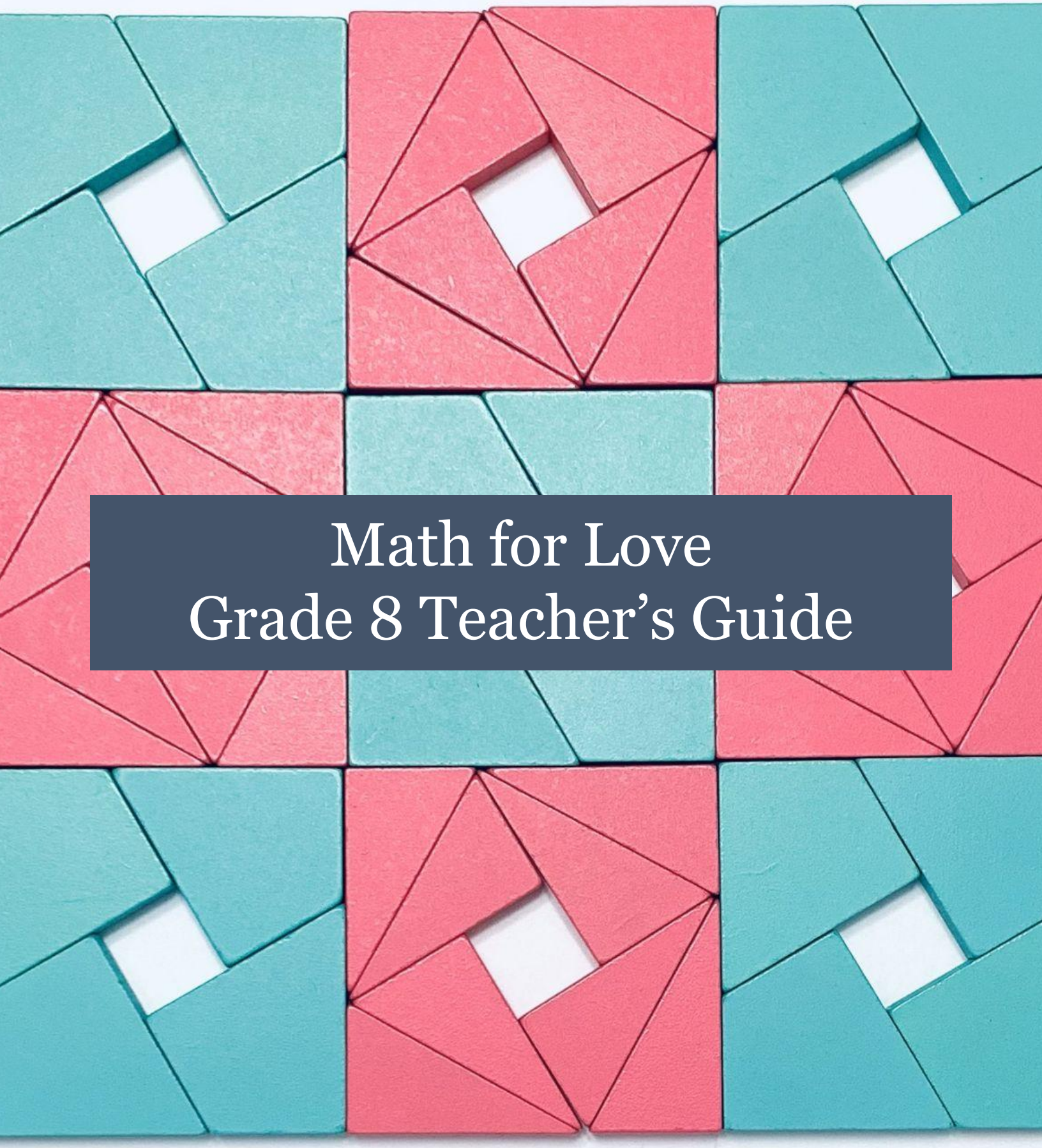


Math for Love
Grade 8 Teacher's Guide

Introduction

Welcome to the Math For Love Supplemental Curriculum! We are thrilled that you will be using this curriculum with your students. Like the lesson plans, we'll make this introduction quick, easy to read, and useful.

We are Math For Love, an organization dedicated to transforming how math is taught and learned. Our passion is connecting students and teachers with opportunities to experience excellent mathematics, deepening everyone's skill and enjoyment in the process.

The Math For Love Supplemental Curriculum is built on our belief that *play* and *rigor* go hand in hand, and that the best of mathematics is accessible to students and teachers who are ready to work hard and have fun. You and your students will learn a lot of math over the next 80 lessons, and by the end we hope you'll see why we think math is one of the best parts of the day.

The Big Picture

We built this curriculum with a few key principles in mind.

Principle 1.
Every student can participate in rigorous mathematical thinking.
Rigorous mathematical thinkers want to understand *why*, not just get the answer. They make connections and seek underlying structure and coherence. They develop powerful tools to solve problems, including fact fluency and procedural efficiency. Rigorous mathematical thinkers ask questions, make conjectures and predictions, test out their ideas relentlessly, and expect to be surprised.

Principle 2.
Play is the engine of learning.
Mathematicians engage in play constantly: exploring, wondering, noticing, and being led by curiosity. Play can transform math class from tedious to joyful, from shallow to deep, from mundane into fascinating. Students at play are more likely to persist, to build tenacity, to remember, and to learn. Play is the secret sauce that helps students come to love and succeed in mathematics.

Principle 3.
Without rigor, mathematical play is formless.
Without play, mathematical rigor is unsustainable.
We need both, together, to get the most out of mathematics.

In this introduction, we'll discuss some specific teacher moves that can help encourage rigorous mathematical play.

But first, some details.

The Details

The Math For Love Supplemental Curriculum is built to provide eighty days of 1-hour (or longer) classes, intended to complement a standard curriculum. It can be used for small groups, enrichment, remediation, after school programs, and summer programs. Every lesson is written to accommodate a wide range of student skill level, making it easy and enjoyable to differentiate and support each individual's learning. Our belief is that beautiful and interesting math problems — when designed to be appropriately accessible — should be offered to everyone, no matter where they are in their math journey.

Materials included with curriculum:

- Teacher Guide
- Student Workbooks
- Manipulative Kit
- Math Games

The Lessons

Each lesson follows a standard format with four sections:

- Opener (through Day 54)
- Main Activity
- Closer
- Choice Time

We sometimes provide a sketch of how a lesson might unfold, with prompts and questions to help you respond organically to what your students bring to the conversation. Any sample dialog is never meant to be a script, and precisely how the lesson goes will depend on you and your students.

We include guidelines for how long we expect each part of the day will take; however, times will vary depending on student engagement and your decisions.

When preparing for a lesson, review all sections of the lesson in advance. This will help you make decisions on how to group students, how to arrange materials, and what images to project. Even a little bit of preparation will help you be ready to emphasize what's important in the lesson and respond naturally to your students' ideas.

Choice Time Days

Occasionally a full lesson - after the Opener - is devoted to Choice Time. These Choice Time Days are intended to give students a chance to dig deeper into any lessons, or relax with some extra time to play the games they already know. As with normal Choice Time, you can use the suggestions we provide, or substitute in other options.

See the sample lesson templates on the next pages for more details about the lesson plans.

SAMPLE DAY Opener Main Activity Closer Choice Time

Overview

Focus Standards

This is where we highlight the main standards we're focusing on for the day, particularly in the main activity. We usually highlight one practice standard and one content standard.

Materials: This section will mention everything you need for the day. The main activity will also include a materials list.

Opener	**We'll say what the opener is here**	10 – 15 minutes
Main Activity	**We'll say what the main activity is here**	20 – 40 minutes
Closer	**We'll summarize what's happening in the Closer here**	5 – 10 minutes
Choice Time	• We'll provide a short list of good options for Choice Time here. • You're always welcome to choose different options!	5 – 25 minutes

Standards Connections

These are additional standards that are connected (or could connect) to today's lesson.

Opener

The lessons cycle through a short collection of our favorite opening routines. The first time you see a particular Opener, there will typically be more detail included. Later, these writeups will become shorter and more succinct. Don't be surprised to see the Opener instructions look almost identical on different days - once you're confident with a given opener, it should take very little time to prep for using it with class.

Here are the main Openers we use in this grade.

➔ Would You Rather

Give students a choice between two options. They debate which one is better, using math to convince each other.

➔ Counterexamples

Make a false claim or conjecture.
Invite students to find an example that proves you wrong.

➔ Broken Calculator

Project a calculator with some broken keys, and a target number.
Prompt: how many ways can you hit the target number using the broken calculator?

➔ Teacher-led Games

Bullseyes and Close Calls, Horseshoes, Penny Nickel Dime Quarter, Don't Break the Bank, and others.

Prompts and Questions

- Look here for useful things to say to students to help them get started or push deeper in their thinking.

Tips for the Classroom

1. Look here for some specific ideas for increasing student interaction, adjusting challenge, and more.
2. If there's an image to project for an opener, it's typically on the next page.
3. We stop using standalone Openers after Day 54 to allow more time for the Main Activities.

Main Activity

Materials and Prep

Here's where we describe what students will need for the main activity (doesn't include choice time materials). You'll need to read the lesson to make some decisions about how to arrange the materials for the day. In general, keep this simple – offer containers of manipulatives rather than exact amounts.

Motivating Question (OR How to Play)

To begin working or playing on their own, students should either have a question that frames the day's exploration— along with the knowledge and skill to begin thinking about it — or know the rules of the game they're about to play. We essentialize that question (or summarize those rules) here.

Launch

This is how to introduce the motivating question and get students excited and curious to think about it, or to teach the game in a way students will understand and find irresistible. In the case of games, demonstrating with a student volunteer is almost always the most powerful way to communicate how the game is played.

In general, the Launch should be as thorough as necessary *and* as short as possible. The goal should always be to have the students spending as much time as possible doing the thinking during math class. Whenever you are speaking to the whole class, pose questions and look for opportunities to ask for student ideas, questions, and contributions.

Work

As soon as they're ready, students go to work on their own or in pairs or small groups. This section will have some ideas of what to look for, the lesson flow, extensions, good hints, and (occasionally) solutions.

While students work, circulate in the room, offering help, prompts, hints, asking questions, making connections between ideas, and getting a sense of your students' strengths and where they could benefit from greater support.

Tips for the Classroom

1. Look here for additional ideas on how to implement this activity.
2. We'll often include extensions or simplifications to help with differentiation.
3. Student workbook pages will typically be included on the page right after the Tips for the Classroom.

Launch Key Points

- We try to include some key points for how to help the launch succeed in getting students excited to work.
- Points about the essential knowledge or skills might be here too.

Prompts and Questions

- This section gives ideas for what you might say to students during the "Work" section of the lesson, when they're working on their own or in small groups.
- Sometimes a prompt, hint, or nudge to talk to someone else is all students need.

Closer

Gather the students together for a whole-class discussion when the Main Activity is done. This is where students reflect, consolidate their learning, and potentially try an extension or variation of the Main Activity. To make sure the engaged thinking continues during this part of the day, rather than just summing up what everyone should have learned, take the opportunity to pose questions, invite student comments, and use partner sharing to give everyone a chance to participate.

Prompts and Questions

- These prompts are for the Closer.
- They might be useful things to say to the class as a whole.
- They also might be helpful replies to anticipated student contributions to a closing discussion.

Choice Time

Choice Time is when students get a chance to revisit games, puzzles, and other material they want to spend more time with. Getting to choose their activity helps with buy-in and self-regulation, and is a chance for students to reflect on what they want to think about more.

Choice Time works like this:

1. Present students with a short list of suggested activities.
2. Students choose the game, worksheet, challenge problem, block set, or other activity they'd like to pursue and commit to sticking with it for at least 5 - 10 minutes.
3. If time permits, students can try more than one activity.

The suggestions for Choice Time are only suggestions. If there is another activity from the curriculum that you think would be a better fit here, or if a student has a strong preference for something not on the suggested list, feel free to make a swap.

You may need to print some materials in advance to prepare for Choice Time. Since the final Choice options are up to you, we don't give a list of materials you'll need for them.

Here are some options that can be freely offered any Choice Time:

- Challenge Problems
- Free Block Play
- Multiplication by Heart (once students know how to play it)
- Prime Climb (once students know how to play it)
- Work on problems from an earlier lesson

Teacher Moves

Here are some useful ways to support your students during these lessons.

- **Model enthusiasm and curiosity.** Ask questions. Statements like “I wonder if…” and “I notice that…” go a long way. If students see you enjoying the work, they’ll be much more likely to enjoy it too.

- **Keep instructions and launches as brief as possible** (but as long as necessary) and look for places to invite student questions or ideas. As much and as often as possible, we want students to be spending classroom time doing mathematics and thinking mathematically.

- When launching games, **play a demo game with a volunteer** to help students learn the rules. When students play games against each other during work time, try these ways of grouping students:
    - Students play one against one and switch opponents often.
    - Students play in groups of three. Two play while one watches as a referee. When the game is over, the referee position rotates.
    - Students play two against two, and have to agree on moves with their teammate.
    - Students play collaboratively with a partner, and try to get the highest score they can, rather than beat an opponent.

- **Resist solving students’ problems for them.** While working on hard problems, it’s natural to feel stuck, or unsure of what to do next. Sometimes a key insight requires a lot of exploration first. Give students the time they need.

- **On the other hand, support students when they need it.** There’s no use in leaving students feeling dispirited or unsuccessful, and the goal is for students to be productive, even if stuck. We provide ideas for questions, prompts, and hints to keep students motivated and engaged. Even when students are playing or exploring, understand your job as looking for opportunities to help students develop greater efficiency, organization, and power in their methods.

- **Have a plan for how to respond to wrong ideas and answers**. One of the strongest ways to handle these moments is to turn them back to the students by treating the idea seriously and asking for counterexamples or supporting arguments. A very good phrase to keep in your back pocket is: “Convince me.”

- **Be willing to be the slowest person in the room**. This means asking for elaboration and clarification if you think there is even one student in the room who doesn’t understand an argument yet.

- **Care and respect**. Show students you care about them, respect their thoughts, and that it matters to you that they learn, and enjoy, mathematics.

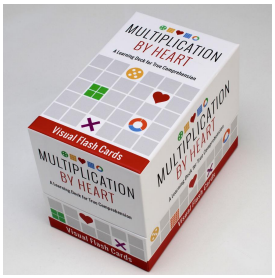
Materials

We provide just about everything you need to use this curriculum with a classroom of 25 (or more) students. The only extras you'll need are scratch paper, pencils, and crayons or colored pencils. You may occasionally need to make some additional photocopies for Choice Time, though students can often turn to earlier pages in their Student Workbook and find what they need. In addition to this Teacher's Guide and the student workbooks, manipulatives and games include:

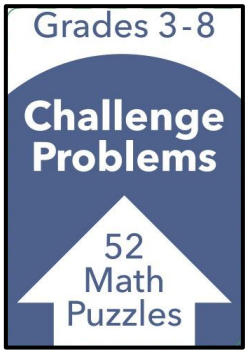
21st Century Pattern Blocks. These blocks include 8 shapes, with enormous possibilities for exploring multiplication, division, fractions, ratios, geometry, and more. These are also great for students to explore with during Choice Time.

Number Rods. Another excellent tool for understanding arithmetic operations, fractions, measurement, and more. Rods go from 1 cm to 10 cm long, in the colors named to the left.

Prime Climb. One of the world's most popular mathematical board games. Includes a unique visual for prime factorizations of numbers that acts as a guide for multiplication and division. Always a good Choice Time option once students learn how to play. Video instructions available at mathforlove.com/prime.



Multiplication by Heart. A visual flash card deck with three subdecks. Great in the classroom for small group fact practice and for simple fluency games. Ideas included in the lessons, and at mathforlove.com/multiply.



Challenge Problems Deck. These extra puzzles and problems are great options for Choice Time. These generally get harder the higher the number.

Also included: **Square Tiles**, **6-sided and 10-sided Dice**, and **Polyominoes** (which include dominoes, triominoes, and tetrominoes).

Other Stuff

- Email errata@mathforlove.com if you notice an error that should be fixed.
- **Additional Material:** We'll gather corrections and additional material at mathforlove.com/curriculum/grade8.
  Password: M4LCurriculum
- Problem with access? Email info@mathforlove.com.

Thanks and Acknowledgements

These lesson plans were built from the lessons we developed over our years working with teachers and students of all ages. However, putting together this more ambitious curriculum required a team, and we were lucky to have an amazing one.

Our curriculum writers were Karen Gallagher, Mark Goldstein, Tara Hofmann, Becky Holden, and Chase Orton. Our editors were Hana Murray and Jen Moffett. We had help with images from Bella Christianne and Hana Murray. Hana Murray also created the cover using photos of 21st Century Pattern Blocks.

This team of writers and editors worked with incredible focus and skill to build the teacher's edition you're holding now — big thanks to all of them for their dedication and contributions to this project.

Some images for this book were made, with permission, using Mathigon Polypad (polypad.amplify.com) - thanks to the good folks there for building such a fantastic tool. Mathigon also partnered with us to create digital versions of the Multiplication by Heart and Addition by Heart cards included in the curriculum kits. If you'd like to use the digital versions, they are free to use at fluency.amplify.com.

Finally, thanks to all the teachers, coaches, students, and staff who have used versions of our materials over the years, and welcomed us into their classrooms.

Daniel Finkel | Founder | Math for Love

Katherine Cook | Creative Director | Math for Love

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Overview

Focus Standards

MP2 Reason abstractly and quantitatively.

4.OA.2 Multiply to solve problems.

Materials: 6-sided dice, Odd Pig Out worksheets, paper, pencil

Opener	**Bullseyes and Close Calls**	10 – 15 minutes
Main Activity	**Odd Pig Out**	20 – 40 minutes
Closer	**Are You More Likely to Roll Even or Odd Products in Odd Pig Out?**	5 – 10 minutes
Choice Time	• Odd Pig Out • Bullseye and Close Calls • Pattern Block Free Play • Challenge Problems	5 – 25 minutes

Standards Connections

MP1 | MP5 | MP6 | MP7 | 7.SP.6

Bullseyes and Close Calls

Secretly choose a number with no repeated digits and write it down where no one can see it.

Students attempt to guess the number. After each guess, respond using the following options.

Outcome of Guess	Feedback
Correct digit in the wrong place	"Close Call"
Correct digit in the correct place	"Bullseye"
No correct digits	"Nothing"

Note: you might need to say "2 Close Calls" or "1 Bullseye and 2 Close Calls" some other combination. DON'T say "first digit Close Call, second digit Bullseye." Your responses apply to the entire numbers, not individual digits.

Play 1 - 3 games. Vary the difficulty depending on student comfort.

Mild: 3-digit numbers
Medium: 3-digit numbers with repeated digits allowed
Spicy: 4-digit numbers
Super spicy: 5-digit numbers!

Tips for the Classroom

1. Note that students DON'T get a Bullseye or Close Call for each digit. The clue applies to the entire number.

2. Write the guesses and the responses somewhere that everyone can see it.

3. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses. For example, after guessing 139 and finding that none of those digits are in the number, cross off the 1, 3, and 9 from the list of possible digits.

4. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

Prompts and Questions

- What numbers can I cross off after that guess?
- Is there anything you know after that guess? Any number that is or isn't in our mystery number?
- Why are you so sure the number doesn't have a 5?

Example Game
Your secret number is 487

Guess	Feedback
139	Nothing
820	Close Call
468	1 Close Call, 1 Bullseye
568	Close Call
482	2 Bullseyes

Odd Pig Out

Materials and Prep

10-sided and 6-sided dice, Odd Pig Out worksheets, pencil

How to Play

Players take turns rolling two 10-sided dice as many times as they like. After each roll, they multiply the numbers shown on the dice.

- If the product is even, they add that number to their current points for the turn.
- If the product is odd, players lose all their points *from that turn* and their turn is over.

A player may choose to end their turn at any time and "bank" their points. The first player with 500 points in their bank wins.

Launch

Choose a volunteer and play a demonstration game using the 10-sided dice. Start by quickly explaining the rules, then clarify as you play during the demonstration game. Play for at least 3 or 4 turns, until students seem to understand the rules.

As you play your turns, solicit advice from students. You can ask them to give a thumbs up if they think you should keep going and a thumbs down if they think you should bank your points.

Work

Students play Odd Pig Out. Try grouping students in pairs to play against each other, one on one, to start. Once they've played a few games, have students play in teams of two on two. Team play will force them to talk to their partner about whether they should roll more or stop.

While students play, walk around the room and check in on how students are thinking about their strategy. This will be a good opportunity for you to get a feel for how comfortable your students are with multiplication facts.

Tips for the classroom

1. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. Help them be good winners and losers.

2. If students need more support, they can play with two 6-sided dice instead of the 10-sided dice. A worksheet and table for this modification is included below. For a game with 6-sided dice, the winner is the first to claim 300 points.

3. Students can play to 1000 for a longer game.

Launch Key Points

- Keep the Launch brief so that students can start playing the game on their own as soon as possible.
- Make sure the process for banking points, or losing them, is clear.
- Take risks during your demonstration game, both to generate excitement and to show what happens when you roll an odd product.

Prompts and Questions

- How many points do you have for this turn, so far?
- Who is ahead?
- Are you sure that's the product for those two numbers?
- Do you find yourself using a strategy? For example, do you roll 3 times, and then bank? Or are you riskier than that?

Day 1

Odd Pig Out

How to Play

Players alternate turns rolling two dice and finding the product as many times as they would like.

a. If the product is **even**, they add that number to their current points. They may choose to roll again or end their turn.
b. If the product is **odd**, they lose all their points from that turn and their turn is over.

The player who reaches 300 points or more in their bank first is the winner!

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Day 1

Odd Pig Out

How to Play

Players alternate turns rolling two 10-sided dice and finding the product as many times as they would like.

a. If the product is **even**, they add that number to their current points. They may choose to roll again or end their turn.
b. If the product is **odd**, they lose all their points from that turn and their turn is over.

The player who reaches 500 points or more in their bank first is the winner!

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Day 1

Odd Pig Out

How to Play

Players alternate turns rolling two dice and finding the product as many times as they would like.

a. If the product is **even**, they add that number to their current points. They may choose to roll again or end their turn.
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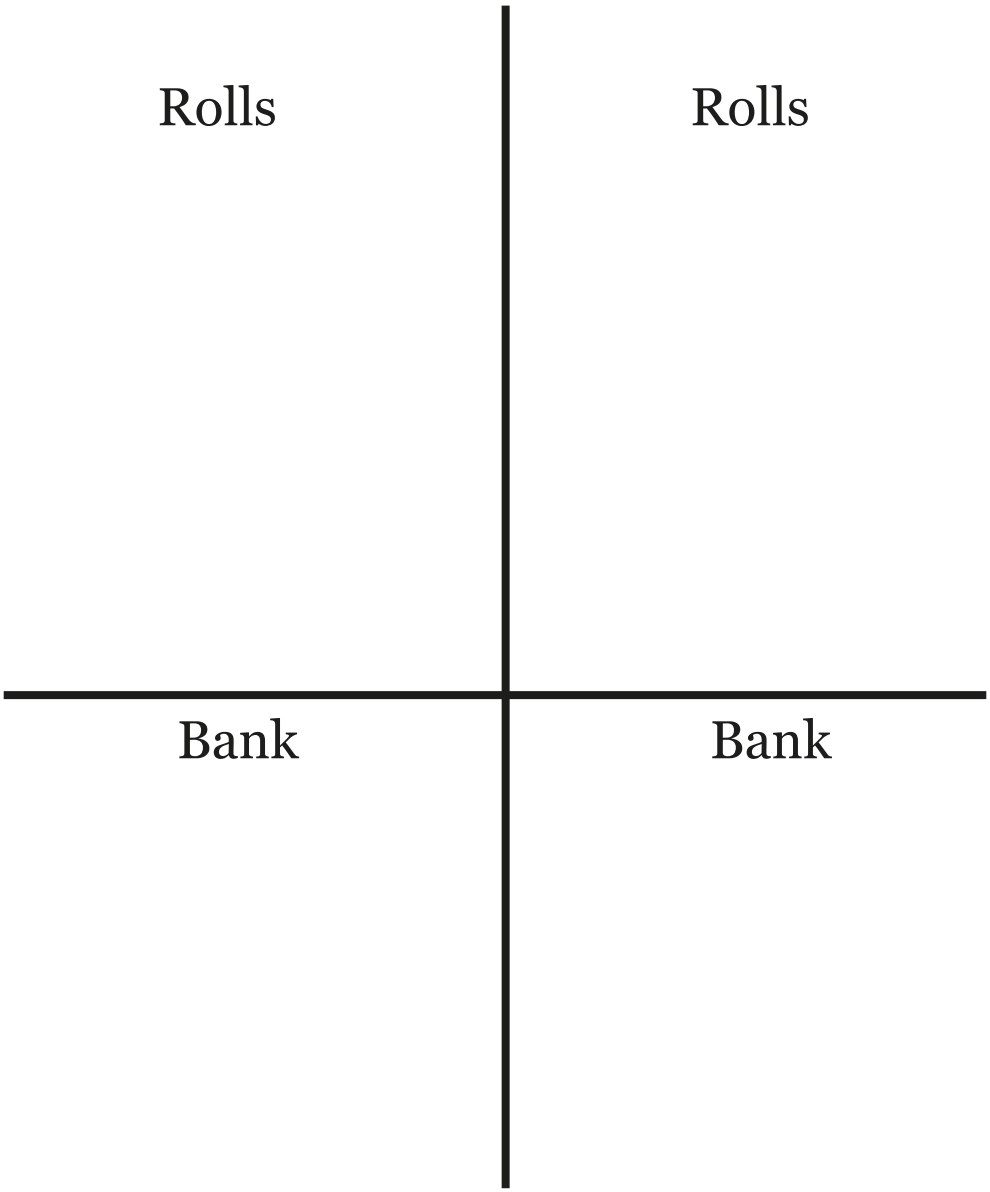
The player who reaches 300 points or more in their bank first is the winner!



Closer

Discuss Odd Pig Out. Ask students whether they developed strategies as they played. If so, did they find themselves playing a riskier or more conservative strategy?

Follow this up by ask students whether they think they are more likely to roll odd products or even products, or whether they think they are equally likely.

To help explore this, pose the following series of questions, and ask students if they can figure out how likely they are to roll even or odd numbers.

- How many odd numbers are there on the multiplication table (up to 6 by 6)?
- How many even numbers?
- How are they distributed?
- Do students see any patterns?

Depending on how the conversation goes, students may start to articulate a conjecture about the products of even and odd numbers.

Even times even = even
Even times odd = even
Odd times even = even
Odd times odd = odd

This gives an argument that the odds of rolling an even product are three times the odds of rolling an odd product.

Alternatively, students might circle the even or odd numbers on a multiplication table and find the same thing!

Prompts and Questions

- Do you prefer to be more risky or more cautious? Which seems like it works better in the long run?
- What's your favorite winning strategy so far?
- Do you have a rule for when you stop rolling and bank your points?

Choice Time

- Odd Pig Out
- Bullseyes and Close Calls
- Pattern Block Free Play
- Challenge Problems

Prepare students for Choice Time by explaining that they will choose from some pre-selected options. Once they choose an activity, they should spend at least 10 minutes on it before trying something else.

Show the Challenge Problem Deck and explain that it will always be among the Choice Time options. Students can simply take a challenge problem card and try to solve it, on their own or with a partner. Be sure to provide scratch paper and pencils.

Pattern Block Free Play is also an option, and helpful for students to explore the possibilities of the 21st Century blocks.

Overview

Focus Standards

MP6 Attend to precision.

6.NS.2 Compute fluently with multi-digit numbers and find common factors and multiples.

Materials: 6-sided dice, scratch paper, pencil

Opener	**Counterexamples**	10 – 15 minutes
Main Activity	**Damult Dice Division**	20 – 40 minutes
Closer	**Damult Dice Division Strategies**	5 – 10 minutes
Choice Time	• Challenge Problems • Odd Pig Out • More Damult Dice Division!	5 – 25 minutes

Standards Connections

MP3 | MP6 | MP7 | 5.NBT.6

Counterexamples

Counterexamples is a fun, quick way to highlight how to disprove conjectures by finding a counterexample. The leader (usually the teacher) makes a false statement that can be proven false with a counterexample. The group tries to think of a counterexample that proves it false.

The best statements usually have the form "All _____s are _____" or "No _____s are _____."

For the first day you play Counterexamples, get the students used to the game with the statement **"All birds can fly."**

Tell students their job is to come up with an example that proves your statement false. In this case, they are likely to suggest penguins as a counterexample. If they do, modify your statement: **"All birds can fly except penguins."** Counterexamples here could include baby birds, injured birds, ostriches, etc.

Next, use the following motivating examples and an "I notice" or "I wonder" statement. This allows students to see you model the process of making a conjecture.

Step 1. Share motivating examples
Write these down where everyone can see them.

- 19 ÷ 5 leaves 4 left over (i.e. 19 = 5 × 3 + 4)
- 19 ÷ 6 leaves 1 left over (i.e., 19 = 6 × 3 + 1)
- 26 ÷ 5 leaves 1 leftover
- 26 ÷ 6 leaves 2 leftover

Share the following observation: "I notice that when I divide the same number by 5 or 6, I don't get the same remainder."

Step 2. Pose a conjecture
State this as an 'obvious' conclusion from the motivating examples. You might say, "I bet it's true that *every* time I divide a number by 5 and 6, I get different remainders." Then write:

Conjecture: No number has the same remainder when you divide it by 5 and by 6.

Step 3. Invite students to try to find counterexamples - that is, examples that prove your conjecture false.
They may need some time. Every time a counterexample is offered, ask the class to consider it fully and see whether it satisfies the goal. Sometimes counterexamples can be deceiving!

Prompts and Questions

- I think I see a pattern here. I'm going to make a conjecture...
- You think my conjecture is wrong. But how can you *show* me it's wrong?

Counterexamples (continued)

Potential Student Counterexamples:
0, 31, 32, 33

These numbers have the same remainder when you divide by 5 or 6. These (and other) examples show that our conjecture is not actually true.

Step 4. At this point, extend the game by offering a revised conjecture that accounts for the counterexamples the students came up with.
Some examples of possible revised conjectures include the following:

Potential Revised Conjectures

- *No number has the same remainder when divided by 4, 5, and 6. (This is false.)*
- *The smallest number that has the same remainder when divided by any collection of numbers will be the product of them plus one (This is also false.)*

How you frame the revised conjecture will depend on what counterexamples the students found.

Once you have a revised conjecture, students can continue searching for new counterexamples. As long as students are engaged, feel free to continue cycling between refining the conjecture and seeking counterexamples, though in general one or two iterations will be sufficient.

In this case, we might have the following counterexamples and refinements:

Counterexamples to the first revised conjecture: 61, 62, 63

Revised Conjecture: no 3-digit numbers have the same remainder when divided by 4, 5, and 6.

Counterexamples: 121, 122, etc.

Damult Dice Division

Materials and Prep

6-sided dice, scratch paper, pencil; 10-sided dice (optional)

How to Play

Basic version (for 2-3 players):
On your turn, roll 3 dice. Choose 2 of the digits you rolled to make a 2-digit number, and divide it by the number on the remaining die. Your score is the quotient, rounded down to the nearest whole number. You get a +10 point bonus if the quotient is a whole number (i.e., if there's no remainder when you perform the division). The winner is the first person to reach 200 points.

Example Turn:
You roll 2, 5, 6. You have the following options for moves:
- 25 ÷ 6 = 4R1: 4 points
- 26 ÷ 5 = 5R1: 5 points
- 52 ÷ 6 = 8R4: 8 points
- 56÷2 = 28. Plus the 10 point bonus is 38 points!
- 62÷5 = 12R2: 12 points.
- 65÷2 = 32R1: 32 points.

All-Play (for 3 or more players): This version gets every student writing equations, and also gives everyone something to do on every turn.

Take turns as the roller. Roll three dice. Everyone writes down a division equation formed by creating a 2-digit number and dividing by the third, and the quotient (either in decimal, fraction, or remainder form). The roller gets to score their move as per the original game (scores are rounded down to the nearest whole number). After the roller has scored, all other players reveal their equations. If a player was the only one to write down a certain equation, they get to keep the score. If two or more players wrote down the same equation, they don't score any points. If a player wrote the same equation as the scorer, they don't score any points.

Example turn: Player 1 rolls 2, 5, 6. Everyone writes down an equation using those numbers.

Player 1 reveals their equation:
56 ÷ 2 = 28.
This comes out to a whole number for a 10 point bonus, so they get 38 points.

Next the other players reveal their equations.
- Player 2: 65 ÷ 2 = 32.5 (0 points)
- Player 3: 65 ÷ 2 = 32.5 (0 points)
- Player 4: 56 ÷ 2 = 28 (0 points)
- Player 5: 26 ÷ 5 = 5.2 (5 points)

Launch

Invite a student volunteer to demonstrate the game. Play a few turns each, making sure the scoring structure is clear. Find an arrangement on one of your turns that lands exactly on a whole number so that you can collect the 10 bonus points.

Once you've demonstrated how to play, students can play the game for themselves in pairs or small groups.

Work

The basic version is good for 2 - 3 player games. For more than that, use the all-play variation.

After students have had about ten minutes to play, bring the group together and introduce the All-Play variation, if you haven't already. Students can play All-Play in groups of 3 - 6.

Closer

Ask students whether they developed any strategies to help them win.

Break into six teams and play a full-class team game of Damult Dice All-Play. Students will need to convince their team of the best move every round, so there should be plenty of conversation. Play for as long as time allows.

Tips for the classroom

1. The All-Play variation is great for classroom play, since everyone engages on every roll. Jumping right into to this variation is a great idea. Demonstrate the game for at least 3-4 turns with the whole class.

2. If students crave additional challenge, try these variations:
   - Play with 10-sided dice instead of 6-sided dice.
   - Roll 4 dice. Your quotient will be a 3-digit number divided by a 1-digit number.

Choice Time

- Challenge Problems
- Odd Pig Out
- More Damult Dice Division!

Launch Key Points

- Damult Dice Division requires students to be able to divide. Make sure this is something students have at least some way of doing before you start. The opener for this lesson is a good way to check for this understanding and review the basics of division if necessary.
- Allow multiplication charts if students struggle with the division.
- Get students playing the game quickly. You only need to play a few turns, not a whole game.
- Make sure that over the course of your turns, you demonstrate every rule and way to score.

Prompts and Questions

- I see you wrote 26÷5. What would happen if you did 56÷2? Would that give you a greater score?
- Could you arrange the digits another way to get a greater quotient?
- Is there another way to arrange these digits to get a whole number?
- How did you do that division?

DAY 3	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP8 Look for and express regularity in repeated reasoning.

6.NS.4 Compute fluently with multi-digit numbers and find common factors and multiples.

Materials: 6-sided die, Prime Climb worksheets, scratch paper, pencil, and colored pencils

Opener	**Don't Break the Bank**	10 – 15 minutes
Main Activity	**Prime Climb Color Chart**	20 – 40 minutes
Closer	**What Patterns Can You Find in the Hundred Chart?**	5 – 10 minutes
Choice Time	• Challenge Problems • Odd Pig Out • Damult Dice Division • More Don't Break the Bank!	5 – 25 minutes

Standards Connections

MP1 | MP3 | MP7 | 4.OA.4

Don't Break the Bank

How to Play

Roll a 6-sided die. After each roll, everyone enters the number in one of the nine spots on the board. After nine rolls, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum without going over 999.

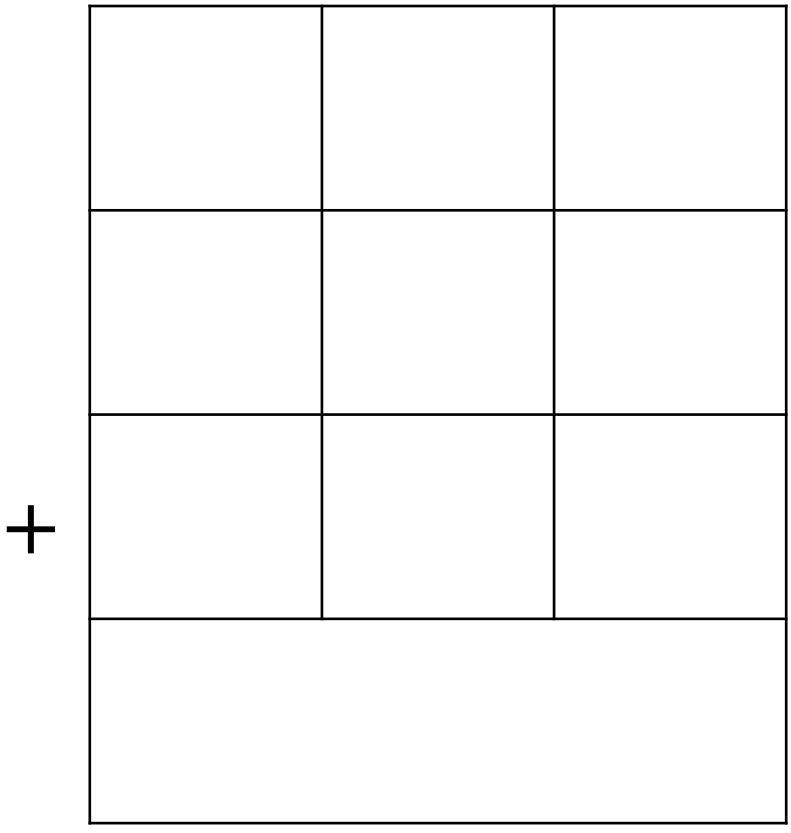
Everyone makes their own board (see image) on their paper. If this is the first time playing, briefly explain the rules. Start play as quickly as you can. Minor confusion over the rules can be worked out during play.

Play nine turns, slowly enough that everyone can make their decisions and keep up, but quickly enough that students can't work out exactly what their current total is, and have to estimate. Once a game is complete, share your score, and ask the class who busted who didn't by a show of hands. For those who didn't, ask how close they got to 999 without busting. It is often nice to have the player with the highest score explain where they put their digits to win - and for the class to check that they didn't make any arithmetic errors!

Play 1 – 3 games, as time allows.

Prompts and Questions

- Where should I put this 5... I think I'll put it in the tens column.
- Where are you going to put that 3? You don't have to place it where I did.
- (After playing) Is there a strategy you used?



Prime Climb Color Chart

Materials and Prep
Prime Climb Coloring Chart, colored pencils, pencil

Motivating Question
What patterns can you find in the hundred chart?

Launch
Show students the chart with the coloring to 20, and ask them what they notice, and what they wonder.

Give students some time in pairs to discuss or write down their ideas. Then discuss as a class. In particular, what are student conjectures to explain the way the numbers are colored?

Possible student observations might include:

- Every even number has orange in it.
- If there is green in the numbers you can reach it if you skip count by 3.
- The circles with blue are numbers end in 5 or 0.
- The circles with red are prime numbers.

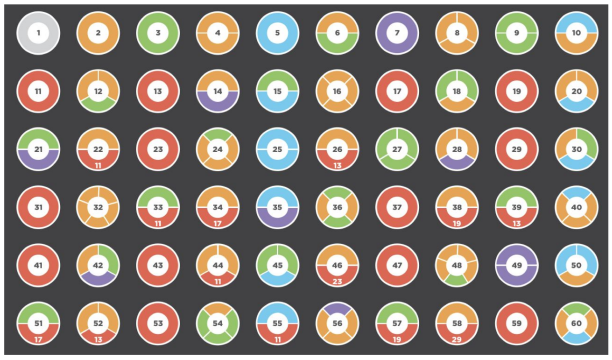
Wondering might include:

- Why does 4 have two orange parts?
- Why does 8 have three orange parts?
- If prime numbers are red, why isn't 7 red?

Work
Once students have discussed some ideas for how the coloring works, challenge them to color in the numbers 21 - 30 so that it extends the pattern.

Let students work in pairs. If they are stuck after five minutes or so, you can gather the class together and discuss how to color 21. If the patterns will still work, 21 should be colored green, because it can be reached if you skip count by 3s. It should also be purple, since you can reach it if you skip count by 7s. Another rationale for the coloring: 3 is green, 7 is purple, and $3 \times 7 = 21$, so 21 should be green and purple.

Once students have the hang of how the coloring works, let them work on their own again. They can color in as much of the chart as they can, but getting to 30 is a good initial goal. In general, multiplying and dividing or skip counting is the key to understanding how the coloring works.



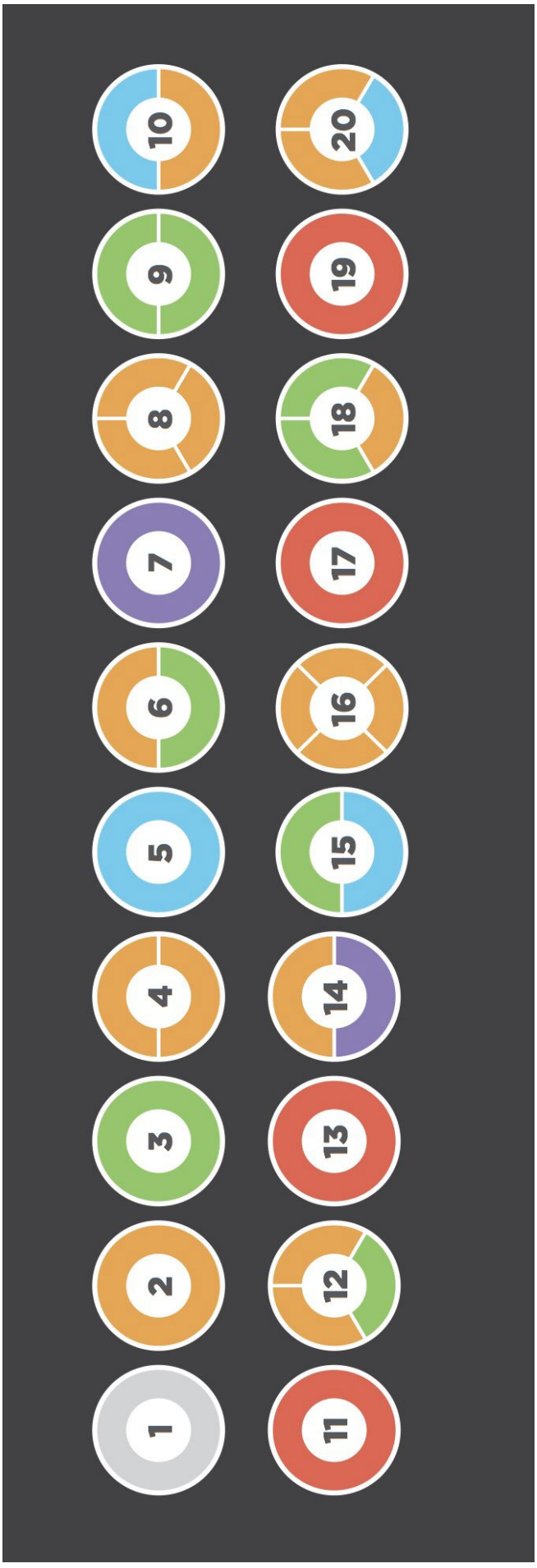
Launch Key Points

- Don't answer questions during the launch - focus on generating them. Plan to come back to unresolved questions and conjectures in the Closer.
- Students should have enough sense of the patterns at play in the numbers from 1 - 20 to work forward. Be ready to gather them again to discuss 21 and 22 if they don't make forward progress.

Prompts and Questions

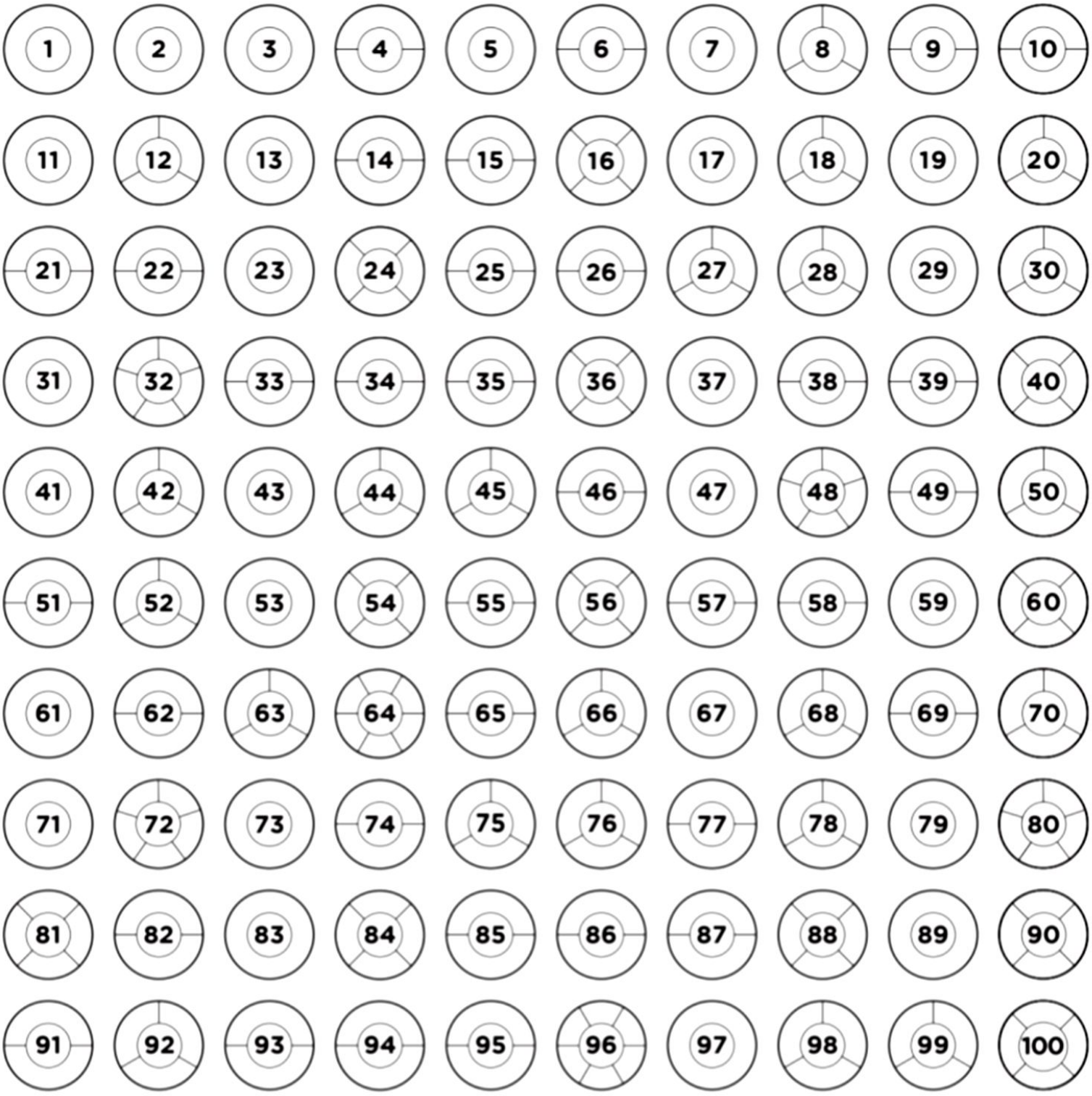
- Look at just one color at a time. What's happening with orange? What's happening with blue?
- What numbers have blue? (5, 10, 15, 20.) What do you think comes next in that pattern? So probably 25 and 30 will have a blue part colored in, right?
- Which numbers have two or more orange parts colored in (4, 8, 12, 16, 20)? What pattern do you see in those numbers?

Day 3



Day 3

Prime Climb Coloring Chart



Tips for Classroom

1. Don't expect students to finish the entire chart in one lesson. They can come back to it in the future. It is a good option for future Choice Times.

2. As much as possible, have students explain their reasoning, question the reasoning of others, and determine the truthfulness of their conjectures.

Closer

Pick a few numbers that everyone has at least thought about: 28, 29, 30, for example, or 22, 23, 24 if students haven't gotten that far. Let students defend their choices for coloring.

- Why should 23 be red?
- Why does 24 have three orange segments and one green segment?
- Based on what we know, what colors will 72 have?

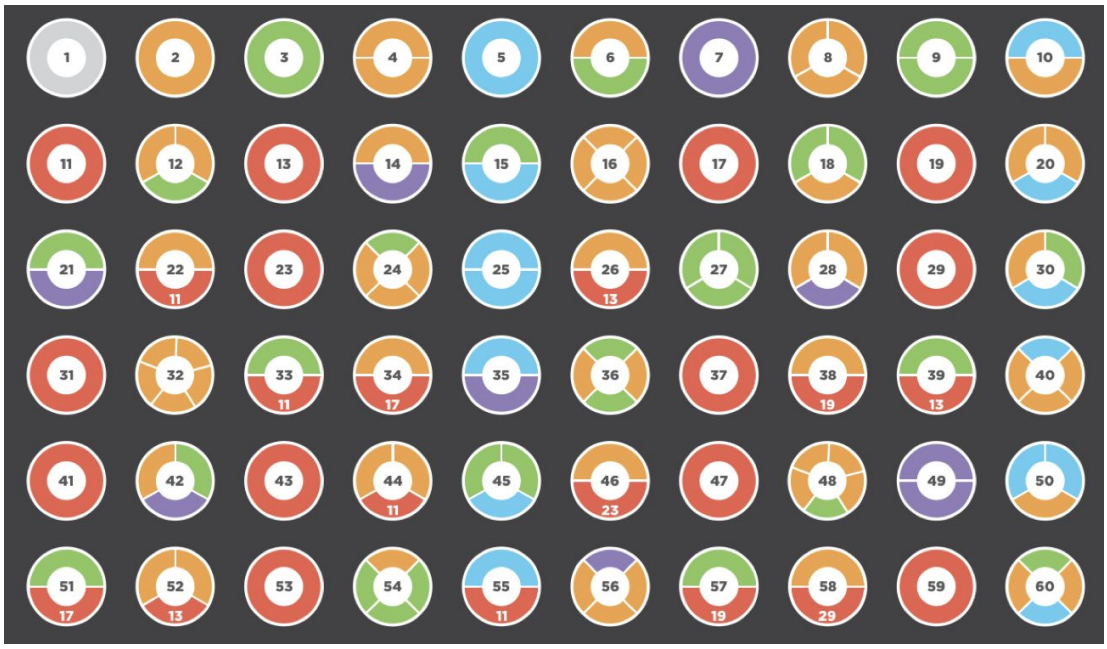
Use the chart colored up to 60 as a reference for yourself; it's best if students can argue why a given coloration works, and convince other students based on multiplication/skip counting arguments.

Prompts and Questions

- What patterns do we see in the Prime Climb Color Chart?
- How can you defend your coloring based on the patterns we know about?
- If I wrote out some multiplication problems like $2 \times 5 = 10$, $3 \times 5 = 15$, $4 \times 5 = 20$, what do you notice about the colors of those products?

Choice Time

- Challenge Problems
- Odd Pig Out
- Damult Dice Division
- Don't Break the Bank



DAY 4	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP7 Look for and make use of structure.

6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

Materials: Prime Climb, scratch paper, pencil

Opener	**Would You Rather**	10 – 15 minutes
Main Activity	**Prime Climb**	20 – 40 minutes
Closer	**Prime Climb Strategies**	5 – 10 minutes
Choice Time	• Challenge Problems • Prime Climb • Odd Pig Out • Damult Dice Division	5 – 25 minutes

Standards Connections

MP6 | MP8 | 5.OA.B

Would You Rather

Would You Rather questions offer a choice between two options. The job of the students is to decide which of the two options they would rather have, and convince their classmates of the wisdom of their choice.

The choice is usually real-world in nature, and may require students to make reasonable guesses to fill in missing information. As a result, Would You Rather provides a protocol that is quick and easy to use as an opener, and gets students to model with mathematics (Math Practice 4) and have mathematical conversations (Math Practice 3).

To begin, project the Would-You-Rather question on the next page, and encourage students to:

1. **Come up with their own answer**
2. **Convince their peers using mathematics**.

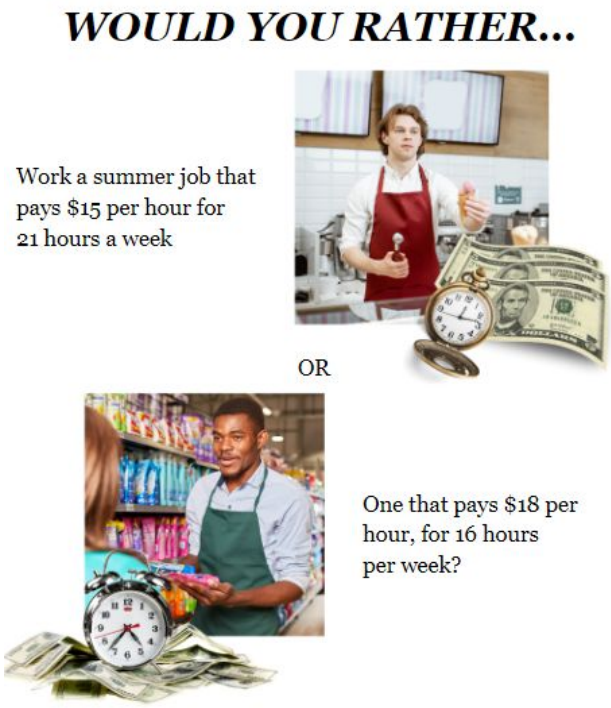
Groups of 3 are generally ideal for small group discussion.

Transition to a full class discussion after groups have talked for 3-5 minutes, and see if a consensus emerges. Students may have questions and requests for more information.

Consensus is not necessary! Students may disagree with the group for any reason. What's important is they defend their reasoning, ideally with mathematics.

Possible student observations and questions.

- $15 per hour for 21 hours is more money.
- $18 per hour for 16 hours is less work for the money.
- The first option is only $27 more for 5 more hours. I'd rather have my time than more money.



Prompts and Questions

- What additional piece(s) of information would help you make a decision?
- What types of jobs that would make either option more appealing for you?

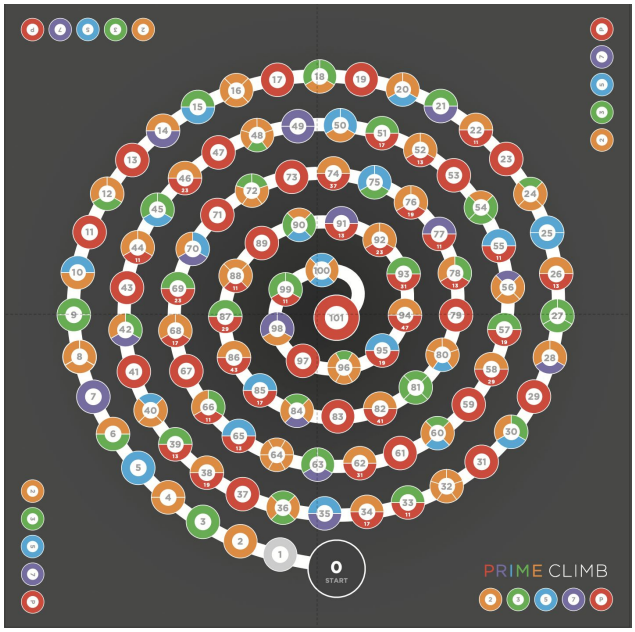
WOULD YOU RATHER...

Work a summer job that pays $15 per hour for 21 hours a week

OR

One that pays $18 per hour, for 16 hours per week?

Prime Climb



Materials and Prep

Prime Climb, scratch paper, pencil

How to Play

Video instructions available at:
mathforlove.com/games/prime-climb/how-to-play.

Here are the "Quick Start" version of the rules:

During a turn, there are four phases.

1. **Roll.** Roll the dice. You get two numbers from 1 to 10 to use for moving. In you roll doubles, you get that number four times instead of two.
2. **Move.** Move your pawn(s). Apply your dice rolls one at a time to the number your pawn(s) is on, using your choice of +, -, ×, or ÷. You can also use Keeper cards if you have them.
3. **Bump.** If you end your Move phase on the same space as another pawn, including your own, send it back to start.
4. **Draw.** If you end your Move Phase on an entirely red space (i.e., a prime greater than 10), draw a Prime card. If it is a Keeper card, save it for a future turn. Otherwise, apply the card now.

After someone lands both pawns exactly on 101, they win the game. (In the quick version of the game, you win after you get just one pawn to 101.)

Example Turn:

With pawns on 4 and 26, you roll a 3 and a 9. You could:

- Add 3 to 4 to move your pawn to 7, then multiply by 9 to move your pawn to 63.
- Multiply 26 by 3 to move your pawn to 78, then add 9 to move it to 87.
- Add 9 to 4 to move one pawn to 13, and multiply 26 by 3 to add the other to 78. Since 13 is completely red, you would them draw a Prime card.

You're never allowed to move to numbers off the board. You CANNOT add the 3 and 9 first and use a 12 to multiply or divide. You have to apply the numbers on the dice one by one.

Launch

Set up the game and explain the rules. Take a student volunteer and demonstrate 3 or 4 turns each, showing, in particular, how Prime cards get drawn when you land on red circles. Also emphasize that dice must be applied to pawn(s) one at a time–they cannot be added (or subtracted) together and then used to multiply (or divide). Once students seem ready, have them play in groups of 4 with two players per team.

Work

Students may have questions that come up during the course of play. You can consult the full rules of the game, or just have students respond by deciding on what seems like the best way to settle the question and keep play going.

Tips for the Classroom

1. Have students roll in the box lid to prevent them from knocking over pawns during the game.

2. Students can use the multiplication table or scratch paper to help themselves with hard multiplication problems. The board's color scheme can help too.

3. Students may dislike getting knocked back to start. However, they'll quickly learn that they can make fast progress if they get a good roll, especially when they roll doubles.

4. You can shorten a game with two pawns by making the objective to get only 1 pawn to "101" instead of both.

Closer

Invite students to reflect on their experience, in pairs trios first, and then as a whole class.

- What strategies did we learn playing this game?
- It can be discouraging being bounced back, but were there ways to catch up quickly? What kind of rolls helped you get near 101 quickly?
- How did you use the Keeper cards? Did you ever use one before you used the numbers you rolled?
- How did the colors help you with the math?

Choice Time

- Challenge Problems
- Prime Climb
- Odd Pig Out
- Damult Dice Division

Launch Key Points

- In the Prime Climb Color Chart lesson, students had an opportunity to explore the color chart used for the numbers for Prime Climb. If necessary, review some of the notice and wonders they made from that lesson.

- As students are having trouble learning the game, consider having them use only 1 pawn instead of both pawns.

Prompts and Questions

- Where do you land if you add each number to the same piece? Where could you land if you added each number to separate pieces?

- Can you get either pawn to a red circle with that roll?

- Can you bump anyone with that roll?

- You rolled a 3 and a 5. What if you added the 3 to your pawn first, then multiplied by 5?

- If you subtract, you could land on a red circle and draw a Prime card.

- Can you divide down and then multiply up?

- What would happen if you used one of your keeper cards first, and then used the numbers you rolled?

DAY 5	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP6 Attend to precision.

7.NS.1 Apply and extend previous understandings of operations with fractions.

Materials: Box the Numbers worksheets, scratch paper, pencil, and colored pencils

Opener	**Broken Calculator**	10 – 15 minutes
Main Activity	**Box the Numbers**	20 – 40 minutes
Closer	**How Can You Box the Biggest Score?**	5 – 10 minutes
Choice Time	• Damult Dice Division • Prime Climb • Odd Pig Out	5 – 25 minutes

Standards Connections

MP7 | MP8 | 6.NS.C

Broken Calculator

This creative exercise in arithmetic is surprisingly dynamic, with a simple constraint that provides interest and rigor.

Tell the students that you have a calculator with some broken buttons. The challenge for students is to **make a target number on the calculator in as many ways as they can**, despite not being able to use the broken keys.

Students work in pairs or trios to write down a list of solutions. Circulate and help students to talk to each other, extend their thinking, and help them get unstuck. If students are starting to slow down after 5 minutes or so, pause to highlight some interesting approach from a student, or let students share solutions they found notable.

If students seem to have exhausted their interest in the original question, try adding challenges.

Example Challenges (optional)

- Solve the problem starting with a 3-digit number.
- Use as many different (working) digits as you can. Can you use them all?

Tips for the Classroom

1. Once students are familiar, you can have the broken calculator image projected, and they can get right to work.

2. Avoid writing faulty "equations" that treat the equals sign as the "compute" button on a calculator, i.e.,

   7 + 8 = 15 + 9 = 24 is false and strange.

   Better is to write a single equation using parentheses if necessary, or rewrite what you've done so far on a new line.

   7 + 8 + 9 = 24 or 7 + 8 = 15 are clear.
   15 + 9 = 24

3. Have a representative from each student group come up at some point while they're working to write one or two of their favorite solutions on the board. This helps other students to get inspired, and also makes the transition to share solutions quicker.

Prompts and Questions

- Good idea, but you used the "1."
- Can you use that same approach to get another solution?
- How many different ways have you come up with so far?
- Do you think it's possible to solve using division?

Target: 24

Box the Numbers

Materials and Prep
Box the Numbers worksheets, pencil, and colored pencils

How to Play
Each player takes turns drawing a horizontal or vertical line connecting two dots. When a player completes a square, they add those points to their score **and they get to go again**. The game is over when all the dots are connected by horizontal or vertical lines. The person with the most points wins.

Launch
Demonstrate a game by choosing a volunteer and playing in front of the class.

Work
After the students have learned the rules, let them play several rounds using the different boards.

Tips for the Classroom
1. Play enough demonstration games with students so that the rules are clear.
2. Use different colored crayons or pencils for a clearer game.
3. Try different boards to re-engage students in the game. Or let them create their own.

Closer
Ask students what they noticed about the game.

If they were going to make their own board with their own numbers inside each box, what numbers would they pick to make the game as fun as possible?

Launch Key Points

- You don't need to play a whole game—just enough so that students understand the rules.

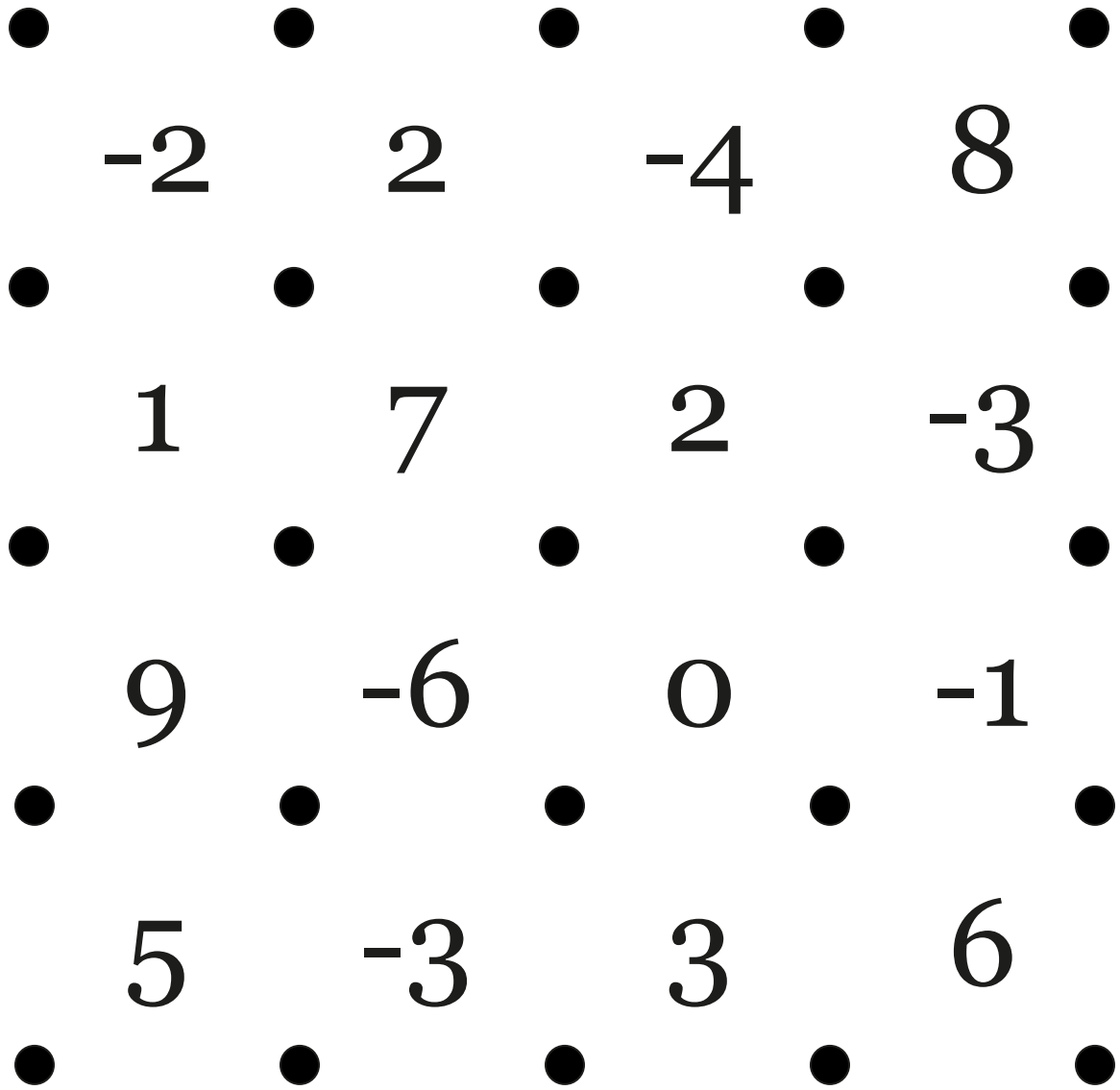
Prompts and Questions

- How did you calculate that?
- What mental math strategies could you use to find your score?
- Are your checking each other's work?
- What are some winning strategies you're figuring out?

Choice Time
- Damult Dice Division
- Prime Climb
- Odd Pig Out

Day 5

Box the Numbers 1



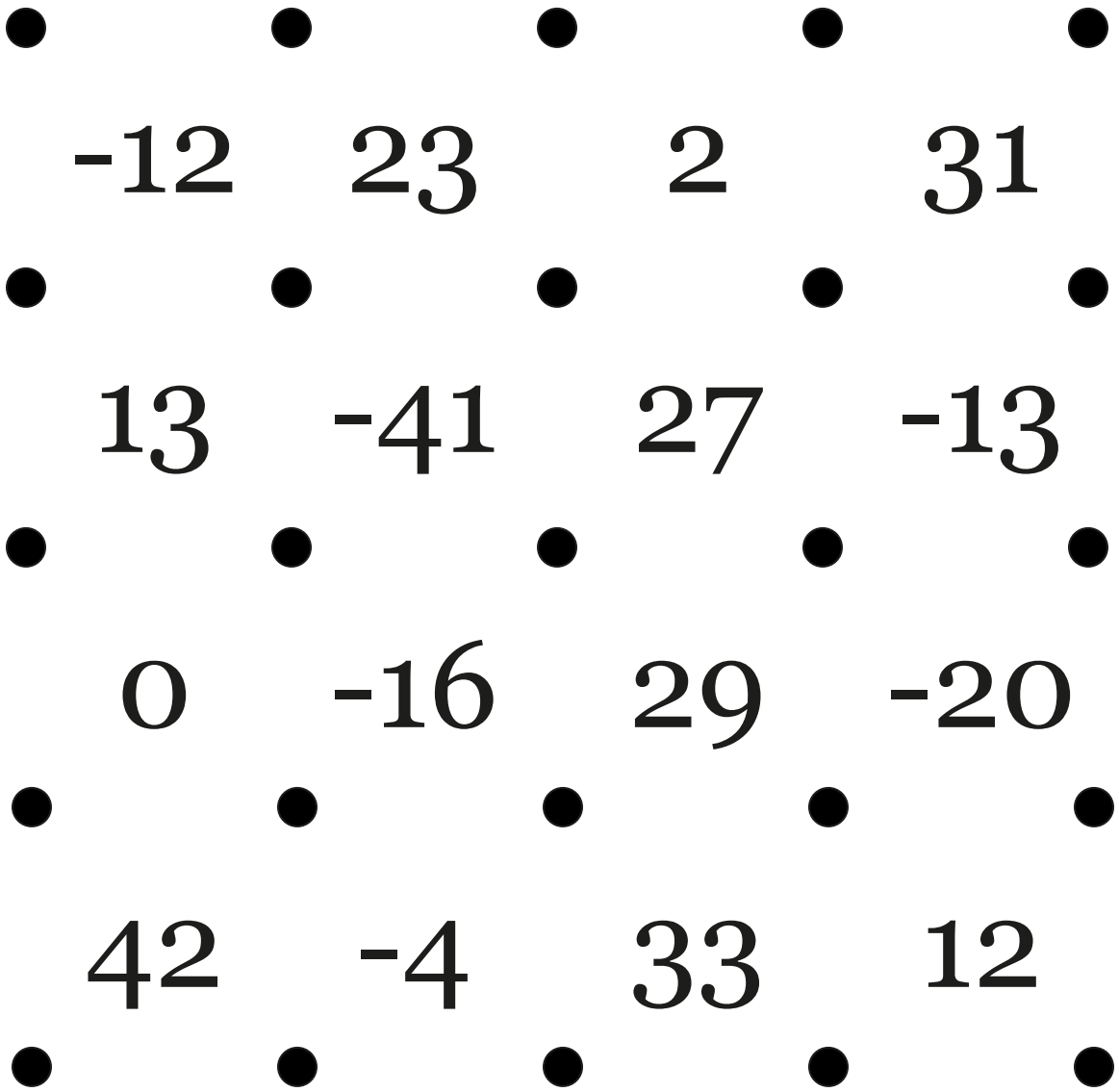
Box the Numbers is a 2-player game.

On your turn, add a vertical or horizontal edge between dots. If you complete a square, you claim the number inside it **and get to go again**.

Your score is the sum of all the numbers you box. Whoever has the highest score at the end wins

Day 5

Box the Numbers 2



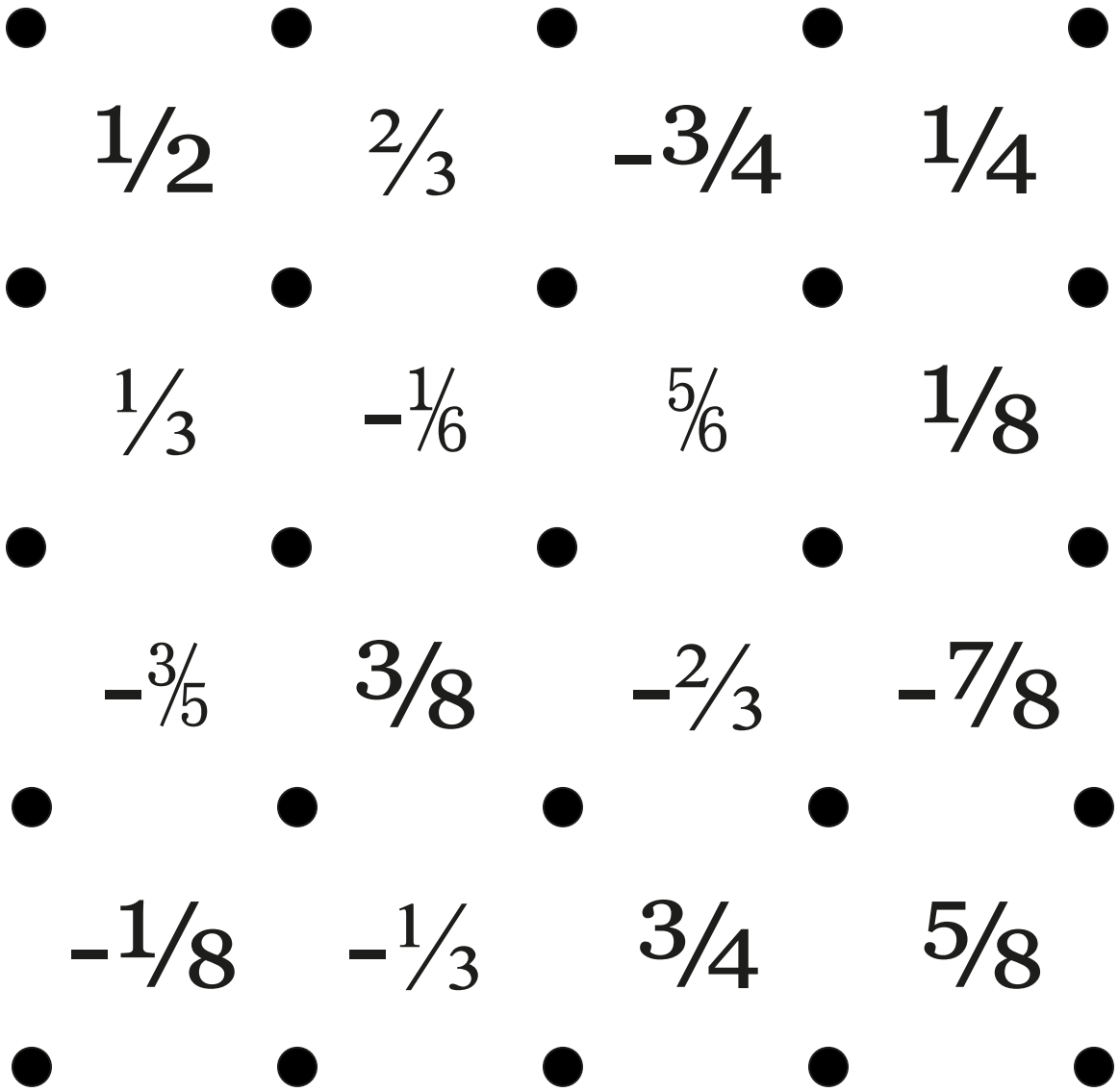
Box the Numbers is a 2-player game.

On your turn, add a vertical or horizontal edge between dots. If you complete a square, you claim the number inside it **and get to go again**.

Your score is the sum of all the numbers you box. Whoever has the highest score at the end wins

Day 5

Box the Numbers 3



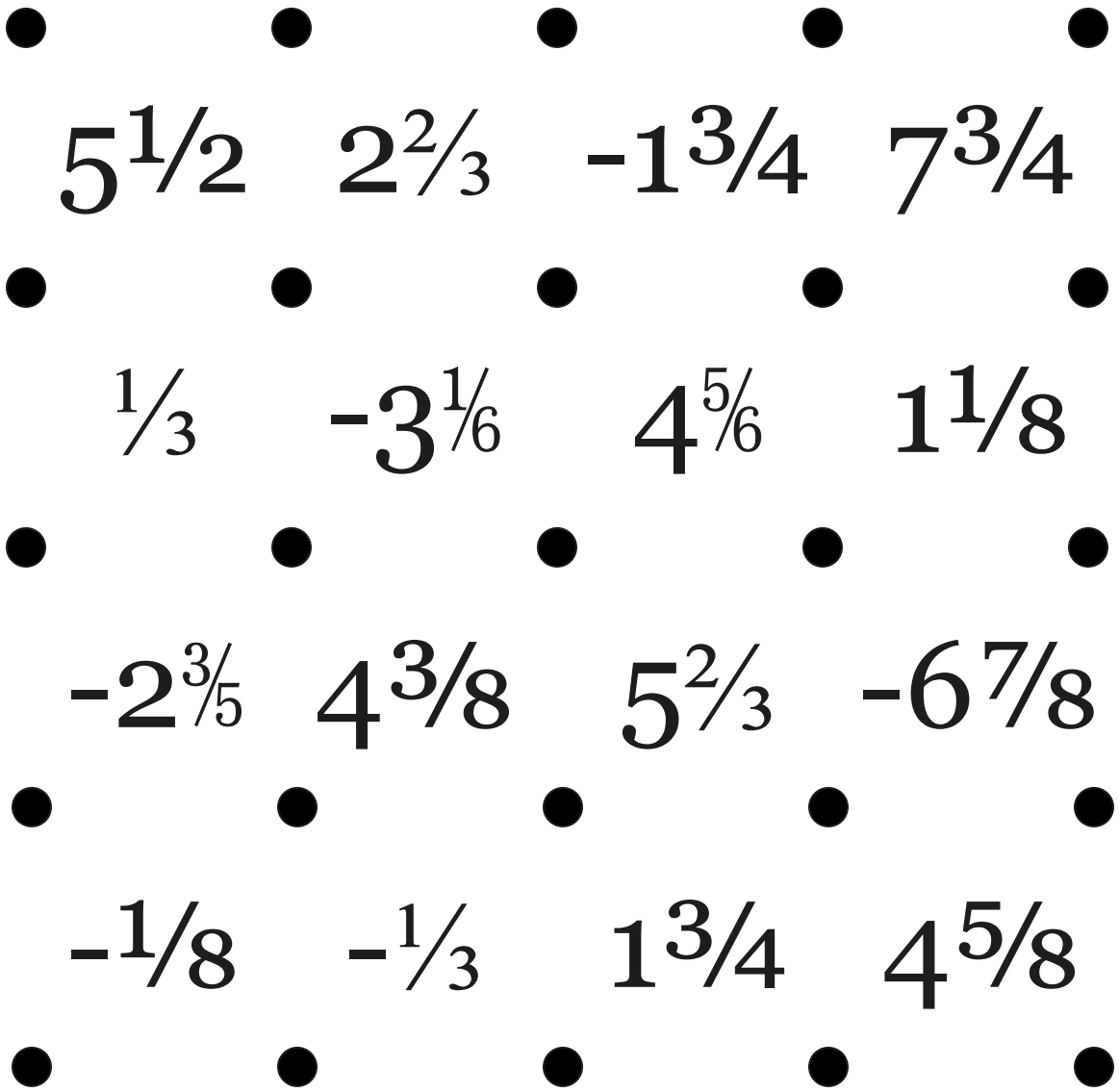
Box the Numbers is a 2-player game.

On your turn, add a vertical or horizontal edge between dots. If you complete a square, you claim the number inside it **and get to go again**.

Your score is the sum of all the numbers you box. Whoever has the highest score at the end wins

Day 5

Box the Numbers 4



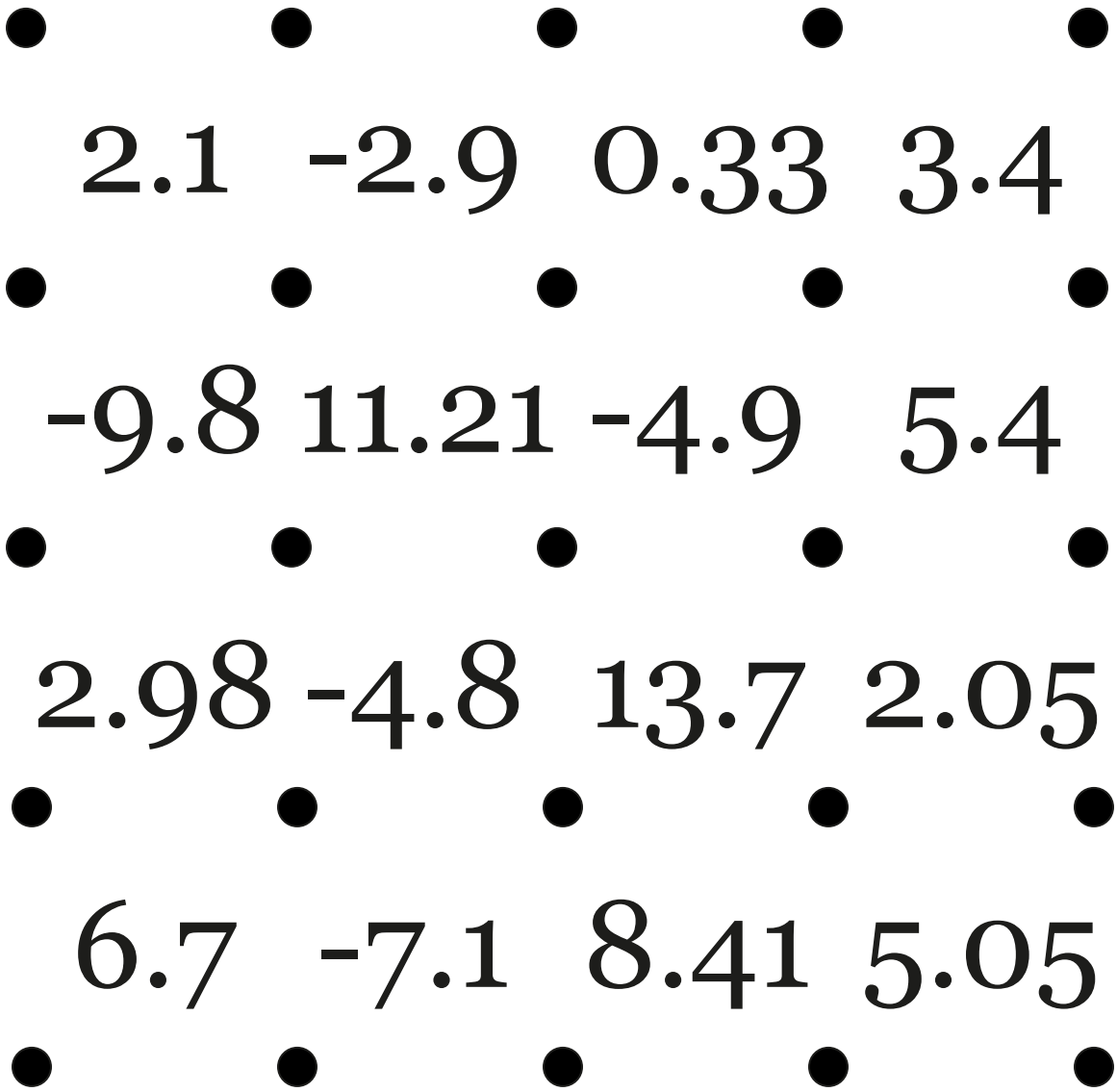
Box the Numbers is a 2-player game.

On your turn, add a vertical or horizontal edge between dots. If you complete a square, you claim the number inside it **and get to go again**.

Your score is the sum of all the numbers you box. Whoever has the highest score at the end wins

Day 5

Box the Numbers 5



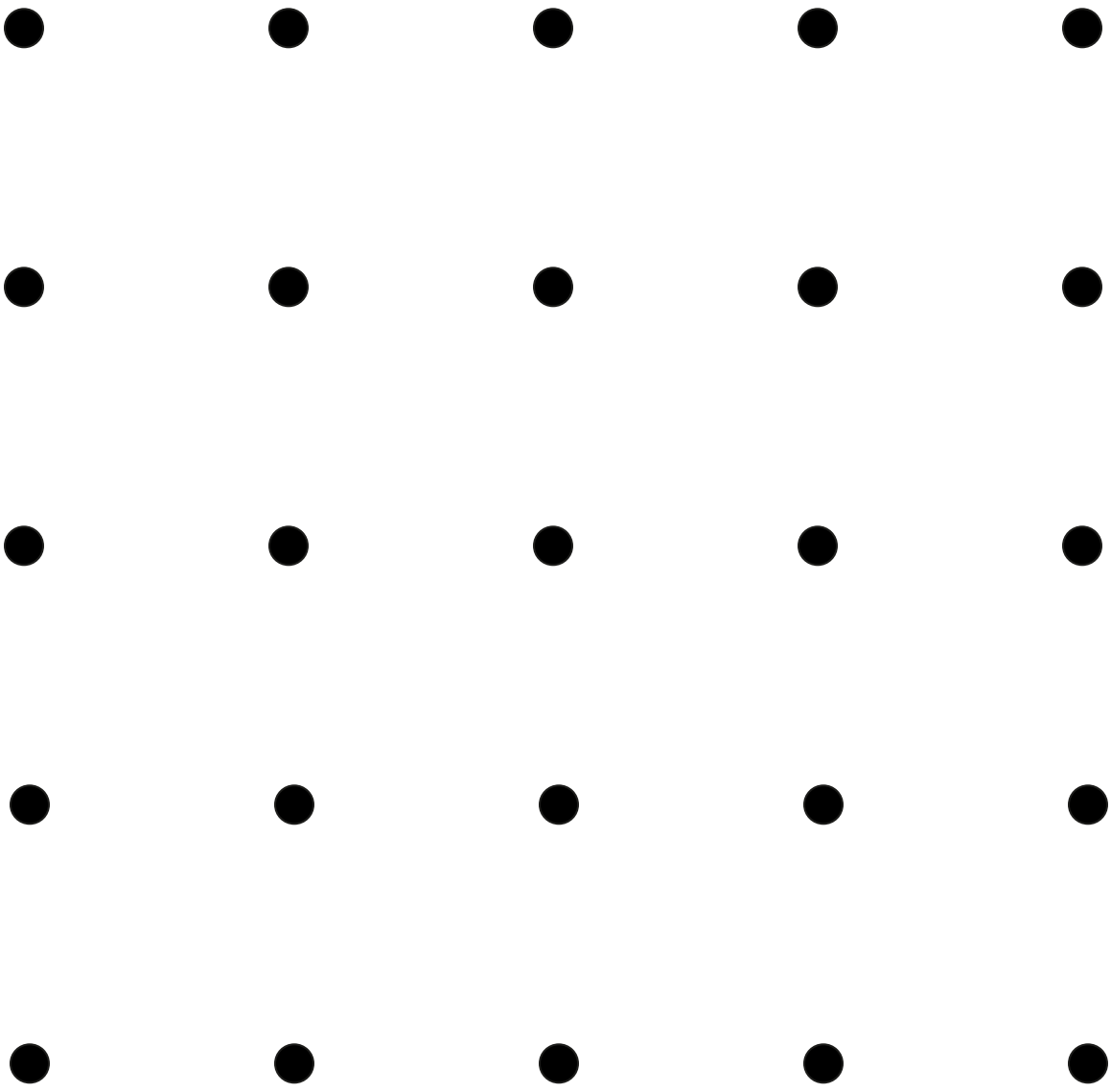
Box the Numbers is a 2-player game.

On your turn, add a vertical or horizontal edge between dots. If you complete a square, you claim the number inside it **and get to go again**.

Your score is the sum of all the numbers you box. Whoever has the highest score at the end wins

Day 5

Box the Numbers
Create Your Own!



Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers.

Materials: 6-sided dice, Magic Squares Challenge worksheets, scratch paper, pencil

Opener	**Don't Break the Bank - Tenths**	10 – 15 minutes
Main Activity	**Magic Squares**	20 – 40 minutes
Closer	**Magic Squares Strategies**	5 – 10 minutes
Choice Time	• Magic Squares • Prime Climb • Damult Dice Division • Odd Pig Out	5 – 25 minutes

Standards Connections

MP7 | MP8 | | 5.NBT.7

Don't Break the Bank - Tenths

How to Play

Roll a 6-sided die. After each roll, everyone enters the number in one of the nine spots on the board.
After nine rolls, the board becomes an addition problem with three 3-digit numbers to add together. The goal is to get the highest sum without going over 99.9.

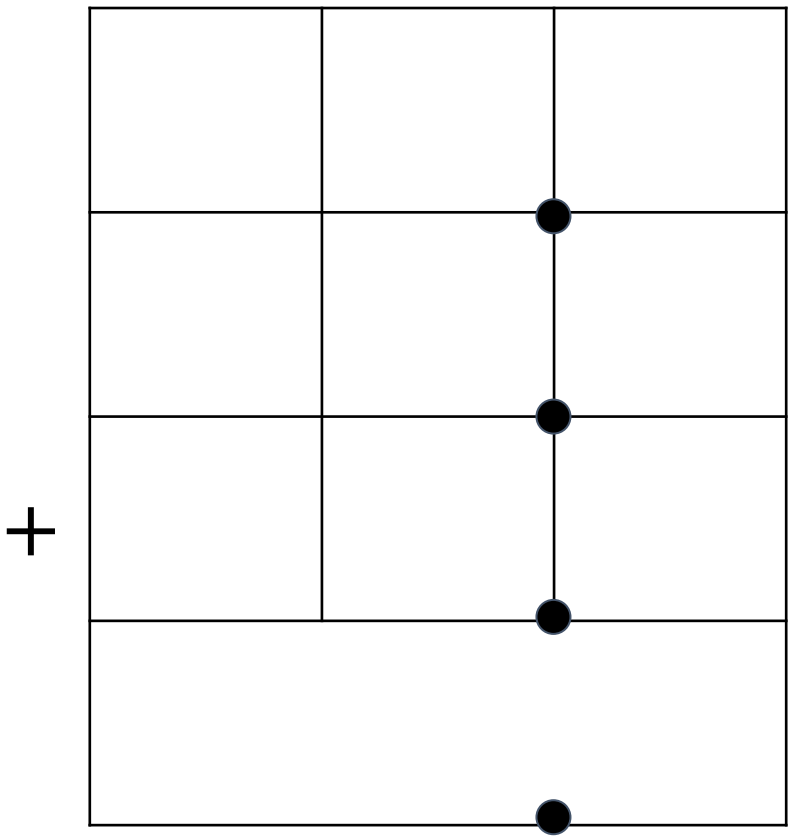
Everyone makes their own board (see image) on their paper. Briefly explains the rules. Start play as quickly as you can. Minor confusion over the rules can be worked out during play.

Play nine turns, slowly enough that everyone can make their decisions and keep up, but quickly enough that students can't work out exactly what their current total is, and have to estimate. Once a game is complete, share your score, ask who busted, and how close students got to 99.99 who didn't bust. It is often nice to have the player with the highest score explain where they put their digits to win - and for the class to check that they didn't make any arithmetic errors!

Play 1 – 3 games, as time allows.

Prompts and Questions

- Where should I put this 5... I think I'll put it in the tens column.
- Where are you going to put that 4? You don't have to place it where I did.
- (After playing) Is there a strategy you used?



Magic Squares

Materials and Prep
Magic Squares Challenges worksheets, pencil

Motivating Question
Can you figure out the Magic Square Challenges?

Launch
Display the figure to the right and ask: what's the sum along each row? (9, 19, and 17). Each column? (15, 10, and 22). Each diagonal? (15, and 13) Since they're ***not all*** the same, this ***not*** a magic square. Can you rearrange the nine numbers to make a magic square where all three rows, all three columns, and both diagonals have the same sum?

Give students a few minutes to do this and share solutions. There is more than one way to build the magic square!

2	1	6
8	4	7
3	5	9

Work
Give students the Magic Square Challenge 1 worksheet and some individual work time to try to figure out the magic squares. Then get them in pairs or trios to share and defend their solutions.

Tips for the Classroom
1. Students can work on either Magic Square Challenges 1 or 2.

2. For early finishers, ask pairs or trios to make up magic square challenges for each other to solve. Milder ones can involve whole numbers only, spicier ones can have decimals, fractions, or negative numbers.

Launch Key Points

- The first example is incorrect to motivate students to try to get a correct one.
- The process should be straightforward enough so that students can get to work quickly on the worksheets.

Prompts and Questions

- How do you know that works?
- Are there other number combinations that work for that one? How many do you think you could find during this class period?
- Do you have a particular strategy?

Closer
Bring the class together to do a quick wrap-up.

Did anyone get any interesting answers to share? Are there any interesting conclusions or conjectures to explore? Did you notice in the four puzzles on the worksheet any relationship between the center number and the target value?

Choice Time
- Magic Squares
- Prime Climb
- Damult Dice Division
- Odd Pig Out

Day 6

Magic Square Challenges 1

Challenge 1
Finish the magic square so that the target is 0.

-3		1
	-2	

Challenge 2
Finish the magic square so that the target is -15.

-4	-9	
-8		

Challenge 3
Place each of the following nine numbers to try to create a magic square. Try at least twice to get as many of the three rows, three columns, and two diagonals that have the same sum as you can.
It is possible to get all eight sums the same!

3 2 1 -5 -6
-7 -13 -14 -15

Day 6

Magic Square Challenges 2

Challenge 4
Finish the magic square so that the target is -12. Hint: all of the numbers are integers, and six of them are negative .

-4		
	-5	

Challenge 5
Finish the magic square so that the target is 1 ½. Hint: four of the numbers are negative.

1 ½		3 ½

Challenge 6
Place each of the following nine numbers to try to create a magic square. Try at least twice to get as many of the three rows, three columns, and two diagonals that have the same sum as you can.
It is possible to get all eight sums the same!

6 ¼ 5 ¼ 4 ¼ 1 ¼ ¼
-¾ -3 ¾ -4 ¾ -5 ¾

DAY 7	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP1 Make sense of problems and persevere in solving them.

Materials: Variable

Opener	**Bullseyes and Close Calls**	10 – 15 minutes
Main Activity	**Choice Time Day**	45 - 50 minutes

Standards Connections

Variable

Bullseyes and Close Calls

Secretly choose a number with no repeated digits and write it down where no one can see it.

Students attempt to guess the number. After each guess, respond using the following options.

Outcome of Guess	Feedback
Correct digit in the wrong place	"Close Call"
Correct digit in the correct place	"Bullseye"
No correct digits	"Nothing"

Note: you might need to say "2 Close Calls" or "1 Bullseye and 2 Close Calls" some other combination. DON'T say "first digit Close Call, second digit Bullseye." Your responses apply to the entire numbers, not individual digits.

Play 1 - 3 games. Vary the difficulty depending on student comfort.

Mild: 3-digit numbers
Medium: 3-digit numbers with repeated digits allowed
Spicy: 4-digit numbers
Super spicy: 5-digit numbers!

Tips for the Classroom

1. Note that students DON'T get a Bullseye or Close Call for each digit. The clue applies to the entire number.

2. Write the guesses and the responses somewhere that everyone can see it.

3. Keep track of digits. The skill in the game is about using the feedback from the guesses to make educated future guesses. For example, after guessing 139 and finding that none of those digits are in the number, cross off the 1, 3, and 9 from the list of possible digits.

4. Pause the game occasionally to ask students what they know for sure. Are there any digits that they are sure are not in the number? Any digits they know are in the number? How do they know?

Prompts and Questions

- What numbers can I cross off after that guess?
- Is there anything you know after that guess? Any number that is or isn't in our mystery number?
- Why are you so sure the number doesn't have a 5?

Example Game
Your secret number is 487

Guess	Feedback
139	Nothing
820	Close Call
468	1 Close Call, 1 Bullseye
568	Close Call
482	2 Bullseyes

Choice Time Day

For Choice Time Day, students have more time than usual to:

- Choose to play any game introduced so far.
- Return to activities they have unanswered questions about.
- Tackle challenge problems, including a video riddle!
- Explore the 21st Century Blocks and other manipulatives

Show the video riddle to everyone at the start of class.
Make sure to pause halfway through! Otherwise the answer will be given away.

> **Link**: mathforlove.com/puzzles
> **Title**: The Cuddly Wuddly Fuddly Duddly Riddle

Let students know the options available to them, and have them commit to one activity for at least the first ten minutes. Students can spend the day trying different options.

If any students are aimless or have trouble committing, you can assign them an activity. Otherwise, encourage students to think about what would be especially fun or interesting for them to try.

This first Choice Time Day is also a good time to share the Multiplication by Heart cards with any students who could benefit from extra fact practice. We include one simple option for a game you can teach quickly to a small group of students, or to the whole class: **The Product of War**. See the next page for how to play. For students who need it, this is always a good option for any future Choice Time or Choice Time Day.

Recommended Choice Time Options

- Odd Pig Out
- Box the Numbers
- Prime Climb
- Damult Dice Division
- Magic Squares with Negatives
- Challenge Problems, including video riddle
- The Product of War

In the last few minutes of class, show the last half of the video riddle, where the solution is given.

Launch Key Points

- Demonstrate any games or activities students may need to be reminded of.
- Make sure the materials students need to play the game or participate in the activity are readily available.

The Product of War

Materials and Prep

Multiplication by Heart cards: any deck or all the decks together.

How to Play

This is a multiplication version of the classic card game War. Divide one or more Multiplication by Heart subdecks evenly among the players.

Everyone plays a card at the same time (without looking at the product), then players go around in the circle and say the product they played. Whoever plays the highest product wins all the cards, provided they named their product correctly.

If a player names their product incorrectly and another player catches the error, they get to steal that players card and add it to their own for a higher play! In case of a tie, the players who tied turn up their next cards as a tiebreaker. Whoever wins that war gets all the cards from the round. Whoever gets the most cards at the end wins.

Example 1
Player 1 plays 3×4 and says "12."
Player 2 plays 5×10 and says "50."
Player 2 wins both cards.

Example 2
Player 1 plays 5×7 and says "35".
Player 2 plays 7×7 and says "42."
Player 3 plays 10×8 and says "80."

Player 1 catches the error and correctly names Player 2's product as "49." Player 1 then steals the 7×7 card and has a total of $35 + 49 = 84$. This beats Player 3's 80, so Player 1 wins all three cards.

DAY 8	Opener	Main Activity	Closer	Choice Time

Overview

Focus Standards

MP7 Look for and make use of structure.

6.EE.A Use properties of operations to generate equivalent expressions.

Materials: Number Rods, scratch paper, pencil

Opener	**Broken Calculator**	10 – 15 minutes
Main Activity	**Number Rod Proofs**	20 – 40 minutes
Closer	**Proof Discussion**	5 – 10 minutes
Choice Time	• Challenge Problems • Prime Climb • Don't Break the Bank	5 – 25 minutes

Standards Connections

MP2 | MP6 | 6.EE.A

Broken Calculator

This creative exercise in arithmetic is surprisingly dynamic, with a simple constraint that provides interest and rigor.

Tell the students that you have a calculator with some broken buttons. The challenge for students is to **make a target number on the calculator in as many ways as they can**, despite not being able to use the broken keys.

Students work in pairs or trios to write down a list of solutions. Circulate and help students to talk to each other, extend their thinking, and help them get unstuck. If students are starting to slow down after 5 minutes or so, pause to highlight some interesting approach from a student, or let students share solutions they found notable.

If students seem to have exhausted their interest in the original question, try adding challenges.

Example Challenges (optional)

- Solve the problem using the division key.
- Solve the problem starting with a 3-digit number.

Tips for the Classroom

1. Avoid writing faulty "equations" that treat the equals sign as the "compute" button on a calculator, i.e.,
   $3 \times 4 = 12 - 2 = 10$ is false and strange.

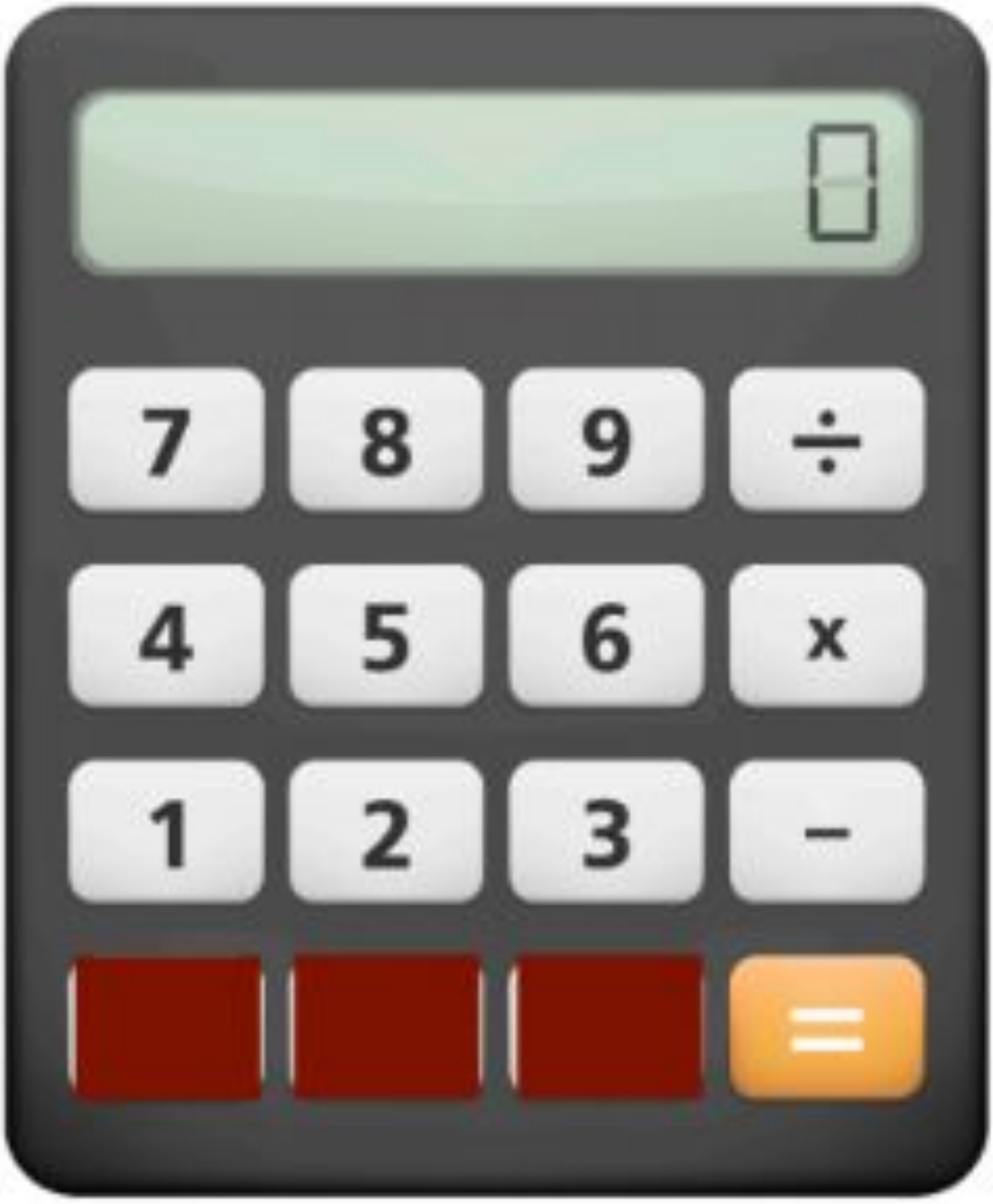
   Better is to write a single equation using parentheses if necessary, or rewrite what you've done so far on a new line.
   $(3 \times 4) - 2 = 10$ or $3 \times 4 = 12$ / $12 - 2 = 10$ are clear.

2. Have a representative from each student group come up at some point while they're working to write one or two of their favorite solutions on the board. This helps other students to get inspired, and also makes the transition to share solutions quicker.

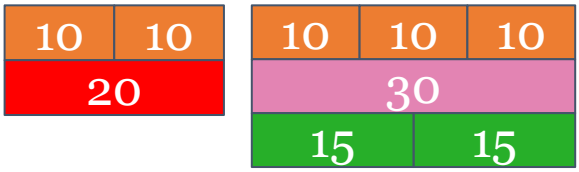
Prompts and Questions

- Good idea, but you used the "0."
- Can you use that same approach to get another solution?
- How many different ways have you come up with so far?
- Do you think it's possible to solve using division?

Target: 10



Number Rod Proofs

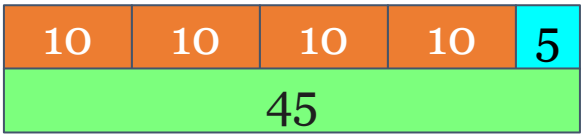


Materials and Prep

Number Rods, scratch paper, pencil

Motivating Question

If you know the value of one Number Rod, how can you find the values of all the others?



Launch

Give about 5 minutes for students to play and explore the rods. Ask for a few student comments about what they notice about them. Then transition to the lesson.

Problem 1. Suppose the orange (2 cm) rod has a value of 10. What are the other Number Rods worth?

Let students offer arguments, and put forward your own if none are forthcoming. For example, the red (4 cm) rod equals 2 orange (2 cm) rods, or 2 tens. That means red equals 20. See images for arguments about the pink and dark green rods. Once students have found what all the rods are worth, ask them to find the value of the light green (9 cm) rod, and prove their answer in more than one way.

Now tell students to let go of all the old values and suppose something totally different:

Problem 2. The red (4 cm) rod equals 32.
Now what is the value of the other rods?

(In this case, every rod will be equal to a multiple of 8.)

Once students can convincingly make or follow these kinds of arguments, pose the additional problems and let them work independently, in pairs or trios.

Launch Key Points

- Go back and forth between proving a rod value (i.e., red equals 20) and letting students work for a minute or two to try their own.
- Do as many examples as needed!
- Release students only after most of them can make or follow an argument that proves a rod value.
- There is no worksheet for students, so be ready to write or project Problems 3 – 6 as students are ready for them.

Work

Students work their way through as many of the problems below as they can. If more problems are needed, let students make up their own, and challenge each other to solve them.

Problem 3. If pink (6 cm) equals 48, what are the other rods?
Problem 4. If yellow (8 cm) equals 72, what are the other rods?
Problem 5. If purple (7 cm) equals 56, what are the other rods?
Problem 6. If gray (10 cm) plus blue (5 cm) equals 60, what are the other rods?

Tips for the Classroom

1. You can pose problems with almost no words by placing/drawing/projecting the Number Rods on a white board and writing the numbers underneath or beside them.

2. Make sure students can build their own version of the problem and solve physically.

3. Adjust the difficulty of the problems as necessary.

4. Students can guess and check as a first strategy.

Prompts and Questions

- You know the yellow (8cm) rod equals 72. What if the turquoise (1cm) rod equaled 6? Would that work, or is that too big or too small?
- How do you know that the blue (5cm) rod has that value?
- Is there any rod that feels easy to do next?
- What makes that one easy?
- What makes that one hard?
- You're done with all the problems? Then you can come up with a new problem and challenge another group to solve it!

Closer

Take Problem 3 (or the last problem all students have attempted) and spend a few minutes letting students share their answers with each other. Have students share their methods with a partner, and then take one or two volunteers to share their method with everyone.

Suppose you discussed Problem 3. The main tactic to underline is that going from the known value of the pink (6 cm) rod (48) to the unknown value of the purple (7 cm) is hard. But it's much more straightforward to figure out the turquoise (1 cm) rod first. It takes 6 of the turquoise to make the pink, so divide 48 by 6 to get that value as 8. Then you can multiply 7 × 8 to get the purple value as 56.

(Note that this ends up being the same situation as Problem 5)

Prompts and Questions

- For the problem we're working on, pick a solution you were able to get, and explain to your partner how you got it.
- What was a harder value to get? Does your partner have any ideas on how to get it?
- What strategy helped make these problems easier?
- Could you always figure out the value of the turquoise rod? Why did doing that help you figure out other values?

Choice Time

- Challenge Problems
- Prime Climb
- Don't Break the Bank