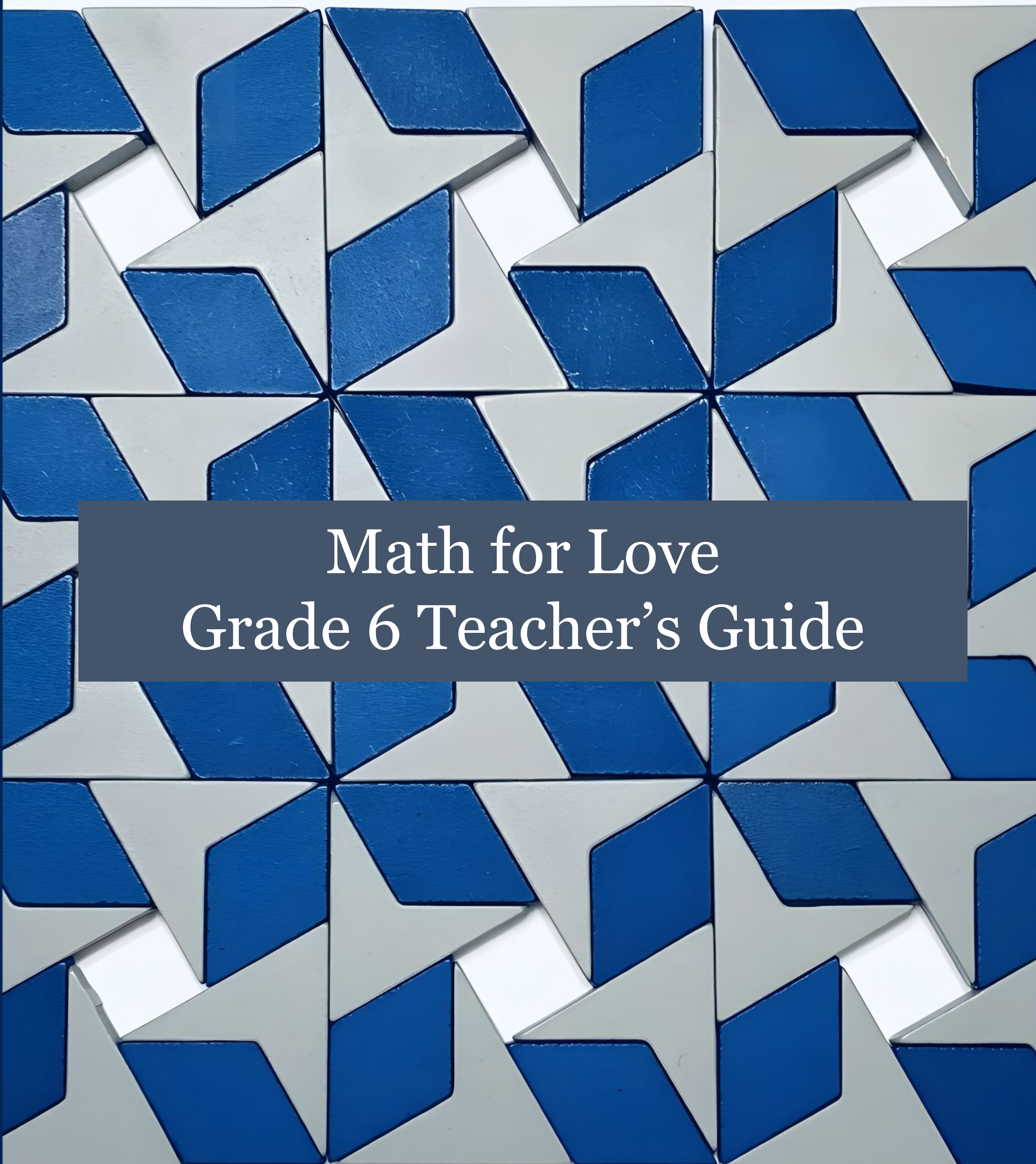


Math for Love
Grade 6 Teacher's Guide

Introduction

Welcome to the Math For Love Supplemental Curriculum! We are thrilled that you will be using this curriculum with your students. Like the lesson plans, we'll make this introduction quick, easy to read, and useful.

We are Math For Love, an organization dedicated to transforming how math is taught and learned. Our passion is connecting students and teachers with opportunities to experience excellent mathematics, deepening everyone's skill and enjoyment in the process.

The Math For Love Supplemental Curriculum is built on our belief that *play* and *rigor* go hand in hand, and that the best of mathematics is accessible to students and teachers who are ready to work hard and have fun. You and your students will learn a lot of math over the next 80 lessons, and by the end we hope you'll see why we think math is one of the best parts of the day.

The Big Picture

We built this curriculum with a few key principles in mind.

Principle 1.
Every student can participate in rigorous mathematical thinking.
Rigorous mathematical thinkers want to understand *why*, not just get the answer. They make connections and seek underlying structure and coherence. They develop powerful tools to solve problems, including fact fluency and procedural efficiency. Rigorous mathematical thinkers ask questions, make conjectures and predictions, test out their ideas relentlessly, and expect to be surprised.

Principle 2.
Play is the engine of learning.
Mathematicians engage in play constantly: exploring, wondering, noticing, and being led by curiosity. Play can transform math class from tedious to joyful, from shallow to deep, from mundane into fascinating. Students at play are more likely to persist, to build tenacity, to remember, and to learn. Play is the secret sauce that helps students come to love and succeed in mathematics.

Principle 3.
Without rigor, mathematical play is formless.
Without play, mathematical rigor is unsustainable.
We need both, together, to get the most out of mathematics.

In this introduction, we'll discuss some specific teacher moves that can help encourage rigorous mathematical play.

But first, some details.

The Details

The Math For Love Supplemental Curriculum is built to provide eighty days of 1-hour (or longer) classes, intended to complement a standard curriculum. It can be used for small groups, enrichment, remediation, after school programs, and summer programs. Every lesson is written to accommodate a wide range of student skill level, making it easy and enjoyable to differentiate and support each individual's learning. Our belief is that beautiful and interesting math problems — when designed to be appropriately accessible — should be offered to everyone, no matter where they are in their math journey.

Materials included with curriculum:

- Teacher Guide
- Student Workbooks
- Manipulative Kit
- Math Games

The Lessons

Each lesson follows a standard format with four sections:

- Opener
- Main Activity
- Closer
- Choice Time

We sometimes provide a sketch of how a lesson might unfold, with prompts and questions to help you respond organically to what your students bring to the conversation. Any sample dialog is never meant to be a script, and precisely how the lesson goes will depend on you and your students.

We include guidelines for how long we expect each part of the day will take; however, times will vary depending on student engagement and your decisions.

When preparing for a lesson, review all sections of the lesson in advance. This will help you make decisions on how to group students, how to arrange materials, and what images to project. Even a little bit of preparation will help you be ready to emphasize what's important in the lesson and respond naturally to your students' ideas.

Choice Time Days

Occasionally a full lesson - after the Opener - is devoted to Choice Time. These Choice Time Days are intended to give students a chance to dig deeper into any lessons, or relax with some extra time to play the games they already know. As with normal Choice Time, you can use the suggestions we provide, or substitute in other options.

See the sample lesson templates on the next pages for more details about the lesson plans.

Overview

Focus Standards

This is where we highlight the main standards we're focusing on for the day, particularly in the main activity. We usually highlight one practice standard and one content standard.

Materials: This section will mention everything you need for the day. The main activity will also include a materials list.

Opener	**We'll say what the opener is here**	10 – 15 minutes
Main Activity	**We'll say what the main activity is here**	20 – 40 minutes
Closer	**We'll summarize what's happening in the Closer here**	5 – 10 minutes
Choice Time	• We'll provide a short list of good options for Choice Time here. • You're always welcome to choose different options!	5 – 25 minutes

Standards Connections

These are additional standards that are connected (or could connect) to today's lesson.

Opener

The lessons cycle through a short collection of our favorite opening routines. The first time you see a particular Opener, there will typically be more detail included. Later, these writeups will become shorter and more succinct. Don't be surprised to see the Opener instructions look almost identical on different days - once you're confident with a given opener, it should take very little time to prep for using it with class.

Here are the main Openers we use in this grade.

➔ Would You Rather

Give students a choice between two options. They debate which one is better, using math to convince each other.

➔ Fraction Talks

Project an image that includes several colors.
Prompt: "What fraction of the image is each color?"

➔ Counterexamples

Make a false claim or conjecture.
Invite students to find an example that proves you wrong.

➔ Broken Calculator

Project a calculator with some broken keys, and a target number.
Prompt: how many ways can you hit the target number using the broken calculator?

➔ Teacher-led Games

For example, Bullseyes and Close Calls, Don't Break the Bank, Penny Nickel Dime.

Tips for the Classroom

1. Look here for some specific ideas for increasing student interaction, adjusting challenge, and more.

2. If there's an image to project for an opener, it's typically on the next page.

Prompts and Questions

- Look here for useful things to say to students to help them get started or push deeper in their thinking.

Main Activity

Materials and Prep

Here's where we describe what students will need for the main activity (doesn't include choice time materials). You'll need to read the lesson to make some decisions about how to arrange the materials for the day. In general, keep this simple – offer containers of manipulatives rather than exact amounts.

Motivating Question (OR How to Play)

To begin working or playing on their own, students should either have a question that frames the day's exploration— along with the knowledge and skill to begin thinking about it — or know the rules of the game they're about to play. We essentialize that question (or summarize those rules) here.

Launch

This is how to introduce the motivating question and get students excited and curious to think about it, or to teach the game in a way students will understand and find irresistible. In the case of games, demonstrating with a student volunteer is almost always the most powerful way to communicate how the game is played.

In general, the Launch should be as thorough as necessary *and* as short as possible. The goal should always be to have the students spending as much time as possible doing the thinking during math class. Whenever you are speaking to the whole class, pose questions and look for opportunities to ask for student ideas, questions, and contributions.

Work

As soon as they're ready, students go to work on their own or in pairs or small groups. This section will have some ideas of what to look for, the lesson flow, extensions, good hints, and (occasionally) solutions.

While students work, circulate in the room, offering help, prompts, hints, asking questions, making connections between ideas, and getting a sense of your students' strengths and where they could benefit from greater support.

Tips for the Classroom

1. Look here for additional ideas on how to implement this activity.

2. We'll often include extensions or simplifications to help with differentiation.

3. Student workbook pages will typically be included on the page right after the Tips for the Classroom.

Launch Key Points

- We try to include some key points for how to help the launch succeed in getting students excited to work.
- Points about the essential knowledge or skills might be here too.

Prompts and Questions

- This section gives ideas for what you might say to students during the "Work" section of the lesson, when they're working on their own or in small groups.
- Sometimes a prompt, hint, or nudge to talk to someone else is all students need.

Closer

Gather the students together for a whole-class discussion when the Main Activity is done. This is where students reflect, consolidate their learning, and potentially try an extension or variation of the Main Activity. To make sure the engaged thinking continues during this part of the day, rather than just summing up what everyone should have learned, take the opportunity to pose questions, invite student comments, and use partner sharing to give everyone a chance to participate.

Prompts and Questions

- These prompts are for the Closer.
- They might be useful things to say to the class as a whole.
- They also might be helpful replies to anticipated student contributions to a closing discussion.

Choice Time

Choice Time is when students get a chance to revisit games, puzzles, and other material they want to spend more time with. Getting to choose their activity helps with buy-in and self-regulation, and is a chance for students to reflect on what they want to think about more.

Choice Time works like this:

1. Present students with a short list of suggested activities.
2. Students choose the game, worksheet, challenge problem, block set, or other activity they'd like to pursue and commit to sticking with it for at least 5 - 10 minutes.
3. If time permits, students can try more than one activity.

The suggestions for Choice Time are only suggestions. If there is another activity from the curriculum that you think would be a better fit here, or if a student has a strong preference for something not on the suggested list, feel free to make a swap.

You may need to print some materials in advance to prepare for Choice Time. Since the final Choice options are up to you, we don't give a list of materials you'll need for them.

Here are some options that can be freely offered any Choice Time:

- Challenge Problems
- Free Block Play
- Multiplication by Heart (once students know how to play it)
- Prime Climb (once students know how to play it)
- Work on problems from an earlier lesson

Teacher Moves

Here are some useful ways to support your students during these lessons.

- **Model enthusiasm and curiosity.** Ask questions. Statements like "I wonder if…" and "I notice that…" go a long way. If students see you enjoying the work, they'll be much more likely to enjoy it too.

- **Keep instructions and launches as brief as possible** (but as long as necessary) and look for places to invite student questions or ideas. As much and as often as possible, we want students to be spending classroom time doing mathematics and thinking mathematically.

- When launching games, **play a demo game with a volunteer** to help students learn the rules. When students play games against each other during work time, try these ways of grouping students:
    - Students play one against one and switch opponents often.
    - Students play in groups of three. Two play while one watches as a referee. When the game is over, the referee position rotates.
    - Students play two against two, and have to agree on moves with their teammate.
    - Students play collaboratively with a partner, and try to get the highest score they can, rather than beat an opponent.

- **Resist solving students' problems for them.** While working on hard problems, it's natural to feel stuck, or unsure of what to do next. Sometimes a key insight requires a lot of exploration first. Give students the time they need.

- **On the other hand, support students when they need it.** There's no use in leaving students feeling dispirited or unsuccessful, and the goal is for students to be productive, even if stuck. We provide ideas for questions, prompts, and hints to keep students motivated and engaged. Even when students are playing or exploring, understand your job as looking for opportunities to help students develop greater efficiency, organization, and power in their methods.

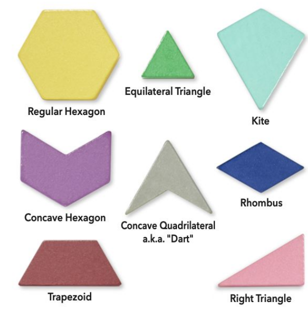
- **Have a plan for how to respond to wrong ideas and answers**. One of the strongest ways to handle these moments is to turn them back to the students by treating the idea seriously and asking for counterexamples or supporting arguments. A very good phrase to keep in your back pocket is: "Convince me."

- **Be willing to be the slowest person in the room**. This means asking for elaboration and clarification if you think there is even one student in the room who doesn't understand an argument yet.

- **Care and respect**. Show students you care about them, respect their thoughts, and that it matters to you that they learn, and enjoy, mathematics.

Materials

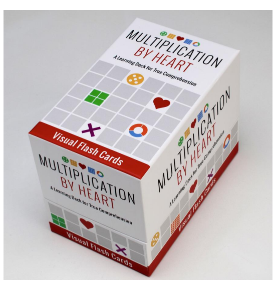
We provide just about everything you need to use this curriculum with a classroom of 25 (or more) students. The only extras you'll need are scratch paper, pencils, and crayons or colored pencils. You may occasionally need to make some additional photocopies for Choice Time, though students can often turn to earlier pages in their Student Workbook and find what they need. In addition to this Teacher's Guide and the student workbooks, manipulatives and games include:



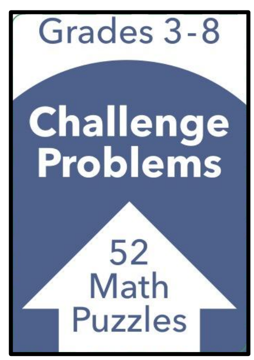
21st Century Pattern Blocks. These blocks include 8 shapes, with enormous possibilities for exploring multiplication, division, fractions, ratios, geometry, and more. These are also great for students to explore with during Choice Time.

Number Rods. Another excellent tool for understanding arithmetic operations, fractions, measurement, and more. Rods go from 1 cm to 10 cm long, in the colors named to the left.

Prime Climb. One of the world's most popular mathematical board games. Includes a unique visual for prime factorizations of numbers that acts as a guide for multiplication and division. Always a good Choice Time option once students learn how to play. Video instructions available at mathforlove.com/prime.



Multiplication by Heart. A visual flash card deck with three subdecks. Great in the classroom for small group fact practice and for simple fluency games. Ideas included in the lessons, and at mathforlove.com/multiply.



Challenge Problems Deck. These extra puzzles and problems are great options for Choice Time. These generally get harder the higher the number.

Also included: **Square Tiles**, **6-sided and 10-sided Dice**, and **Polyominoes** (which include dominoes, triominoes, and tetrominoes).

Other Stuff

- Email errata@mathforlove.com if you notice an error that should be fixed.
- **Additional Material:** We'll gather corrections and additional material at mathforlove.com/curriculum/grade6.
  Password: M4LCurriculum
- Problem with access? Email info@mathforlove.com.

Thanks and Acknowledgements

These lesson plans were built from the lessons we developed over our years working with teachers and students of all ages. However, putting together this more ambitious curriculum required a team, and we were lucky to have an amazing one.

Our curriculum writers were Karen Gallagher, Mark Goldstein, Tara Hofmann, Becky Holden, and Chase Orton. Our editors were Hana Murray and Jen Moffett. We had help with images from Bella Christianne and Hana Murray. Hana Murray also created the cover using photos of 21st Century Pattern Blocks.

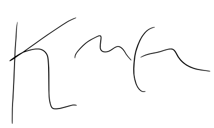
This team of writers and editors worked with incredible focus and skill to build the teacher's edition you're holding now — big thanks to all of them for their dedication and contributions to this project.

Some images for this book were made, with permission, using Mathigon Polypad (polypad.amplify.com) - thanks to the good folks there for building such a fantastic tool. Mathigon also partnered with us to create digital versions of the Multiplication by Heart and Addition by Heart cards included in the curriculum kits. If you'd like to use the digital versions, they are free to use at fluency.amplify.com.

Finally, thanks to all the teachers, coaches, students, and staff who have used versions of our materials over the years, and welcomed us into their classrooms.

Daniel Finkel | Founder | Math for Love

Katherine Cook | Creative Director | Math for Love



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Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

MP6 Attend to precision.

Materials: 21st Century Pattern Blocks, scratch paper, pencil.

Opener	**Bullseyes and Close Calls**	10 – 15 minutes
Main Activity	**Forty Faces**	20 – 40 minutes
Closer	**Strategies for Calculating Values of Faces**	5 – 10 minutes
Choice Time	• Block Free Play • Bullseyes and Close Calls • Challenge Problems	5 – 25 minutes

Standards Connections

MP3 | MP7 | 3.OA.3 | 3.NBT.2

Bullseyes and Close Calls

Secretly choose a number with no repeated digits and write it down where no one can see it. Play with a 2-digit number for the first game, and 3-digit numbers for games after that.

Students attempt to guess the number. After each guess, give feedback using a combination of "Close Call" and "Bullseye", or possibly "Nothing".

Note: you might need to say "2 Close Calls," or "1 Bullseye and 2 Close Calls," or some other combination. DON'T say "first digit Close Call, second digit Bullseye." Your responses apply to the entire number, not individual digits.

As soon as the rules are mostly clear, begin to play the game. Any confusion about the rules will get worked out during play itself.

Outcome of Guess	Feedback
Correct digit in the wrong place	"Close Call"
Correct digit in the correct place	"Bullseye"
No correct digits	"Nothing"

Example Game
Your secret number is 487.

Guess	Feedback
139	Nothing
820	Close Call
468	1 Close Call, 1 Bullseye
568	Close Call
482	2 Bullseyes

Tips for the Classroom

1. Note that students DON'T get a Bullseye or Close Call for each digit. The clue applies to the entire 2- or 3-digit number.
2. Write the guesses and the responses somewhere that everyone can see them.
3. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses. For example, after guessing 139 and finding that none of those digits are in the number, cross off the 1, 3, and 9 from the list of possible digits.
4. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

Prompts and Questions

- What digits can I cross off after that guess?
- Is there anything you know after that guess? Any digit that is or isn't in our mystery number?
- Why are you so sure the number doesn't have a 5?

Forty Faces

Materials and Prep

21st Century Pattern Blocks, paper, pencil, Number Rods (optional).

Motivating Question

How can you make a face with an area equal to exactly forty green triangles?

Launch

Hold up a pattern block hexagon and a green triangle, and ask students if they know how many green triangles it would take to build the hexagon. Repeat for the blue diamond, the red trapezoid, and the purple concave hexagon. (Stop there–leave the other 21st Century Pattern Blocks for later). Students who have worked with Pattern Blocks before will be able to answer. You can confirm demonstrate by building.

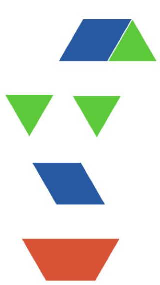
Project the images below (and on the next page), or build them where everyone can see. Ask for guesses about how many green triangles' worth of area each face is.

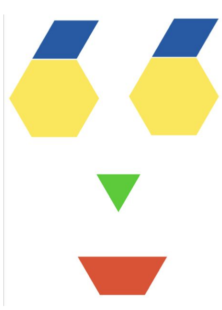
After discussing the guesses, show how the faces are worth 10 or 20 triangles. For the second face, for example, there are 2 hexagons, 2 rhombuses, one trapezoid, and one triangle. In terms of triangle area, the total "value" would be 12 (in hexagons)+ 3 (in trapezoids) + 4 (in rhombuses) + 1 (in triangles) = 12 + 3 + 4 + 1 = 20 triangles worth of area. Demonstrate writing this equation down.

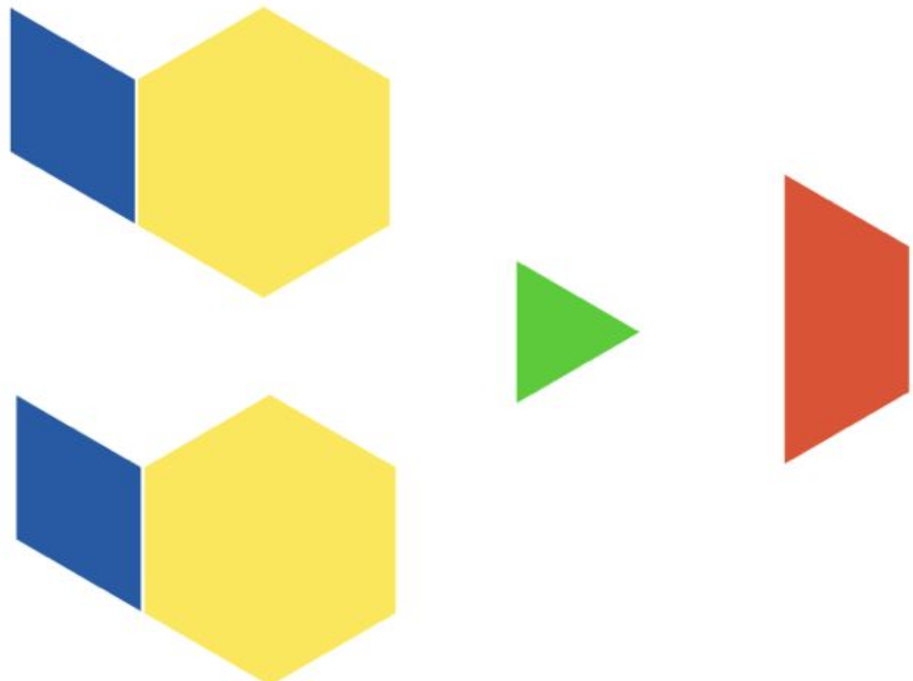
Once students understand how to count the "value" of the face, challenge them to create their own faces from pattern blocks that have value (i.e., area) 30 and 40.

Launch Key Points

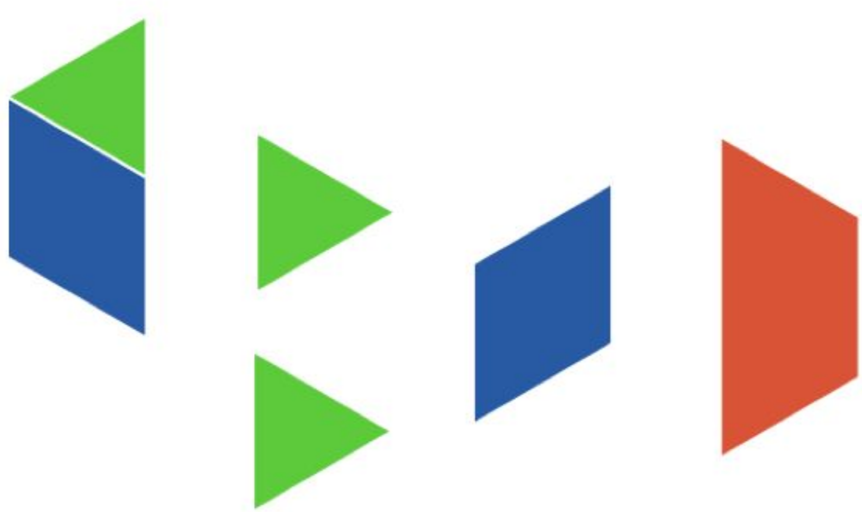
- Each pattern block can be built from triangles–pose a quick series of questions to students to find the values of the hexagon, blue diamond, and red trapezoid.
- Ask students to guess the value of the faces before discussing them.
- Make sure students understand how to model a face with an equation.







20 Face



10 Face

Work

Circulate the room as students construct their faces. Encourage them to check their neighbors' work to make sure the faces everyone builds are actually worth 30 and 40.

Tips for the Classroom

1. Let students challenge themselves when they're ready. Can they make a "100 face"?

2. Encourage students to use pencil and paper to track the arithmetic. It gets difficult to find the answer without making a mistake once the faces get larger.

3. The new shapes of 21st Century Pattern Blocks (the gray dart, pink triangle, and teal kite) create an additional challenge: what are those blocks worth? Figuring out the answer requires some subtle arguments. This will come up in the Closer, and in future lessons.

Prompts and Questions

- How much more area do you need to add to get to 30?
- Show me how you found the area.
- Let's count how much the hexagons are worth.
- The trapezoids came to 18 area? Let's write that down.
- Do you think the two of you could make a face with an area of 75?

Closer

Make sure students don't clean up their faces before the Closer. Ask students to tour the room and find a face that looks like it has area 50 but actually doesn't. Can they find any errors? (You can plant a face with area 49 somewhere, though chances are there will be errors in the room).

Once students get a chance to appreciate each other's creativity and check their work, gather students and show a face that has area close to 50 (you'll need to build this ahead of time). Ask students to find its value, and be ready to defend their thinking about it. Ask students to share their approaches and discuss some of the different ideas with the class. Possible strategies include skip-counting, adding, and multiplying. Once everyone agrees on the area of the figure, take student suggestions for how it could be adjusted to come to 50 exactly.

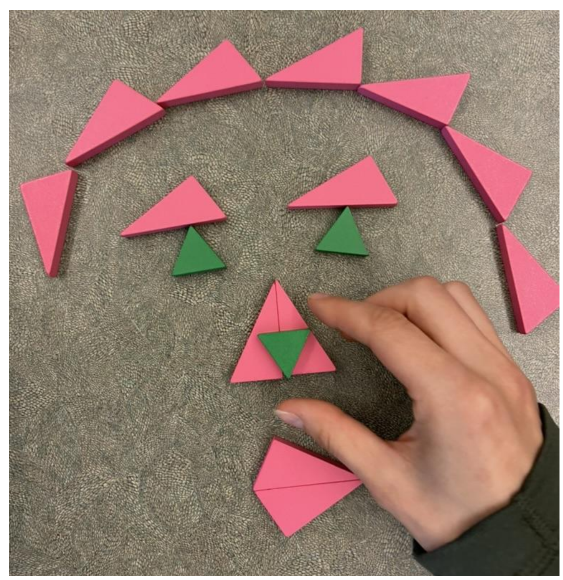
Have students consider a different value for the face they created. What would the value of their face be if the rhombus (blue block) had an area of 1? Students discuss in pairs how defining the rhombus as 1 changes the value of their face (it is half as much).

Prompts and Questions

- Can you explain why this face has an area that is almost 50 triangles?
- How might we change this face so that the area is exactly 50?
- What is the value of your face if the rhombus has an area of 1? How do you know?

Choice Time

Today's Choice Time Options
- Block free play
- Bullseyes and Close Calls
- Challenge Problems



Prepare students for Choice Time by explaining that they will choose from some pre-selected options. Tell students that once they choose an activity, they should spend at least 10 minutes on it before trying something else.

Show the Challenge Problem Deck and explain that it will always be among the Choice Time options. Students can simply take a challenge problem card and try to solve it, on their own or with a partner. Be sure to provide scratch paper and pencils to support student work.

Block free play is also an option. This is an opportunity for students to explore the Pattern Blocks or other manipulatives.

Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

MP3 Construct viable arguments and critique the reasoning of others.

Materials: 6-sided dice, Big Pig Scoring Sheet, scratch paper, pencil.

Opener	**Counterexamples**	10 – 15 minutes
Main Activity	**Big Pig**	20 – 40 minutes
Closer	**Big Pig Strategy**	5 – 10 minutes
Choice Time	• Block Free Play • Big Pig • Challenge Problems	5 – 25 minutes

Standards Connections

MP6 | MP7 | 4.NBT.4

Counterexamples

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All _______s are ________" or "No _______s are ________."

For the first day you play Counterexamples, get the students used to the game with the statement **"All birds can fly."**

Tell students their job is to come up with an example that proves your statement false. In this case, they are likely to suggest penguins as a counterexample. If they do, modify your statement: **"All birds can fly except penguins."** Counterexamples here could include baby birds, injured birds, ostriches, etc.

Next, use the following motivating examples and an "I notice" or "I wonder" statement. This allows students to see you model the process of making a conjecture.

Step 1. Share motivating examples
Write these down where everyone can see them.

- 4 + 5 = 9
- 10 + 11 = 21
- 12 + 13 + 14 = 39
- 1 + 2 + 3 + 4 + 5 = 15

Share the following observation: "I notice that every time I add consecutive numbers, the sum is an odd number." (*Consecutive* means "next to each other.")

Step 2. Pose a conjecture
State this as an 'obvious' conclusion from the motivating examples. You might say, "I bet it's true that every time I add consecutive numbers, the sum is odd. I'm going to *conjecture* that this is true." Then write:

Conjecture. Any sum of consecutive numbers is odd.

Step 3. Invite students to try to find counterexamples - that is, examples that prove your conjecture false.
They may need some time. Every time a counterexample is offered, ask the class to consider it fully and see whether it satisfies the goal. Sometimes counterexamples can be deceiving!

Possible counterexamples students might share include:

Prompts and Questions

- I think I see a pattern here. I'm going to make a conjecture...
- You think my conjecture is wrong. But how can you *show* me it's wrong?

Counterexamples (continued)

Potential Student Counterexamples:
3 + 4 + 5 = 12
4 + 5 + 6 + 7 = 22

These are counterexamples because they represent consecutive numbers with a sum that is not odd. This contradicts the original conjecture, so the conjecture must be false!

Step 4. At this point, extend the game by offering a revised conjecture that accounts for the counterexamples the students came up with.
Some examples of possible refined conjectures include the following:

Revised Conjectures

- *Any sum of two consecutive numbers is odd.* (This is true.)
- *Any sum of an even number of consecutive numbers is even.* (This is false.)

How you frame the refined conjecture will depend on what counterexamples the students found.

Once you have a refined conjecture, students can continue searching for new counterexamples. As long as students are engaged, feel free to continue cycling between refining the conjecture and seeking counterexamples, though in general one or two iterations will be sufficient.

Big Pig

Materials and Prep

6-sided dice, Big Pig scoring sheet, pencil.

How to Play

Big Pig is a game for 2 or more players. Players take turns rolling two dice as many times as they like until their turn ends. The sum of the two dice are the points for the roll. After each roll, they can keep rolling to get more points, or end their turn and "bank" their points. The first player with 100 points or more in their bank wins.

Other important rules:

- If the dice show a **combination of the numbers 2, 3, 4, 5, or 6**, the player adds that many points to their score for the turn. For example, roll a 2 and a 4 to get 6.
- If a roll is **doubles**, they get twice as many points as normal. For example, double 5s are worth 10 × 2 = 20.
- A roll of **double 1s** is worth 25 points.
- If you roll **a single 1 (roll a 1 on only one of the dice)** lose all your unbanked points and your turn is over.

Launch

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students advise you on whether they think you should keep rolling by giving a thumbs up/down.

Play until it feels like most students are clear on the rules - usually 3 - 4 turns for you and your volunteer.

Work

Students play Big Pig. Group students in pairs to play against each other, one on one, to start. Once they've played a few games and started getting a feel for strategy, combine the pairs to play in teams of two on two. Team play will force them to talk to their partner about whether they should roll more or stop.

Tips for the Classroom

1. For students who are less confident with addition, suggest that if they start to accumulate points on a turn, they add them at each roll to keep a running total.

2. Shorten or lengthen games by going to 50 or 200. If time is a factor, the player with the most points when time runs out is the winner.

3. Whoever isn't rolling is in charge of writing. That way, every student has something to do every turn.

Launch Key Points

- Take risks and ask students to give a thumbs up/down on whether you should stop rolling.
- Demonstrate how to use the score sheet, where to keep track of rolls, and when to bank.
- It can be useful to roll recklessly until you get a 1 for one of your turns to demonstrate losing all the points for that turn.

Prompts and Questions

- What's your strategy of when to stop or keep going? Does it ever change?
- Do you have a strategy for keeping track of your score?

Big Pig

How to Play
Take turns rolling two dice. You can roll as many times as you like, until you BANK or ROLL A ONE.

To bank, move all the points you've rolled this turn to your bank, and end your turn.

If you roll a one, you lose all the points not in your bank, and your turn is over.

Scoring Points

- If the dice show a combination of the numbers 2, 3, 4, 5, or 6, add that many points to your score for the turn. For example, roll a 2 and a 4 to get 6. Keep going or bank points and stop your turn.
.
- If you roll doubles, get twice as many points as normal. For example, double 5s are worth 10 × 2 = 20.
- If you roll double 1s, get 25 points! This doesn't end your turn.
- **Remember!** If you roll a **single 1** (roll 1 on only one of the dice) you lose all your unbanked points and your turn is over.

The player who reaches 100 points or more in their bank first is the winner!

Day 2

Big Pig

Rolls	Rolls
Bank	Bank

Closer

Gather the whole class together for a discussion of Big Pig. Ask students if they noticed themselves using any strategies as they played. For example, a student might roll the same number of times every turn, then bank (if they can), or roll till they have a certain number of points, then bank.

You may find that some students play a riskier game than others. If time permits, consider selecting a student who plays a risky game (rolls many times before banking) and one who plays a conservative game (banks very quickly) and have them play against each other in a demonstration game. You can take a vote ahead of time about which strategy is more likely to win.

There is an opportunity here to discuss the difference between a good strategy and a single loss–luck plays a big role here!

Prompts and Questions

- Do you have a rule for when you stop rolling and bank your points?
- Is your strategy working for you so far?
- Do you prefer to be more risky or more cautious? Which seems like it works better in the long run?

Choice Time

- Block Free Play
- Big Pig
- Challenge Problems

Overview

Focus Standards

MP7	Look for and make use of structure.
6.RP.2	Understand the concept of a unit rate.

Materials: Number Rods, scratch paper, pencil.

Opener	**Would You Rather**	10 – 15 minutes
Main Activity	**Number Rod Proofs**	20 – 40 minutes
Closer	**Proof Discussion**	5 – 10 minutes
Choice Time	• Big Pig • Close Calls and Bullseyes • Block Free Build • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP2 | MP3 | MP8 | 6.RP.1 | 6.RP.3

Would You Rather

Would You Rather questions offer a choice between two options. The job of the students is to decide which of the two options they would rather have, and convince their classmates of the wisdom of their choice.

The choice is usually real-world in nature, and may require students to make reasonable guesses to fill in missing information. As a result, Would You Rather provides a protocol that is quick and easy to use as an opener, and gets students to model with mathematics (Math Practice 4) and have mathematical conversations (Math Practice 3).

To begin, project the image of the Would-You-Rather question, and encourage students to:

1. **Come up with their own answer**
2. **Convince their peers using mathematics**.

Groups of 3 are generally ideal for small group discussion. Transition to a full class discussion after groups have talked for 3 – 5 minutes, and see if a consensus emerges. Students may have questions and requests for more information. This is a good time to share extra info (i.e., how much does a banana weigh) and see if that helps students come to a conclusion.

Consensus is not necessary! Students may disagree with the group for any reason ("I only want one banana"). What's important is they defend their reasoning, ideally with mathematics.

Possible student observations and questions.

- What does one banana weigh?
- How big/small are the bananas in question?
- How many bananas in a pound?
- Option 2 is better if you get 7 or more bananas.
- I only want 1 banana, so I'd take option 1.

Extra info, to share if requested.

- On average, a banana weighs about a quarter pound.
- Typically, bananas weigh between 1/5 and 1/3 of a pound.

WOULD YOU RATHER…

Option 1
Buy **1 banana** for **29¢**

OR

Option 2
Buy **1 pound of bananas** for **$1.89?**

Prompts and Questions

- What additional piece(s) of information would help you make a decision?
- How many bananas would you guess are in a pound? What's a reasonable range?
- What would bananas have to weigh to make Option 1 a good deal?

WOULD YOU RATHER…

Option 1

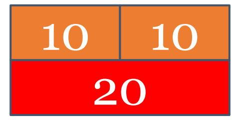
Buy **1 banana** for **29¢**

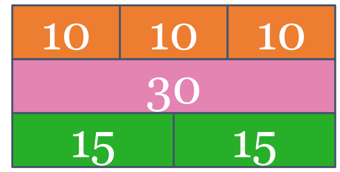
OR

Option 2

Buy **1 pound of bananas** for **$1.89?**

Number Rod Proofs



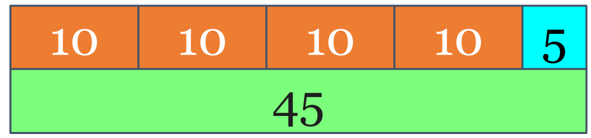


Materials and Prep

Number Rods, scratch paper, pencil.

Motivating Question

If you know the value of one Number Rod, how can you find the values of all the others?



Launch

Give about 5 minutes for students to play and explore the rods. Ask for a few student comments about what they notice about them. Then transition to the lesson.

Problem 1. Suppose the orange (2 cm) rod has a value of 10. What are the other Number Rods worth?

Let students offer arguments, and put forward your own if none are forthcoming. For example, the red (4 cm) rod equals 2 orange (2 cm) rods, or 2 tens. That means red equals 20. See images for arguments about the pink and dark green rods. Once students have found what all the rods are worth, ask them to find the value of the light green (9 cm) rod, and prove their answer in more than one way.

Now tell students to let go of all the old values and suppose something totally different:

Problem 2. The red (4 cm) rod equals 32.
Now what is the value of the other rods?

(In this case, every rod will be equal to a multiple of 8.)

Once students can convincingly make or follow these kinds of arguments, pose the additional problems and let them work independently, in pairs or trios.

Launch Key Points

- Go back and forth between proving a rod value (i.e., red equals 20) and letting students work for a minute or two to try their own.
- Do as many examples as needed!
- Release students only after most of them can make or follow an argument that proves a rod value.
- There is no worksheet for students, so be ready to write or project Problems 3 – 6 as students are ready for them.

Work

Students work their way through as many of the problems below as they can. If more problems are needed, let students make up their own, and challenge each other to solve them.

Problem 3. If pink (6 cm) equals 48, what are the other rods?
Problem 4. If yellow (8 cm) equals 72, what are the other rods?
Problem 5. If purple (7 cm) equals 56, what are the other rods?
Problem 6. If gray (10 cm) plus blue (5 cm) equals 60, what are the other rods?

Tips for the Classroom

1. You can pose problems with almost no words by placing/drawing/projecting the Number Rods on a white board and writing the numbers underneath or beside them.

2. Make sure students can build their own version of the problem and solve physically.

3. Adjust the difficulty of the problems as necessary.

4. Students can guess and check as a first strategy.

Prompts and Questions

- You know the yellow (8 cm) rod equals 72. What if the turquoise (1 cm) rod equaled 6? Would that work, or is that too big or too small?

- How do you know that the blue (5 cm) rod has that value?

- Is there any rod that feels easy to do next?

- What makes that one easy?
  What makes that one hard?

- You're done with all the problems? Then you can come up with a new problem and challenge another group to solve it!

Closer

Take Problem 3 (or the last problem all students have attempted) and spend a few minutes letting students share their answers with each other. Have students share their methods with a partner, and then take one or two volunteers to share their method with everyone.

Suppose you discussed Problem 3. The main tactic to underline is that going from the known value of the pink (6 cm) rod (48) to the unknown value of the purple (7 cm) is hard. But it's much more straightforward to figure out the turquoise (1 cm) rod first. It takes 6 of the turquoise to make the pink, so divide 48 by 6 to get that value as 8. Then you can multiply 7 × 8 to get the purple value as 56.

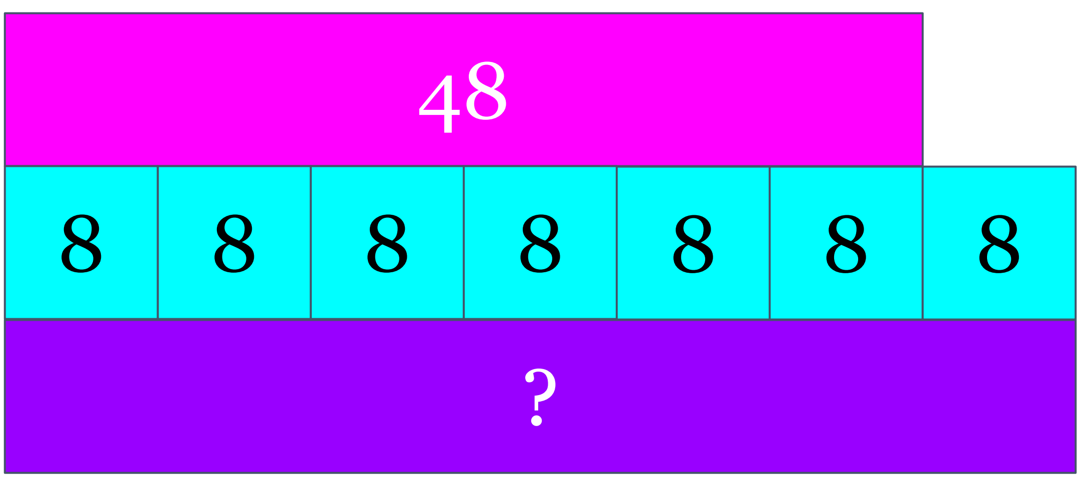
(Note that this ends up being the same situation as Problem 5)

Prompts and Questions

- For the problem we're working on, pick a color you were able to get, and explain to your partner how you got it.
- What was a harder color to get? Does your partner have any ideas on how to get it?
- What strategy helped make these problems easier?
- Could you always figure out the value of the turquoise rod? Did doing that help you figure out other values?

Choice Time

- Big Pig
- Close Calls and Bullseyes
- Block Free Build
- Challenge Problems



Overview

Focus Standards

MP2	Reason abstractly and quantitatively.
4.OA.2	Multiply or divide to solve problems.

Materials: 6-sided dice, 10-sided dice, Odd Pig Out worksheets, scratch paper, pencil.

Opener	**Broken Calculator**	10 – 15 minutes
Main Activity	**Odd Pig Out**	20 – 40 minutes
Closer	**What's more likely, odd or even products?**	5 – 10 minutes
Choice Time	• Big Pig • Odd Pig Out • Pattern Block Free Play • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP3 | MP5 | 3.NBT.2 | 3.OA.7

Broken Calculator

This creative exercise in arithmetic is surprisingly dynamic, with a simple constraint that provides interest and rigor.

Tell the students that you have a calculator with some broken buttons. The challenge for students is to **make a target number on the calculator in as many ways as they can**, despite not being able to use the broken keys.

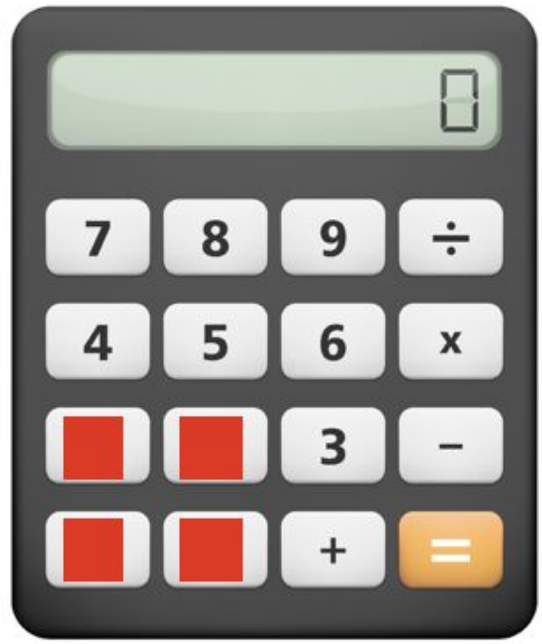
Students work in pairs or trios to write down a list of solutions. Circulate and help students to talk to each other, extend their thinking, or get unstuck. If students are starting to slow down after 5 minutes or so, pause to highlight some interesting approaches, or let students share solutions they found notable.

If students seem to have exhausted their interest in the original question, you can add challenges.

Example Challenges (optional)

- Solve the problem using the division key.
- Solve the problem starting with a 3-digit number.

Target: 21



Tips for the Classroom

1. Avoid writing faulty "equations" that treat the equals sign as the "compute" button on a calculator, i.e.,

   $5 \times 6 = 30 - 5 = 25$ is false.

   Write a single equation using parentheses if necessary, or rewrite what you've done so far on a new line.

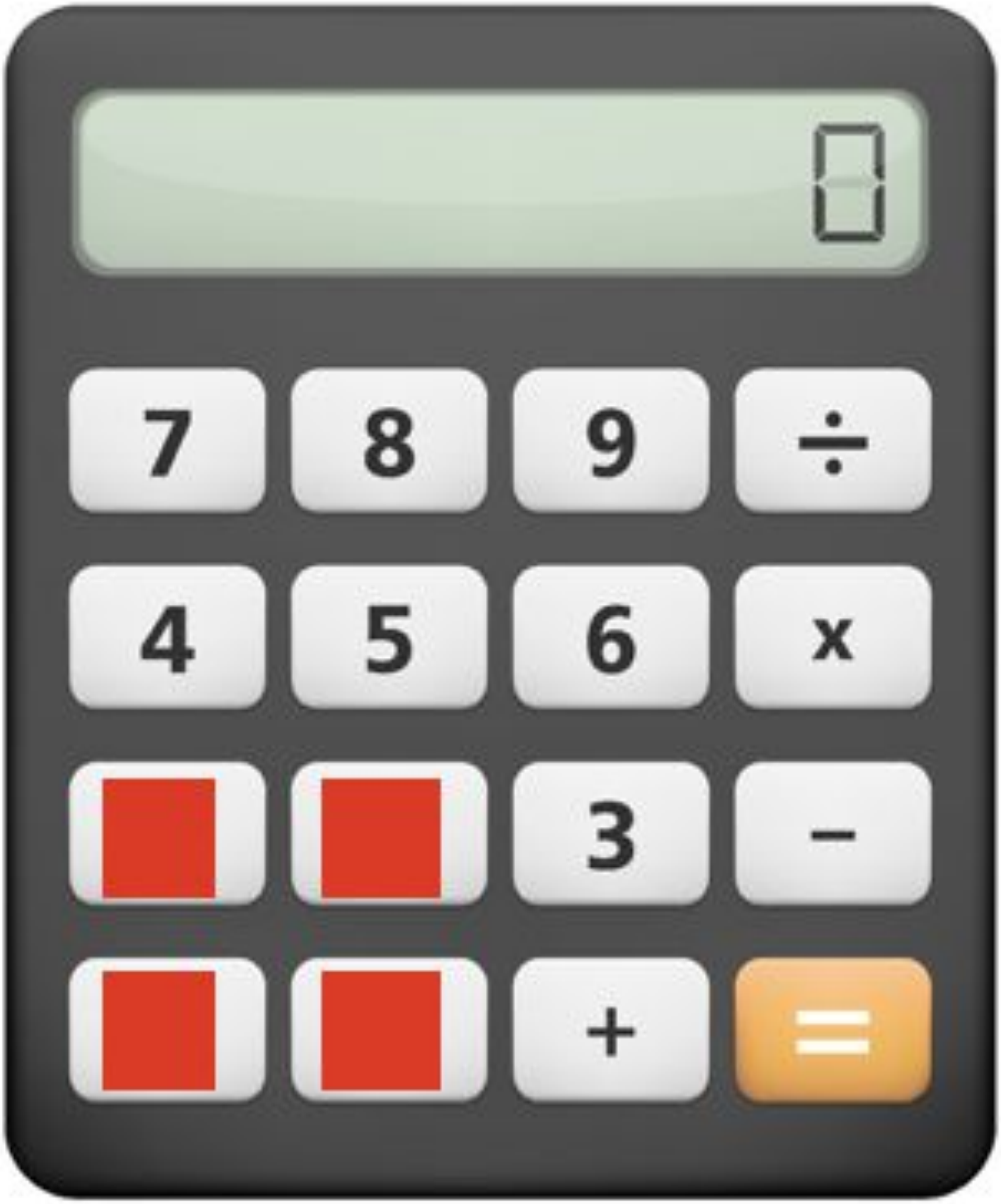
   Examples $(5 \times 6) - 5 = 25$ or $5 \times 6 = 30$
   $30 - 5 = 25$

2. Have a representative from each student group come up during their work time to write one or two of their favorite solutions on the board. This helps other students to get inspired, and also makes the transition to share solutions quicker.

Prompts and Questions

- Good idea, but you used the "1."
- Can you use that same approach to get another solution?
- How many different ways have you come up with so far?
- Do you think it's possible to solve using division?

Target: 21



Odd Pig Out

Materials and Prep

10-sided and 6-sided dice, Odd Pig Out worksheets, pencil.

How to Play

Players take turns rolling two 10-sided dice as many times as they like. After each roll, they multiply the numbers shown on the dice.

- If the product is even, they add that number to their current points for the turn.
- If the product is odd, players lose all their points *from that turn* and their turn is over.

A player may choose to end their turn at any time and "bank" their points. The first player with 500 points wins.

Launch

Odd Pig Out is a natural extension of Big Pig, but for multiplication. Introduce the game, choose a volunteer, and play a demonstration game using the 10-sided dice. If students already know Big Pig, this game should be relatively quick and intuitive to learn. You'll only need to play a few rounds before releasing the class to play their own games.

Launch Key Points

- Students should have some backup method of confirming their products if they need it. Either drawing arrays, "groups of," or using a multiplication table are all possibilities
- If students know Big Pig already, the launch can go quickly.

Work

Students play Odd Pig Out. Try grouping students in pairs to play against each other, one on one, to start. Once they've played a few games, combine the pairs to play in teams of two on two. Team play will force them to talk to their partner about whether they should roll more or stop.

Prompts and Questions

- How many points do you have for this turn, so far?
- Who is ahead?
- Are you sure that's the product for those two numbers?
- What strategy are you using?

Tips for the classroom

1. Solicit advice from the class about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.

2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. Help them be good winners and losers.

3. If students need more support, they can play with two 6-sided dice instead the 10-sided dice. A worksheet and table for this modification is included below. For a game with 6-sided dice, the winner is the first to claim 300 points.

4. Students can play to 1000 for a longer game.

Day 4

Odd Pig Out

How to Play

Players alternate turns rolling two dice and finding the product as many times as they would like.

a. If the product is **even**, they add that number to their current points. They may choose to roll again or end their turn.
b. If the product is **odd**, they lose all their points from that turn and their turn is over.

The player who reaches 300 points or more in their bank first is the winner!

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Day 4

Odd Pig Out

How to Play

Players alternate turns rolling two dice and finding the product as many times as they would like.

a. If the product is **even**, they add that number to their current points. They may choose to roll again or end their turn.
b. If the product is **odd**, they lose all their points from that turn and their turn is over.

The player who reaches 300 points or more in their bank first is the winner!

Rolls	Rolls
Bank	Bank

Closer

Do students know if they're more likely to roll odd products or even products? Some questions to discuss to build up to an answer:

- How many odd numbers are there on the multiplication table (up to 6 by 6)?
- How many are even numbers?
- How are the numbers distributed?
- Do students see any patterns?

Depending on how the conversation goes, students may start to articulate a conjecture about the products of even and odd numbers.

Even times even = even
Even times odd = even
Odd times even = even
Odd times odd = odd

This gives an argument that the chance of rolling an even product are three times the chance of rolling an odd product.

Alternatively, students might circle the even or odd numbers on a multiplication table and find the same thing!

Prompts and Questions

- Do you prefer to be more risky or more cautious? Which seems like it works better in the long run?
- What's your favorite winning strategy so far?
- Do you have a rule for when you stop rolling and bank your points?

Choice Time

- Big Pig
- Odd Pig Out
- Pattern Block Free Play
- Challenge Problems

Overview

Focus Standards

MP1	Make sense of problems and persevere in solving them.
6.EE.6	Use variables to represent numbers and write expressions when solving problems.

Materials: 21st Century Pattern Blocks, 6-sided dice, scratch paper, pencil.

Opener	**Don't Break the Bank**	10 – 15 minutes
Main Activity	**Pattern Block Proofs**	20 – 40 minutes
Closer	**Squares and Light Gray Rhombuses**	5 – 10 minutes
Choice Time	• Odd Pig Out • Don't Break the Bank • 21st Century Pattern Block Free Build • Challenge Problems	5 – 25 minutes

Standards Connections

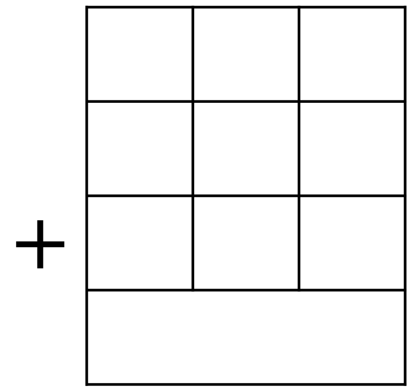
MP3 | MP4 | MP5 | MP7 | MP8 | 6.G.1

Don't Break the Bank

How to Play

Don't Break the Bank is a fun game to play with the whole class.

Ask everyone to draw a table on their own paper that looks like the one to the right. Draw one for yourself as well. You'll need one six-sided die to play.



Roll the die. Whatever number it lands on, every player enters it in one of the nine spots on their board. After nine turns, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum **without going over 999**. Note that EVERYONE uses the same collection of numbers - they just place them differently on their own board.

For example, consider the game to the side. After eight turns, there's just one more roll left to go. If a 1 is rolled, this player would have a near perfect score of 998. Any other roll would result in busting - that is, going over 999.

Once the game is complete, find out who busted, and who got the best scores. It's usually fun to play 2 - 3 games.

Tips for the Classroom

1. Make sure students are actually placing their numbers after each roll.

2. **Play slowly enough to allow students to think, but quickly enough that they can't do too many calculations**. They should be estimating rather than calculating precisely.

3. Choose your own placement slowly enough for each roll that you don't overly influence where students choose to place the numbers on their own boards.

4. In playing early games, you might choose to bust on purpose, so students know it's okay to do.

Prompts and Questions

- Where should I put this 5... I think I'll put it in the tens column.

- Where are you going to put that 4? You don't have to place it where I did.

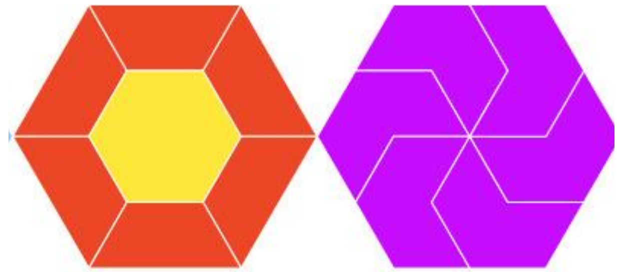
- (After playing) Did you use a particular strategy?

DAY 5	Opener	Main Activity	Closer	Choice Time

Pattern Block Proofs

Materials and Prep
21st Century Pattern Blocks, scratch paper, pencil.

Motivating Question
How can you determine the relative areas of all the 21st Century Pattern Blocks?



Launch
Refer students to all 8 of the 21st Century Pattern Blocks. Pose a series of questions about their areas–specifically, the yellow hexagon, red trapezoid, and blue rhombus (in terms of the green triangle). This should be quick, especially if students have worked with pattern blocks before. Now hold up the gray dart, the teal kite, and the pink right triangle. Tell students that today we'll try to find their areas!

You can start by taking student guesses on what the missing values are. Students may have intuitions, but math will allow everyone to create arguments and know for sure.

Build large hexagons (see image on the right) in two different ways, so all students can see them. Ask students how these two large hexagons could help with finding the value of the purple concave hexagon.

Students can discuss in trios, then as a class. Students will, ideally, be able to articulate an argument: we can find the area of the shape on the left in terms of green triangles (24) and the right in terms of the "unknown" shape (6 concave hexagons).

So 6 purple = 24 green, which means 1 purple = 4 green.

That's not exactly surprising, since you can build the purple shape from 4 green triangles. But this is still a powerful approach, since it could work for shapes we CAN'T make out of triangles. Thus, the challenge: **Find the areas of the gray dart, the pink right triangle, and the teal kite even though none can be built from equilateral triangles.**

Launch Key Points

- If students can build a shape twice, once with blocks they know, once with blocks they don't, they'll be able to make these kinds of arguments.
- It's tempting to write full equations (6p = 24g) and use algebra, but applying the rules of algebra abstractly will confuse students more than enlighten. Keep working visually, and emphasize the equations as connected to the images.

Work

Students work in pairs or trios to find the areas of the three unknown shapes in the 21st Century Pattern Blocks. Circulate to give hints, help students write equations to match their drawings, suggest next steps in their arguments, or just listen.

A good first hint: the same large hexagon shape we used in the beginning can be used to relate all the shapes to each other. (See image below.)

If students can successfully find all the missing areas, encourage them to find smaller shapes to create simpler relationships between blocks.

Tips for the Classroom

1. Tracking the unit (the area of 1 green triangle) is important. You can shorten it to simply "g".

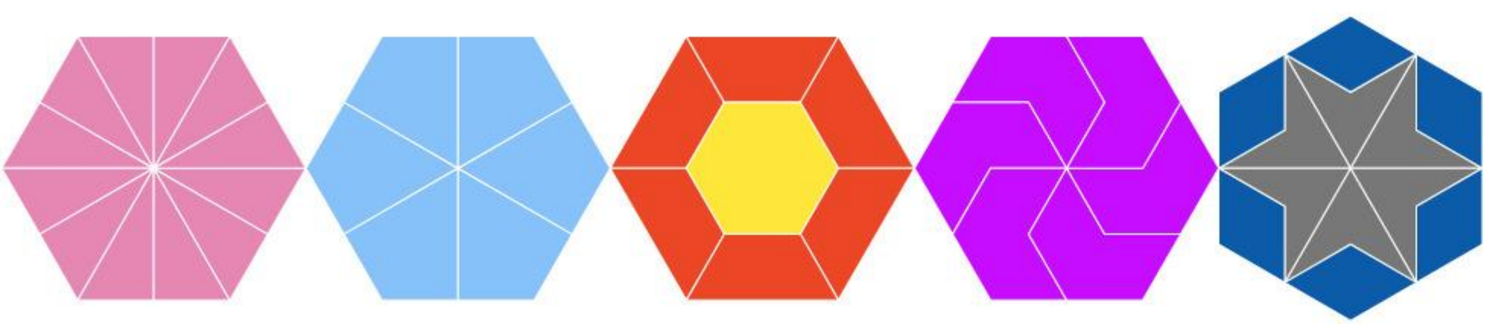
2. This activity isn't just about getting the answers. It's also about connecting geometry with equations to solve for unknowns. Don't let students satisfy you with right answers only. Clear arguments are a major goal.

3. Here are the answers, but try not to provide them for students. It's better if they can find these themselves and convince each other what they are.
   Dart = 2g
   Kite = 4g
   Pink Triangle = 2g

4. Make sure each member of each group is engaged. This usually means that everyone should be manipulating blocks themselves. If you see kids who aren't touching blocks at all, intervene to help them engage.

Prompts and Questions

- Can you make a shape out of blocks you know, and also out of blocks you don't know?
- What if we try the same hexagon shape again. Could you make it out of kites?
- What do you know based on what you've built?
- Can you find another shape that allows you to make a simpler argument?



Closer

Show the image below on a projector or document camera, and ask students to find an argument to relate the orange square and the light gray rhombus.

Students can work in pairs or trios for a few minutes, and then present ideas to the class.

The argument to expect from this image is: the shapes are the same area, and both include 12 triangles. So what's left when you remove 12 triangles from each must also be the same. That means 6 squares has the same area as 12 rhombuses. So the area of each square is double the area of a rhombus.

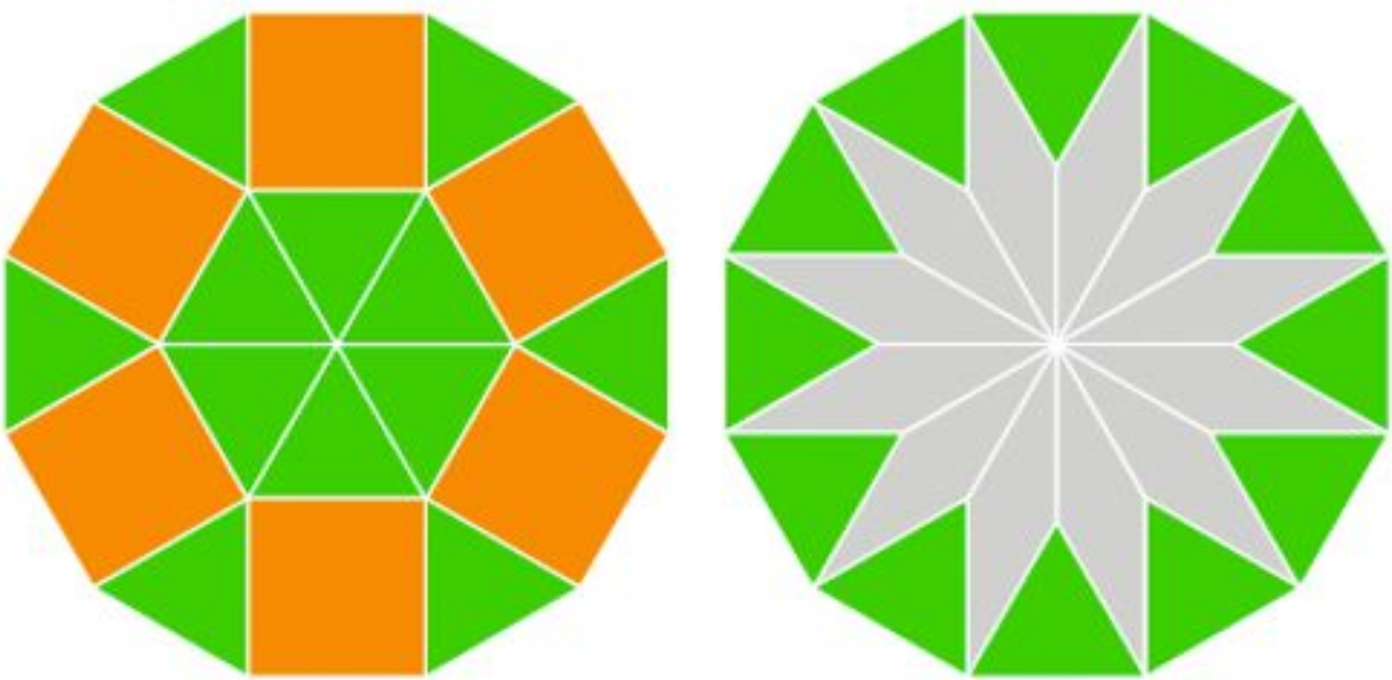
If you wrote this out algebraically, you might write:
6 squares + 12 triangles = 12 rhombuses + 12 triangles
So 6 squares = 12 rhombuses.
That means 1 square = 2 rhombuses.

Prompts and Questions

- What can you figure out from this image?
- Do the two shapes have the same area?
- How many triangles are in each?
- How many squares/rhombuses?
- If you took the triangles away, would what's left still be equal?

Choice Time

- Odd Pig Out
- Don't Break the Bank
- 21st Century Pattern Block Free Build
- Challenge Problems



Overview

Focus Standards

MP8	Look for and express regularity in repeated reasoning.
4.NBT.5	Multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Materials: Big Blockout game board, 6-sided dice, scratch paper, colored pencils, pencil.

Opener	**Would You Rather**	10 – 15 minutes
Main Activity	**Area Model for Multiplication; Big Blockout**	20 – 40 minutes
Closer	**Big Blockout Analysis**	5 – 10 minutes
Choice Time	• Big Blockout • Odd Pig Out • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP3 | 3.MD.7

Would You Rather

Would You Rather questions offer a choice between two options. The job of the students is to decide which of the two options they would rather have, and convince their classmates of the wisdom of their choice.

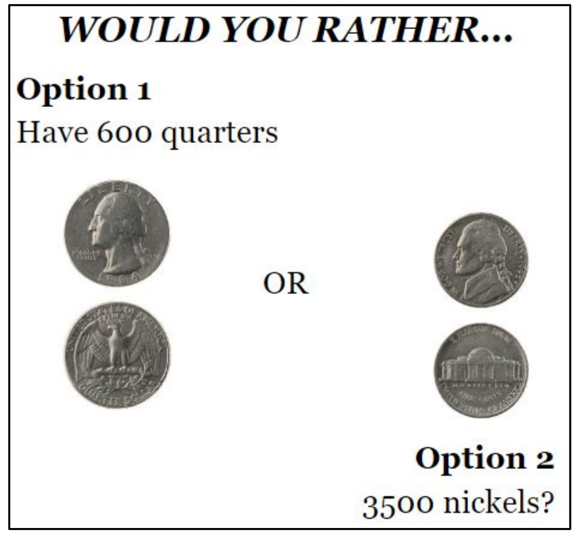
Project image of the Would You Rather question, and encourage students to:

1. **Come up with their own answer**
2. **Convince their peers using mathematics**.

Groups of 3 are generally ideal for small group discussion. Transition to a full class discussion after groups have talked for 3 – 5 minutes, and see if a consensus emerges. Students may have questions and requests for more information. This is a good time to share extra info and see if that helps students come to a conclusion.

Possible student observations and questions.

- Which gives you more money?
- How heavy are this many quarters versus this many nickels (or how much space do they take up)?
- How hard is it to exchange all these coins for paper money?



Prompts and Questions

- What additional piece(s) of information would help you make a decision?
- What is 600 quarters worth?
- What is 3,500 nickels worth?
- How many of each could you have so that the amounts would be the same?

WOULD YOU RATHER...

Option 1

Have 600 quarters

OR

Option 2

3500 nickels?

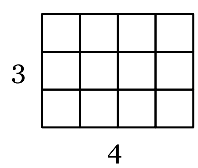
Area Model for Multiplication
(Main Activity part 1)

Materials and Prep
Scratch paper, pencil.

Motivating Question
How can a rectangle help you multiply large numbers?

Launch
The Launch today involves leading the students through using the area model of multiplication before transitioning to playing the game Big Blockout.

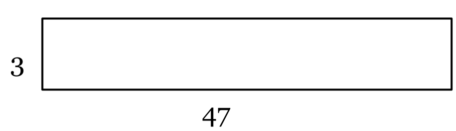
Show students a 3x4 array and ask: How does the 3 by 4 rectangle connect to multiplication?

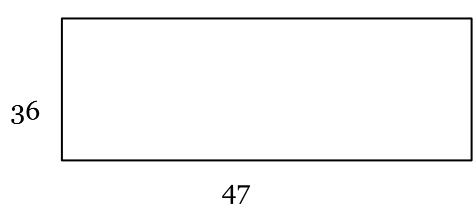


Students might recognize that the product of the side lengths is the area: $3 \times 4 = 12$ (students will likely recognize how the rectangle above connects to the idea of three groups of four). We can use this to jump to the area model for multiplication.

Using this rectangle idea, how could you draw pictures to represent 3×47 or 36×47?

Students will see that it's cumbersome to draw these rectangles to scale, but we could draw "generic" rectangles.





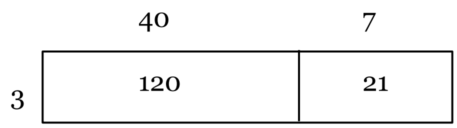
How can you use these generic rectangles to help you multiply? In other words, can we cut them into parts so that numbers are easier to multiply?

Launch Key Points

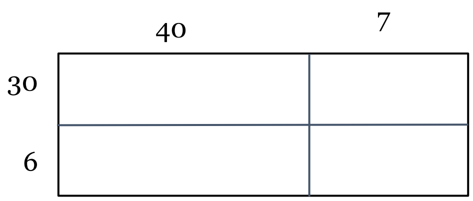
- Allow some productive struggle with the early examples, but move students on to breaking numbers into tens and ones pretty quickly if they're making things difficult for themselves.
- Some students may interpret breaking numbers apart as simply cutting them in half. They will quickly see that breaking a number like 47 in half doesn't make it easy to work with!
- Reinforce the fact that the area model is not to scale. However, it is a great way to organize the work visually for a multiplication problem.

Launch (continued)

Students might see that it's probably easiest to use place value, namely tens and ones, to break apart the numbers into more manageable chunks. Then the generic rectangles can be used as a tool to calculate the partial products, and you simply add up the parts. Here are some reasonable possibilities:



So, $3 \times 47 = (3 \times 40) + (3 \times 7)$
$= 120 + 21$
$= 141$



And, $36 \times 47 = (30 \times 40) + (30 \times 7) + (6 \times 40) + (6 \times 7)$
$= 1{,}200 + 210 + 240 + 42$
$= 1{,}692$

Work

Ask students to try this method for 9×68 and 54×81, and then check their work with partners.

Follow up challenge: What's the greatest product that can be made using the digits 1, 2, 3, and 4 exactly once each?

After some struggle, students may realize that it's $41 \times 32 = 1{,}312$.

Spend a few minutes discussing student work on these problems before moving on to the second Main Activity for the day: Big Blockout.

Tips for the Classroom

1. Allow students to break apart numbers any way they choose at first. They may realize quickly that their method isn't very helpful compared to using tens and ones, which allows for quicker mental calculations with the friendlier numbers.

2. If the opportunity arises, reinforce the fact that breaking apart numbers this way for multiplication is using the distributive property.

Prompts and Questions

- What's a way to break apart that number into two smaller numbers that are easier to multiply?
- How could using tens and ones help?
- For a 2-digit times 2-digit problem:
  - What's the sum of each column in your array? What's the sum of those two sums?
  - What's the sum of each rows in your array? What's the sum of those two sums?
  - Are your answers to these two things the same or different? Why?

Big Blockout
(Main Activity part 2)

Materials and Prep
Big Blockout game board, 6-sided dice, colored pencils, pencil.

How to Play
Players take turns rolling three dice, choosing two numbers to add and multiplying by the third. The resulting expression defines a rectangle, which the player shades onto the board in any place they choose, so long as it entirely fits on the board and doesn't overlap an existing rectangle. If a player can't find a way to place their rectangle, they must pass for their turn. The game ends when both players must pass, one after the other. Whoever claims the most area by the end of the game wins.

Example:
- First player rolls 2, 3, and 4, chooses to take $(4 + 3) \times 2$, shades in a 7 by 2 rectangle, and records 14 points.
  Was that the best choice? (No)
- Second player rolls 1, 3, and 5, chooses $(1 + 5) \times 3$, shades in a 6 by 3 rectangle, and records 18 points.
  Was that the best choice? (No)

Launch
With a volunteer, demonstrate the first two turns. Players choose colors, then take turns rolling the dice. Make sure the rules and how to win are clear before releasing students to play on their own.

Work
Give students time to play a few games and observe the choices they're making. If they try an expression whose corresponding rectangle can't be fit on the board, encourage them to try reconfiguring their numbers to find one that can.

Launch Key Points

- Allow students to make "bad" choices as they start the game. There's time later to analyze strategy and generalize best choices.
- Make sure that students know that even if they can figure out the 'best' choice in terms of maximizing area, if it doesn't fit on the board they can't use it.
- Rectangle shading should be done quickly and doesn't need to look perfect.

Prompts and Questions

- How are you making your choices on each roll?
- When is the best score *not* the best move?

Tips for the Classroom

1. For students who struggle with multiplication, consider giving them a multiplication table, or having them create one on their own. Many students really only struggle with some of the facts while knowing others, so consider having them write the troublesome facts to keep in front of them for quick reference.

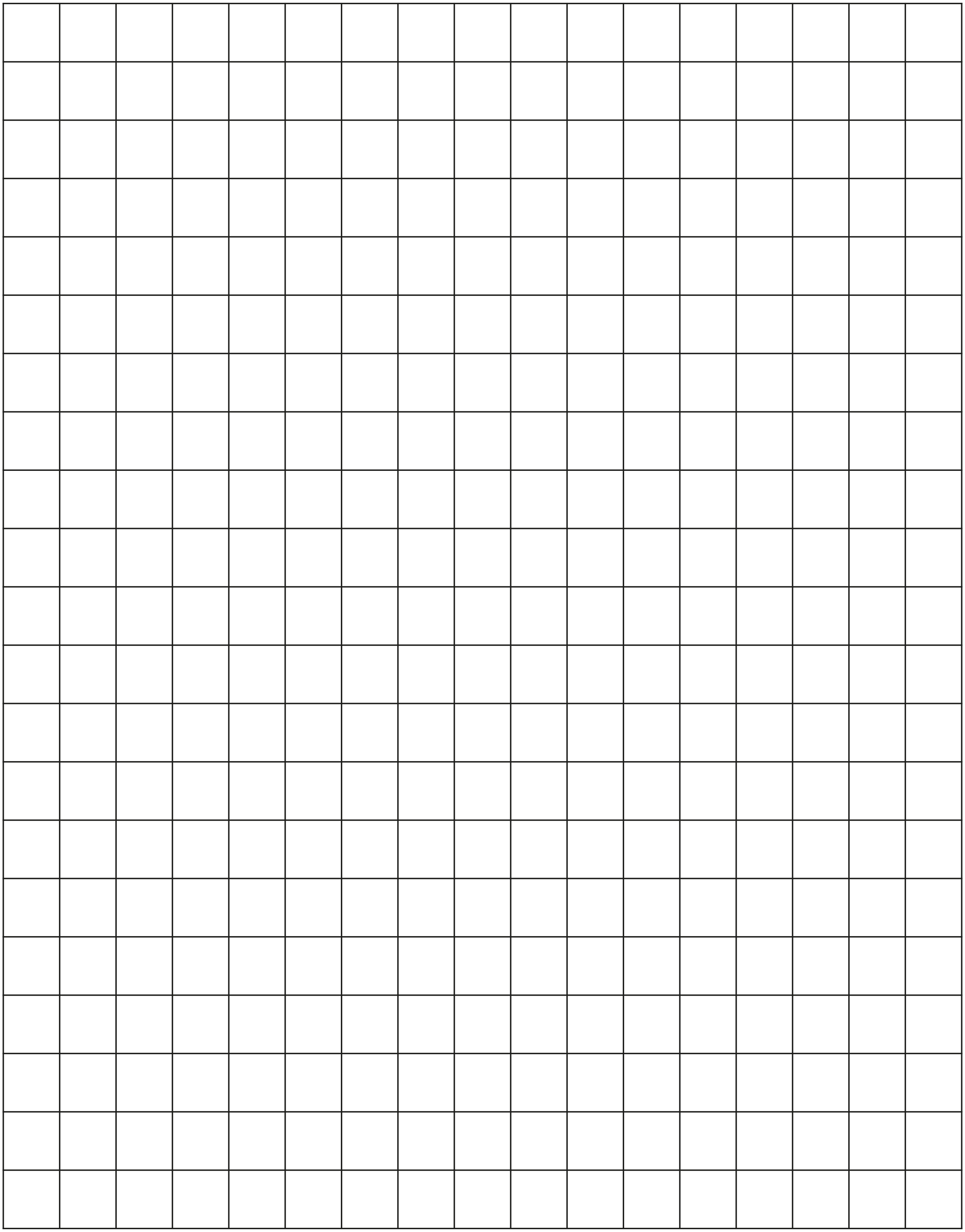
2. Rather than shading in rectangles with their color, students might record their scores inside their rectangles instead.

3. Consider these game variations:
   - Instead of adding two dice values, make a 2-digit number out of them; so, 2, 3, and 4 could become 23 × 4 = 92.
   - Roll four dice; add three numbers and multiply by the fourth.
   - Use 8, 10, or 12-sided dice.

   The first and third variations may create large numbers that make the board unusable. For these variations, ditch the board and the rectangles and instead just track score arithmetically. Players would just record their scores and try to reach a target number, such as 500. This way of playing loses the interest provided by constraining where rectangles can be places on the board, but it provides a lot of fact practice.

Day 6

Big Blockout Game Board



Closer

Bring the class together to do a quick wrap-up for analyzing Big Blockout.

If you roll 1, 4, and 5, what are your choices?
Which is the best, in terms of maximizing area?

$(1 + 4) \times 5 = 25$ $(1 + 5) \times 4 = 24$ $(4 + 5) \times 1 = 9$
The first one is the best.

Use the following rolls in the blank:
2, 4, 5 3, 4, 5 4, 4, 5 5, 4, 5 6, 4, 5

What if you roll ________? What's your best choice?

For *any* three numbers rolled, what's your best choice?

Possible student response:
It looks like you add the two smaller numbers and multiply by the largest.

If time, invite students to try to defend this by arguing why it might be true in general.

Prompts and Questions

- Do some values seem to come up more often than others?
- How did you determine your rule for what the best choice is?

Choice Time

- Big Blockout
- Odd Pig Out
- Challenge Problems

Overview

Focus Standards

MP8 Look for and express regularity in repeated reasoning.

6.NS.4 Compute fluently with multi-digit numbers and find common factors and multiples.

Materials: Penny Nickel Dime recording sheet, Prime Climb Coloring Chart, 6-sided dice, crayons or colored pencils, pencil

Opener	**Penny Nickel Dime**	10 – 15 minutes
Main Activity	**Prime Climb Color Chart**	20 – 40 minutes
Closer	**Prime Climb Color Chart**	5 – 10 minutes
Choice Time	• Challenge Problems • Odd Pig Out • Don't Break the Bank	5 – 25 minutes

Standards Connections

MP1 | MP3 | MP7 | 4.OA.4

Penny Nickel Dime

Materials and Prep

One 6-sided dice, Penny Nickel Dime recording sheet, pencil.

How to Play

Roll the die. Whatever number it lands on, each player can choose to take that many pennies, that many nickels, or that many dimes. Each student records their numbers on their own recording sheet as the game continues. Repeat for seven rolls in all. The winner is the person who comes the closest to $1 without going over.

Play a game with your students. Go slowly enough that everyone can make their decisions and keep up, but quickly enough that students can't work out exactly what their current total is, and have to make estimates. Once a game is complete, have students share their score. Who busted? How close were you to getting to $1 without busting?

Invite the player with the best score to explain where they put their coin choices to win - and for the class to check that they didn't make any arithmetic errors!

Play 1 - 3 games, as time allows. Then have students share strategies.

Tips for the Classroom

1. Start play quickly, and assume that by the end of the first game students will have absorbed the rules.

2. While kids may bust their first few games, they'll inevitably start estimating and choosing good strategies for themselves.

3. To keep the game novel, ask students what they'd do differently if there were only 5 rolls per game, or six. Or eight! Try those variations and see what happens.

4. Downlevel the game by removing nickels as an option. Uplevel the game by adding quarters and change the target number to $2.50 or higher.

Prompts and Questions

- We rolled a 5. You can choose 5 dimes, 5 nickels, or 5 pennies. Which one do you want to choose?
- If you could go back and change one choice, which would it be and why?
- It seems like no one got $1 this time. Is it possible to get $1 with the rolls we had?

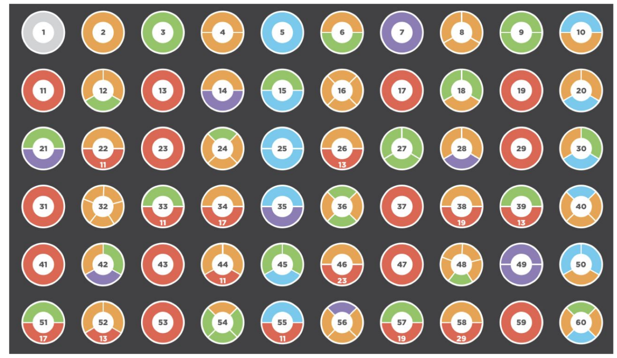
Day 7

Penny, Nickel, Dime Recording Sheet

Roll	Dimes	Nickels	Pennies
1			
2			
3			
4			
5			
6			
7			
Totals			
Total Score: _______________			

DAY 7	Opener	Main Activity	Closer	Choice Time

Prime Climb Color Chart



Materials and Prep
Prime Climb Coloring Chart, colored pencils or crayons, pencil.

Motivating Question
What patterns can you find in the hundred chart?

Launch
Show students the chart with the coloring to 20, and ask them what they notice and wonder.

Give students some time in pairs or trios to discuss ideas. Then discuss as a class. In particular, what are student conjectures to explain the way the numbers are colored?

Possible student observations might include:

- Every even number has orange in it.
- If there is green in a number you can reach it if you skip count by 3.
- The numbers with blue end in 5 or 0.
- The numbers with red are prime.

Wonderings might include:

- Why does 4 have two orange parts?
- Why does 8 have three orange parts?
- If prime numbers are red, why isn't 7 red?

Launch Key Points

- Make sure to leave their questions unresolved. They will be useful questions to come back to in the Closer.
- Similarly, don't be the answer key when it comes to their conjectures. Allow students to determine if a conjecture is always true, sometimes true, or untrue. This is also something that can be discussed in the Closer.

Work

Once students have discussed some ideas for how the coloring works for numbers 1 - 20, challenge them to color in the numbers 21 - 30 so that it extends the pattern(s).

Let students work in pairs. If they are stuck after five minutes or so, you can gather the class together and discuss how to color 21. A rationale for the coloring: 3 is green, 7 is purple, and 3 × 7 = 21, so 21 should be green and purple.

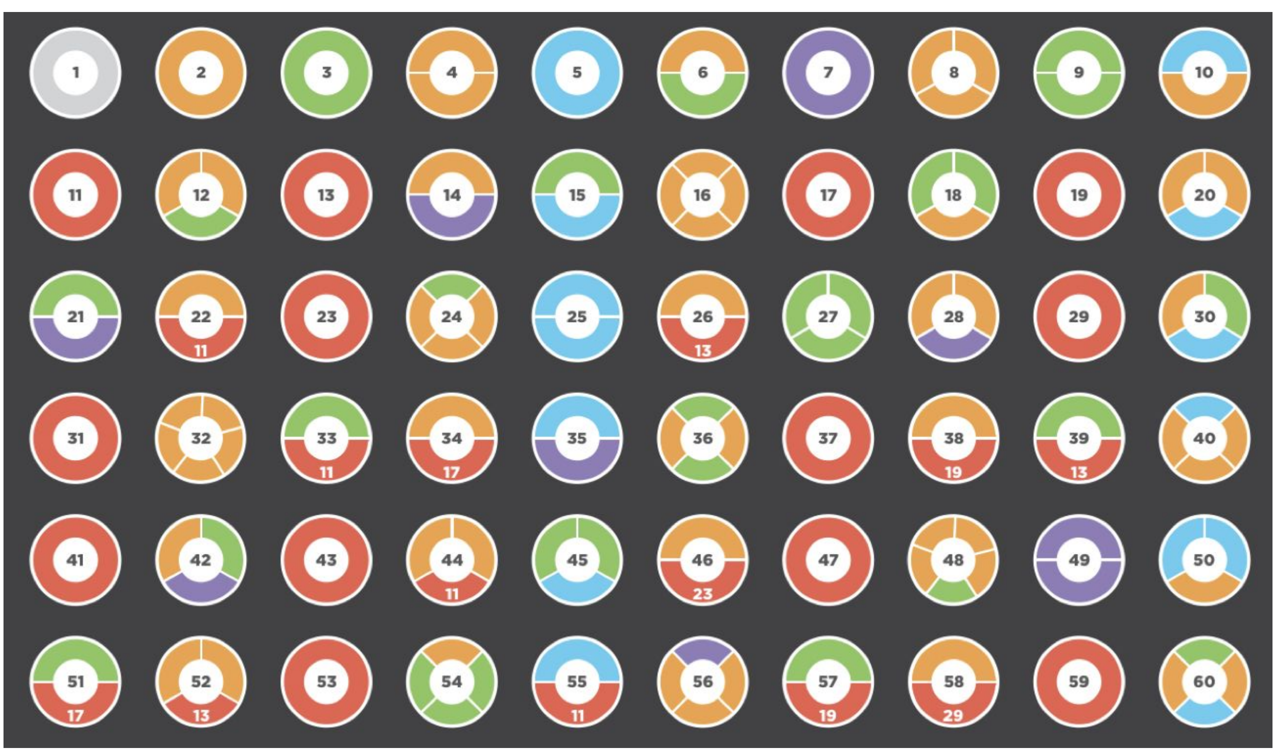
Once students have the hang of how the coloring works, let them work on their own again. They can color in as much of the chart as they can, but getting to 50 is a good initial goal. In general, multiplying and dividing or skip counting is the key to understanding how the coloring works.

Prompts and Questions

- Look at just one color at a time. What's happening with orange? What's happening with blue?
- What numbers have blue? (5, 10, 15, 20.) What do you think comes next in that pattern? So. should we colored in 25 and 30 with a blue part?
- Which numbers have two or more orange parts colored in (4, 8, 12, 16, 20)? What pattern do you see in those numbers?

Tips for Classroom

1. Don't expect students to finish the entire chart in one lesson. They can come back to it in the future.
2. As much as possible, have students explain their reasoning, question the reasoning of others, and determine the truthfulness of their conjectures.
3. For composite numbers divisible by primes larger than 10, the number will have a red segment in its coloration which can be distinguished by writing in the prime it represents on the segment. See the colored-in chart to 60.



Closer

Pick a few numbers that everyone has at least thought about: 28, 29, 30, for example. Let students defend their choices for coloring. Prompt them with questions, such as the ones below (modify them for the numbers your class chooses to discuss).

- Why should 29 be red?
- Why does 30 have orange, green, and blue?
- Based on what we know, what colors will 72 have?

You can use the chart colored up to 60 as a reference, but it's best if students can argue why a given coloration works, and convince other students based on multiplication arguments.

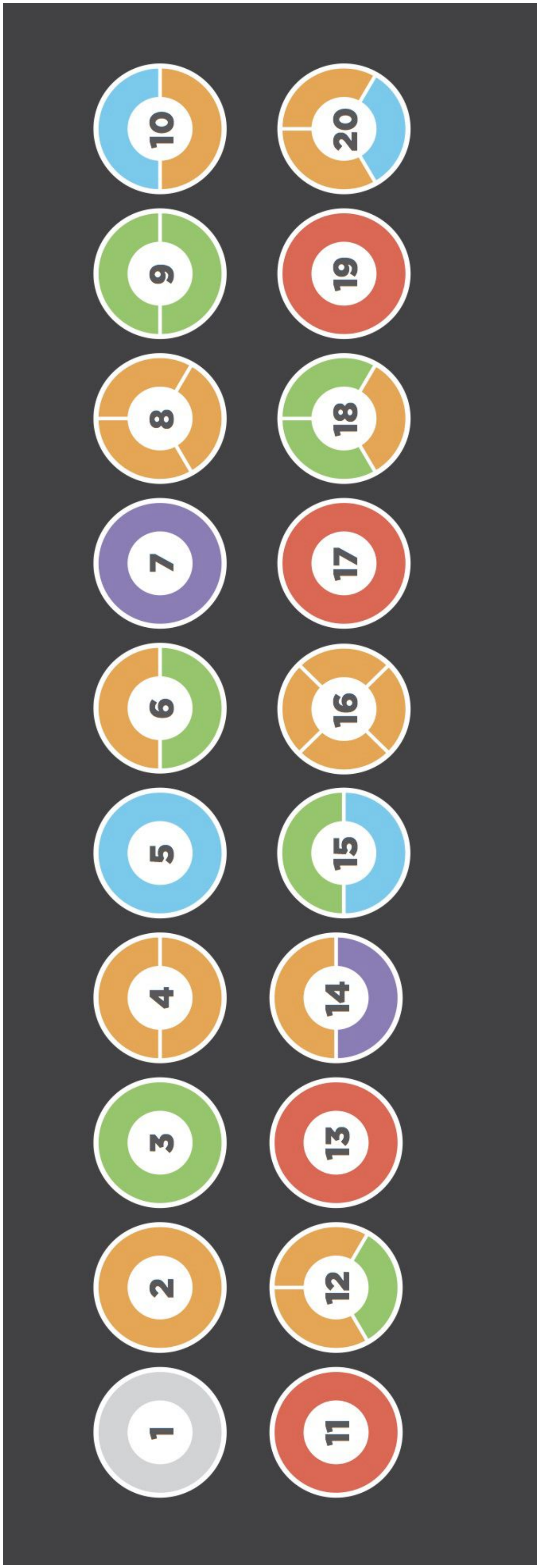
Prompts and Questions

- What patterns do we see in the Prime Climb Color Chart?
- If I write out some multiplication problems like $2 \times 5 = 10$, $3 \times 5 = 15$, and $4 \times 5 = 20$, what do you notice about the colors of all the numbers in these equations?

Choice Time

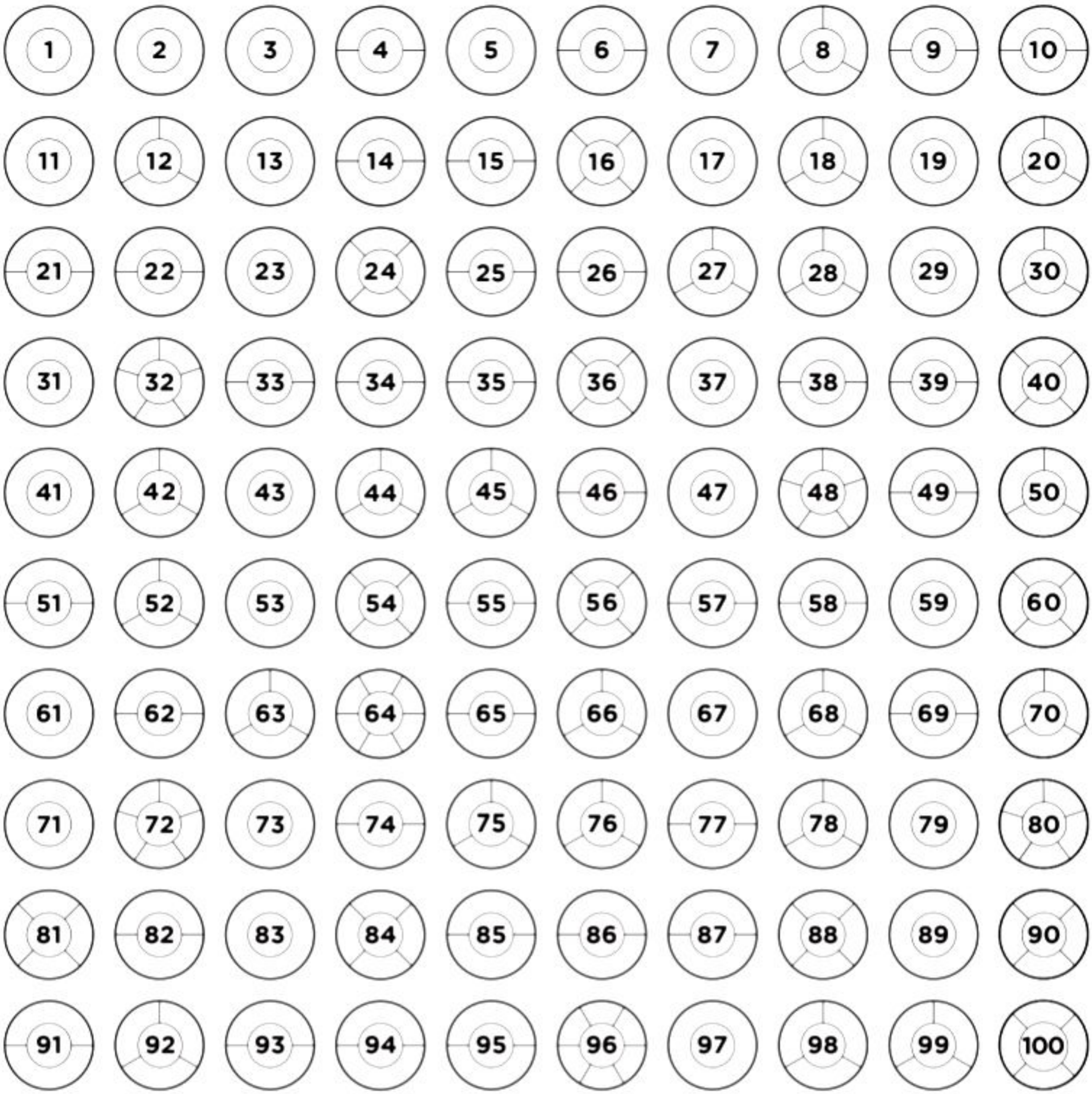
- Challenge Problems
- Odd Pig Out
- Don't Break the Bank

Day 7



Day 7

Prime Climb Coloring Chart



Overview

Focus Standards

MP7 Look for and make use of structure.

6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

Materials: Prime Climb game board, 10-sided dice, scratch paper, pencil.

Opener	**Counterexamples**	10 – 15 minutes
Main Activity	**Prime Climb**	20 – 40 minutes
Closer	**Prime Climb Strategies**	5 – 10 minutes
Choice Time	• Challenge Problems • Prime Climb • Odd Pig Out • Penny Nickel Dime	5 – 25 minutes

Standards Connections

MP6 | MP8 | 5.OA.B

Counterexamples

Consider some motivating examples, and make a (false) conjecture to draw students into playing. Then revise your conjecture to account for the counterexamples students offer. See Day 2 for a more detailed explanation.

Motivating examples

- 222/(2+2+2) = 37
- 333/(3+3+3) = 37
- 444/(4+4+4) = 37

I notice that this pattern always equals 37.

Conjecture.
Any number with the same digit in each place value divided by the sum of those digits is always equal to 37.

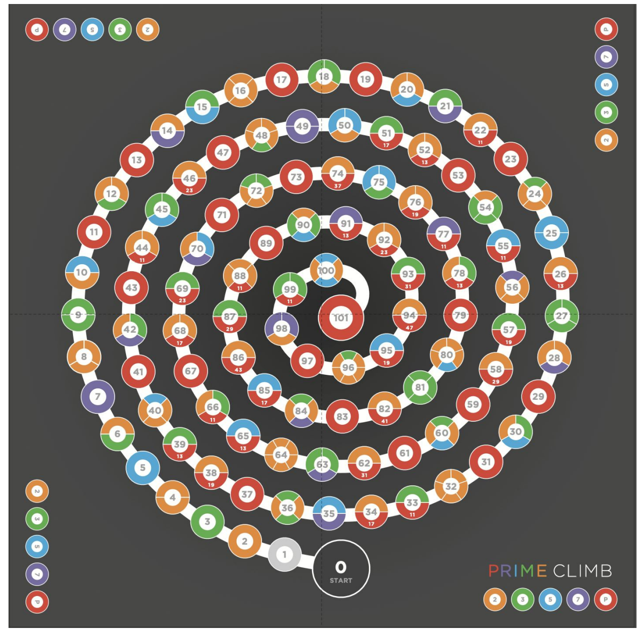
Possible student counterexamples.
For 2-digit numbers, the quotient is 5.5.
For example, 22/(2+2) = 5.5

There are all manner of possible revisions, including specifying the number of digits (i.e., with five digit numbers, the answer is always 2222.2, as in 55555/(5+5+5+5+5)), or playing with combining numbers with more than a single digit (for example: 232323/(23+23+23) = 3367).

Prompts and Questions

- I think I see a pattern here. I'm going to make a conjecture…
- You think my conjecture is wrong. But how can you *show* me it's wrong?
- Why is the quotient always 37 for 3-digit numbers? Can we prove it?
- Why is the quotient always 5.5 for 2-digit numbers? Can we prove it?

Prime Climb



Materials and Prep

Prime Climb game board, 10-sided dice, scratch paper, pencil.

How to Play

Video instructions available at:
mathforlove.com/games/prime-climb/how-to-play.

Here is the "quick start" version of the rules:

During a turn, there are four phases.

1. **Roll.** Roll the dice. You get two numbers from 1 to 10 to use for moving. In you roll doubles, you get that number four times instead of two.
2. **Move.** Move your pawn(s). Apply your dice rolls one at a time to the number your pawn(s) is on, using your choice of +, -, ×, or ÷. You can also use Keeper cards if you have them.
3. **Bump.** If you end your Move phase on the same space as another pawn, including your own, send it back to start.
4. **Draw.** If you end your Move phase on an entirely red space (i.e., a prime greater than 10), draw a Prime card. If it is a Keeper card, save it for a future turn. Otherwise, apply the card now.

After someone lands both pawns exactly on 101, they win the game. (In the quick version of the game, you win after you get just one pawn to 101.)

Example Turn:

With pawns on 4 and 26, you roll a 3 and a 9. You could:

- Add 3 to 4 to move your pawn to 7, then multiply by 9 to move your pawn to 63.
- Multiply 26 by 3 to move your pawn to 78, then add 9 to move it to 87.
- Add 9 to 4 to move one pawn to 13, and multiply 26 by 3 to add the other to 78. Since 13 is completely red, you would then draw a Prime card.

You're never allowed to move to numbers off the board. You CANNOT add the 3 and 9 first and use a 12 to add, subtract, multiply or divide. You have to apply the numbers on the dice one by one.

Launch

Set up the game and explain the rules. Take a student volunteer and demonstrate 3 or 4 turns each, showing, in particular, how Prime cards get drawn when you land on red circles. Also emphasize that dice must be applied to pawn(s) one at a time–they cannot be added (or subtracted) together and then used to multiply (or divide). Once students seem ready, have them play in groups of 4 with two players per team.

Launch Key Points

- Make sure the process for drawing and using Keeper Cards is clear.
- Make sure students understand dice must be applied to pawns one at a time.

Work

Students may have questions that come up during the course of play. You can consult the full rules of the game, or just have students respond by deciding on what seems like the best way to settle the question and keep play going.

Tips for the Classroom

1. Have students roll in the box lid to prevent them from knocking over pawns during the game.
2. Students can use the multiplication table or scratch paper to help themselves with hard multiplication or division problems. The board's color scheme can help too.
3. Students may dislike getting knocked back to start. However, they'll quickly learn that they can make fast progress if they get a good roll, especially when they roll doubles.
4. You can shorten a game with two pawns by making the objective to get only 1 pawn to "101" instead of both.

Prompts and Questions

- Where do you land if you add each number to the same piece? Where could you land if you added each number to separate pieces?
- Can you get either pawn to a red circle with that roll?
- Can you bump anyone with that roll?
- You rolled a 3 and a 5. What if you added the 3 to your pawn first, then multiplied by 5?
- If you subtract, you could land on a red circle and draw a Prime card.
- Can you divide down and then multiply up?
- What would happen if you used one of your keeper cards first, and then used the numbers you rolled?

Closer

Invite students to reflect on their experience, in pairs or trios first, and then as a whole class. Some questions to guide the discussion might include:

- What strategies did we learn playing this game?
- It can be discouraging being bounced back, but were there ways to catch up quickly? What kind of rolls helped you get near 101 quickly?
- How did you use the Keeper cards? Did you ever use one before you used the numbers you rolled?
- How did the colors help you with the math?

Choice Time

- Challenge Problems
- Prime Climb
- Odd Pig Out
- Penny Nickel Dime