The Tax Collector (Part 1)

Materials and Prep

Index cards (or sticky notes) labeled $1 - $12, scratch paper, pencil.

How to Play

Start with a collection of paychecks (cards), from $1 to $12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over, and the tax collector gets all the remaining paychecks. The goal of the game is to beat the tax collector and get as much money as you can.

Launch

This is the first time that students will play Tax Collector. They will have additional opportunities to play in later lessons.

Display the 12 numbered cards and demonstrate how to play by involving your students and playing the role of the tax collector yourself. Here's how it could play out...

Teacher: Here are 12 paychecks, each with their value written on them. This is your money and I you want to get as much as you can. Which would you like first?

Students: $12!

Teacher: OK. If you get $12, the tax collector gets $1, $2, $3, $4, and $6. Why did I take those numbers?

Students: Those are the numbers that are factors of ("go into") 12.

Teacher: Oops. I forgot to tell you this important rule. When you take an paycheck the tax collector takes any remaining factors. So if you get $12, the tax collector gets $1, $2, $3, $4, and $6. Why did I take those numbers?

Students: 11.

Teacher: Here's another rule. If you to take a paycheck, the tax collector must get some money too. Taking $11 isn't a legal move because the tax collector can't take any money (because the $1 paycheck is already gone). Is there another paycheck you could take instead?

Launch Key Points

- Explain the rules of the game by playing it with your students.
- While the goal of the game is to get as much money as you can, lower the floor of the task by inviting students to simply get more money than $22 (or whatever the result from playing with your students).

Launch (continued)

Students: We can take $10.

Teacher: If you take $10, what does the tax collector get?

Students: $5 (because the $1 and $2 were already taken).

Teacher: Right. Now if you look at what's left, you have no more legal moves, so the game is over, and the Tax Collector gets the remaining paychecks ($7, $8, $9, and $11). Let's find our totals. You got $22. And the tax collector got $56.

If you played this game again, do you think you could do better than $22?

Work

Students work in pairs or trios playing the game, alternating who is the taxpayer. When groups feel like they are satisfied with their approach, they can discuss their strategies and solutions with another group. Do they agree? What is the same/different about their strategies and/or solutions?

If the class (or individual groups) are satisfied that their solution is indeed the best possible, they can try to extend their reasoning by playing the game with more paychecks such as $1-14.

Tips for the Classroom

1. Playing the part of the tax collector is a fun way to launch and engage students in this activity.

2. Avoid the temptation to be the answer key for your students. Instead, invite them to discuss with each other that they've found the best score possible.

3. When appropriate, focus on using vocabulary such as factors, prime numbers, and composite numbers.

Prompts and Questions

- Were you able to get more than $22?
- The tax collector got a lot of money on that turn. I wonder if there's a better move.
- Is that the most amount of money you can get? How do you know?
- Does the order you choose the paychecks matter? Sometimes? Always?
- Which number did you save until the end? Why?

Closer

To close this lesson, facilitate a discussion about the strategies students have discovered while playing this game.

Begin by asking: does it matter what order you take the envelopes?

Ask students to share their conjectures, but do not be the answer key as a teacher. Allow the uncertainty to exist.

Then write the following solutions on the board:

- 11, 9, 8, 10, 12
- 11, 10, 8, 9, 12
- 11, 8, 10, 9, 12

"Here are three different solutions I saw in the classroom. What is the same about these solutions? Why do they all start with 11? Why do they all end with 12? When does the order you take the envelopes matter?"

Prompts and Questions

- Why do we want to start with the largest prime number?
- How many prime numbers were you able to get? Why can't it be more?
- Why do you want to save 12 until the very end?
- What's special about 9?

Choice Time

- Challenge Problems
- Prime Climb
- Big Blockout
- Tax Collector Extensions

The Tax Collector (Part Two)

Materials and Prep
Index cards (or sticky notes) numbered $1 to $20, scratch paper, pencil.

How to Play
The rules are the same as Tax Collector Part 1, but with paychecks from $1 to $20.

Launch
Ask students what rules they remember from playing the Tax Collector in the previous lesson. Clarify as needed. Then ask them what strategies they recall that were successful.

Work
Distribute cards/sticky notes. Students work in pairs or trios playing the game and trying to get as much money as they can. When groups feel like they are satisfied with their approach, they can discuss their strategies and solution s with another group. Do they agree? What is the same/different about their solutions? How are they building on the strategies they with Tax Collector Part 1?

If the class (or individual groups) are satisfied that their solution is indeed the best possible, they can extend their reasoning by playing the game with paychecks $1 to $21 or $1 to $36.

Tips for the Classroom
1. Focus on the strategies students use (i.e., take the greatest prime number first or take numbers in an order so that they only give one factor to the Tax Collector.)

2. Avoid the temptation to be the answer key for your students. Instead, invite them to discuss with each other that they've found the best score possible.

3. When appropriate, focus on using vocabulary such as factors, prime numbers, and composite numbers.

Launch Key Points

- If students recall the rules, jump right into the new game. Play a round with them only if needed.

Prompts and Questions

- What number do you want to take first?
- How many prime numbers can you get?
- Which numbers are not great first moves?
- Which number did you save until the end?
- Is that the most amount of money you can get? How do you know?
- Does the order you choose the envelopes matter? Sometimes? Always?

Closer

Facilitate a discussion around the student strategies and how they have evolved after today's games.

Invite 2-3 students to write their solutions on the board. Then ask students to compare and contrast these solutions.

What is the same/different about these solutions?
What strategies did we need to revise or change?

Prompts and Questions

- Why do we want to start with the largest prime number?
- How many prime numbers were you able to get? Why can't it be more?
- On Tax Collector Part 1, you took 9 early because it was a perfect square and only had a factor of 3. 16 is also a perfect square. Why didn't you take 16 early?
- When does the order matter? When does the order not matter?

Choice Time

- Challenge Problems
- Odd Pig Out
- Prime Climb
- Tax Collector Extensions

Fate Duel (Tax Collector Part 3)

Materials and Prep
Index cards (or sticky notes) numbered 1 - 23, scratch paper, and pencil.

How to Play
The rules for Fate Duel are the same as for Tax Collector, but with some added twists:

- If you win and take the 10, you will have great fortune.
- If you win and take the 6, you will find true love.
- If you win and take the 2, you will break the family curse forever.

Launch
To explain the new rules, show the following video:
mathforlove.com/puzzle/fate-and-the-tarot-duel-riddle/

Make sure to pause the video at 2:22 (or you will reveal the solution to students).

Alternatively, you can tell the following story:

"Many, many years ago, your ancestors stole a deck of fortune-telling tarot cards from Fate herself. This deck of cards has brought your family great fortune over the centuries, but it comes at a terrible cost. Every 23 years, Fate pays your family a visit and someone in your family must play Fate in a duel to the death. No one in the family has ever beaten Fate. They have all died and no one knows the rules.

"And this year, Fate says that it's your turn to play. As you sit down to play, Fate lays out 23 fortune-telling cards from the deck and explains the rules:

- When you take a card, Fate takes all the remaining factors of that card.
- In order to win, the total of your cards must be more than the total of Fate's cards.
- And in order to keep playing, it must be possible for Fate to take a card when you take a card. So if you run out of moves, Fate will get all the remaining cards.

"You've been playing the Tax Collector game in math class, so you're feeling confident that you can beat Fate at this game.

Launch Key Points

- Explain the rules of the game with the added twists.
- While the goal of the game is to beat Fate, encourage students to see if they can beat Fate and get all (or some) of the special cards (2, 6, and 10).

Launch (continued)

"But then Fate adds a twist that makes you pause. She tells you that if you win and take the 10, you would have great fortune. If you win and take the 6, you would find true love. And if you win and take the 2, you would break the curse on your family forever.

"Can you beat Fate and grab any of these special cards?"

Work

Students work in pairs or trios trying to beat Fate and take any (or all) of the special cards.

Optional extension: "When you sit down, Fate suggests you take the 16 first. Is it possible to beat Fate and take the 16 first?"

Prompts and Questions

- What strategies can you use from the Tax Collector to beat Fate?
- Can you beat Fate and take more than one of these special cards? All of them?

Tips for the Classroom

1. Avoid the temptation to be the answer key for your students. Instead, invite them to discuss with each other that they've found the best score possible.

2. When appropriate, focus on using vocabulary such as factors, prime numbers, and composite numbers.

Closer

To close this lesson, ask: **can you beat Fate and grab any of the special cards (the 2, 6, or 10)?**

Invite students to discuss their strategies and write their solutions on the board. Discuss similarities and differences among the strategies and solutions.

Prompts and Questions

- Is it possible to win and take the 10?
- Is it possible to win and take the 6?
- Is it possible to win and take the 2?
- Is it possible to win and take more than one of these cards? How do we know?
- How did your strategy change from playing the Tax Collector to playing Fate Duel?

Choice Time

- Challenge Problems
- Odd Pig Out
- Prime Climb
- Big Blockout