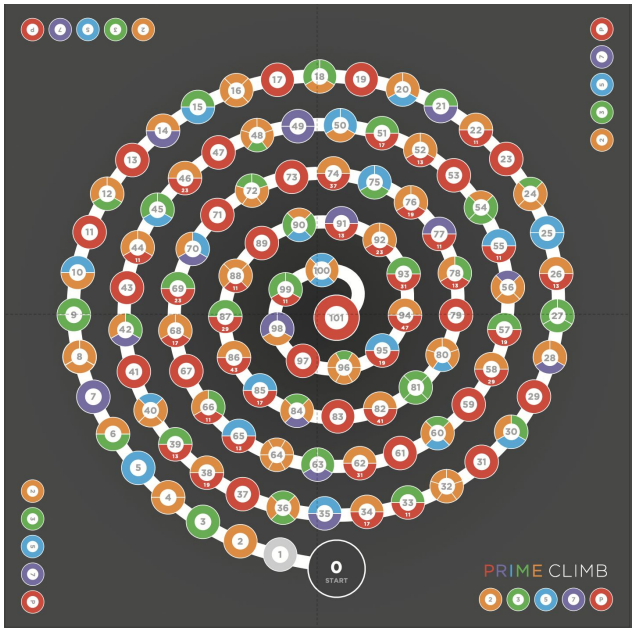
Prompts and Questions

- Who would like to defend this answer?
- I don't quite follow. Do you mean I should count this group first?
- How did you do that/know that?
- Does anyone else think they can explain what Shawn is saying?
- Turn to the person next to you and explain how you counted.





Prime Climb



Materials and Prep

Prime Climb board game.

How to Play

Video instructions available at:
mathforlove.com/games/prime-climb/how-to-play.

Here are the "Quick Start" version of the rules:

During a turn, there are four phases.

1. **Roll.** Roll the dice. You get two numbers from 1 to 10 to use for moving. In you roll doubles, you get that number four times instead of two.
2. **Move.** Move your pawn(s). Apply your dice rolls one at a time to the number your pawn(s) is on, using your choice of +, -, ×, or ÷. You can also use Keeper cards if you have them.
3. **Bump.** If you end your Move phase on the same space as another pawn, even your own, send it back to start.
4. **Draw.** If you end your Move Phase on an entirely red space (i.e., a prime greater than 10), draw a Prime card. If it is a Keeper card, save it for a future turn. Otherwise, apply the card now.

After someone lands both pawns exactly on 101, they win the game. (In the quick version of the game, you win after you get just one pawn to 101.)

You're never allowed to move to numbers off the board. And you CANNOT add the 3 and 9 first and use a 12 for anything. You have to apply the numbers on the dice one by one.

Example Turn:

With pawns on 4 and 26, you roll a 3 and a 9. You could:

- Add 3 to 4 to move your pawn to 7, then multiply by 9 to move your pawn to 63.
- Multiply 26 by 3 to move your pawn to 78, then add 9 to move it to 87.
- Add 9 to 4 to move one pawn to 13, and multiply 26 by 3 to add the other to 78. Since 13 is completely red, you would them draw a Prime card.

Launch

Set up the game and explain the rules. Take a student volunteer and demonstrate 3 or 4 turns each, showing, in particular, how Prime cards get drawn when you land on red circles. Also emphasize that dice must be applied to pawn(s) one at a time–they cannot be added (or subtracted) together and then used to multiply (or divide). Once students seem ready, have them play in groups of 4 with two players per team.

Work

Students may have questions that come up during the course of play. You can consult the full rules of the game, or just have students respond by deciding on what seems like the best way to settle the question and keep play going.

Tips for the Classroom

1. Have students roll in the box lid to prevent them from knocking over pawns during the game.

2. Students can use the multiplication table or scratch paper to help themselves with hard multiplication problems. The board's color scheme can help too.

3. Students may dislike getting knocked back to start. However, they'll quickly learn that they can make fast progress if they get a good roll, especially when they roll doubles.

4. You can shorten a game with two pawns by making the objective to get only 1 pawn to "101" instead of both.

Closer

Invite students to reflect on their experience, in pairs or small groups first, and then as a whole class.

- What strategies did we learn playing this game?
- It can be discouraging being bounced back, but were there ways to catch up quickly? What kind of rolls helped you get near 101 quickly?
- How did you use the Keeper cards? Did you ever use one before you used the numbers you rolled?
- How did the colors help you with the math?

Choice Time

- Prime Climb
- Pig
- Odd Pig Out
- Challenge Problems

Launch Key Points

- In Day 7, students had an opportunity to explore the color chart used for the numbers for Prime Climb. If necessary, review some the notice and wonders they made from that lesson.

- As students are having trouble learning the game, consider having them use only 1 pawn instead of both pawns.

Prompts and Questions

- Where do you land if you add each number to the same piece? Where could you land if you added each number to separate pieces?

- Can you get either pawn to a red circle with that roll?

- Can you bump anyone with that roll?

- You rolled a 3 and a 5. What if you added the 3 to your pawn first, then multiplied by 5?

- If you subtract, you could land on a red circle and draw a Prime card.

- Can you divide down and then multiply up?

- What would happen if you used one of your keeper cards first, and then used the numbers you rolled?