

The Tax Collector and the Tarot Card Duel

A 3-part lesson plan from mathforlove.com

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curriculum.

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DAY 63**Opener****Main Activity****Closer****Choice Time**

The Tax Collector (Part 1)

Materials and Prep

Index cards or sticky notes labeled \$1 - \$12, scratch paper, pencil

How to Play

Start with a collection of paychecks (the index cards) labeled \$1 to \$12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over, and the tax collector gets all the remaining paychecks. The goal of the game is to beat the tax collector and get as much money as you can.

Launch

This is the first time that students will play Tax Collector. They will have additional opportunities to play in later lessons.

Display the 12 numbered cards and demonstrate how to play by inviting your students to play, and taking the role of the tax collector yourself. Here's how it could play out:

Teacher: Here are 12 paychecks, each with their value written on them. This is your money and you should get as much of it as you can. Which paycheck would you like first?

Students: \$12!

Teacher: OK. If you get \$12, the tax collector gets \$1, \$2, \$3, \$4, and \$6. Why did I take those numbers?

Students: Those numbers all go into 12. (Or, those numbers are the factors of 12)

Teacher: Oops. I forgot to tell you this important rule. When you take a paycheck the tax collector takes any remaining factors. So if you take \$12, the tax collector takes \$1, \$2, \$3, \$4, and \$6. Why did I take those numbers? Because they're the factors. What would you like to take next?

Students: 11.

Teacher: Here's another rule. If you to take a paycheck, the tax collector must get some money too. Taking \$11 isn't a legal move anymore because the tax collector can't take any money - the \$1 paycheck is already gone. Is there another paycheck you could take instead?

Launch Key Points

- Explain the rules of the game by playing it with your students.
- The dialog in the launch gives possible language for one demonstration game. You can play a second with students if it seems like it would help clarify the rules for them.
- While the goal of the game is to get as much money as possible, you can make the task more accessible by encouraging students to simply get more money than \$22 (or whatever the result from the demonstration game was).

DAY 63**Opener****Main Activity****Closer****Choice Time****Launch (continued)**

Students: We can take \$10.

Teacher: If you take \$10, what does the tax collector get?

Students: \$5 (because the \$1 and \$2 were already taken).

Teacher: Right. Now if you look at what's left, you have no more legal moves, so the game is over, and the Tax Collector gets the remaining paychecks (\$7, \$8, \$9, and \$11). Let's find our totals. You got \$22. And the tax collector got \$56. That's pretty rough, but this was just the game where we learned the rules.

If you played this game again, do you think you could do better than \$22?

Work

Students work in pairs or trios playing the game, alternating who is the taxpayer. When groups feel like they are satisfied with their approach, they can discuss their strategies and solutions with another group. Do they agree? What is the same/different about their strategies and/or solutions?

If the class (or individual groups) are satisfied that their solution is indeed the best possible, they can try to extend their reasoning by playing the game with more paychecks such as \$1-14.

Prompts and Questions

- Were you able to get more than \$22?
- The tax collector got a lot of money on that turn. I wonder if there's a better move.
- Is that the most amount of money you can get? How do you know?
- Does the order you choose the paychecks matter? Sometimes? Always?
- What's the best first move? Why?
- Which number did you save until the end? Why?

Tips for the Classroom

1. Playing the part of the tax collector is a fun way to launch and engage students in this activity.
2. Avoid the temptation to explain an optimal strategy to students. Instead, invite them to discuss with each other how they tried to get the best scores in their games.
3. When appropriate, use vocabulary such as factors, prime numbers, and composite numbers.

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Closer

To close this lesson, facilitate a discussion about the strategies students have discovered while playing this game.

Begin by asking: does it matter what order you take the paychecks?

Ask students to share their conjectures, but do not be the answer key as a teacher. Allow the uncertainty to exist.

Then write the following solutions on the board:

- 11, 9, 8, 10, 12
- 11, 10, 8, 9, 12
- 11, 8, 10, 9, 12

“Here are three different solutions I saw in the classroom. What is the same about these solutions? Why do they all start with 11? Why do they all end with 12? When does the order you take the paychecks matter?”

Choice Time

- Challenge Problems
- Prime Climb
- Big Blockout
- Tax Collector Extensions

Prompts and Questions

- Why do we want to start with the largest prime number?
- How many prime numbers were you able to get? Why can't it be more?
- Why do you want to save 12 until the very end?
- What's special about 9?

DAY 64**Opener****Main Activity****Closer****Choice Time**

The Tax Collector (Part Two)

Materials and Prep

Index cards or sticky notes numbered \$1 to \$20, scratch paper, pencil

How to Play

The rules are the same as Tax Collector Part 1, but with paychecks from \$1 to \$20.

Launch

Ask students what rules they remember from playing the Tax Collector in the previous lesson. Clarify as needed. Then ask them what strategies they recall that were successful. Tell them that today they'll be tackling the same problem, but with paychecks that go from \$1 to \$20.

Work

Distribute cards/sticky notes. Students work in pairs or trios playing the game and trying to get as much money as they can. When groups feel like they are satisfied with their approach, they can discuss their strategies and solutions with another group. Do they agree? What is the same/different about their solutions? How are they building on the strategies from Tax Collector Part 1?

If the class (or individual groups) are satisfied that their solution is indeed the best possible, they can extend their reasoning by playing the game with paychecks \$1 to \$21 or \$1 to \$36.

Tips for the Classroom

1. Focus on the strategies students use (i.e., take the greatest prime number first or take numbers in an order so that they only give one factor to the Tax Collector.)
2. There may be controversy over what the highest amount of money it's possible to take in the game. Lean in to that controversy, and make students defend their claims.
3. When appropriate, use vocabulary such as factors, prime numbers, and composite numbers.

Launch Key Points

- If students recall the rules, jump right into the new game. Play a round with them only if needed.

Prompts and Questions

- What number do you want to take first?
- How many prime numbers can you get?
- Which numbers are not great first moves?
- Which number did you save until the end?
- Is that the most amount of money you can get? How do you know?
- Does the order you choose the paychecks matter? Sometimes? Always?

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Closer

Facilitate a discussion around the student strategies and how they have evolved after today's games.

Invite 2-3 students to write their solutions on the board. Then ask students to compare and contrast these solutions.

What is the same/different about these solutions?
What strategies did we need to revise or change?

Choice Time

- Challenge Problems
- Odd Pig Out
- Prime Climb
- Tax Collector Extensions

Prompts and Questions

- Why do we want to start with the largest prime number?
- How many prime numbers were you able to get? Why can't it be more?
- On Tax Collector Part 1, you took 9 early because it was a perfect square and only had a factor of 3. 16 is also a perfect square. Why didn't you take 16 early?
- When does the order matter? When does the order not matter?

DAY 65

Opener

Main Activity

Closer

Choice Time

Fate Duel (Tax Collector Part 3)

Materials and Prep

Index cards or sticky notes numbered 1 - 23, scratch paper, pencil

How to Play

The rules for Fate Duel are the same as for Tax Collector, but with some added twists:

- If you win and take the 10, you will have great fortune.
- If you win and take the 6, you will find true love.
- If you win and take the 2, you will break the family curse forever.

Launch

To explain the new rules, show the following video:

mathforlove.com/puzzle/fate-and-the-tarot-duel-riddle/

Make sure to pause the video at 2:22 (or you will reveal the solution to students).

Alternatively, you can tell the following story:

“Many, many years ago, your ancestors stole a deck of fortune-telling tarot cards from Fate herself. This deck of cards has brought your family great fortune over the centuries, but it comes at a terrible cost. Every 23 years, Fate pays your family a visit and someone in your family must play Fate in a duel to the death. No one in the family has ever beaten Fate. They have all died and no one knows the rules.

“And this year, Fate says that it’s your turn to play. As you sit down to play, Fate lays out 23 fortune-telling cards from the deck and explains the rules:

- When you take a card, Fate takes all the remaining factors of that card.
- In order to win, the total of your cards must be more than the total of Fate’s cards.
- And in order to keep playing, it must be possible for Fate to take a card when you take a card. So if you run out of moves, Fate will get all the remaining cards.

“You’ve been playing the Tax Collector game in math class, so you’re feeling confident that you can beat Fate at this game.

Launch Key Points

- Explain the rules of the game with the added twists.
- While the goal of the game is to beat Fate, encourage students to see if they can beat Fate and get all (or some) of the special cards (2, 6, and 10).

DAY 65

Opener

Main Activity

Closer

Choice Time

Launch (continued)

“But then Fate adds a twist that makes you pause. She tells you that if you win and take the 10, you would have great fortune. If you win and take the 6, you would find true love. And if you win and take the 2, you would break the curse on your family forever.

“**Can you beat Fate *and* take any of these special cards?**”

Work

Students work in pairs or trios trying to beat Fate and take any (or all) of the special cards.

Optional extension: “When you sit down, Fate suggests you take the 16 first. Is it possible to beat Fate and take the 16 first?”

Tips for the Classroom

1. Try to pair groups together that have arrived at different, contrary conclusions. Encourage them to explain and defend their approaches.
2. When appropriate, use vocabulary such as factors, prime numbers, and composite numbers.

Closer

To close this lesson, ask: **can you beat Fate and grab any of the special cards (the 2, 6, or 10)?**

Invite students to discuss their strategies and write their solutions on the board. Discuss similarities and differences among the strategies and solutions.

Note: it turns out to be possible to take at most one of those three cards. If you try to take more than one, Fate wins the duel.

Choice Time

- Challenge Problems
- Odd Pig Out
- Prime Climb
- Big Blockout

Prompts and Questions

- What strategies can you use from the Tax Collector to beat Fate?
- Can you beat Fate and take more than one of these special cards? All of them?

Prompts and Questions

- Is it possible to win and take the 10?
- Is it possible to win and take the 6?
- Is it possible to win and take the 2?
- Is it possible to win and take more than one of these cards? How do we know?
- How did your strategy change from playing the Tax Collector to playing Fate Duel?